Application of Derivatives

Question 1

$f(x) = x(x-1)^2 = x(x^2+1-2x)$

$= x^3 - 2x^2 + x$

$f'(x) = 3x^2 - 4x + 1$

$3x^2 - 4x + 1 = 0$

$x = \frac{4 \pm \sqrt{16 - 4 \times 3 \times 1}}{6}$; $x = 1, x = 1/3$

$f(1/3) = 4/27$, $f(1) = 0$

$= (1, 0)$

Quartion 2

$f(x) = x^2(x-1) = x^3 - x^2$

$f'(x) = 3x^2 - 2x$

$3x^2 - 2x = 0$

$x = \frac{2 \pm \sqrt{4}}{6}$; $x = 0, x = 2/3$

$f(0) = 0$, $f(2/3) = -4/27$

local maximum: $(0, 0)$

Question 3

$$f(x) = \frac{1}{x+1}$$

$$f'(x) = \frac{-1}{(x+1)^2} = 0$$

None

Question 4

Vertical asymptotes $f(x) = \frac{x}{x^2-4}$

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

$$\lim_{x \to -2^+} \frac{x}{x^2-4} = \infty$$

$$\lim_{x \to 2^+} \frac{x}{x^2-4} = \infty.$$

Vertical asymptotes: $x = -2$

$x = 2$

Question 5

Vertical asymptotes $\frac{1}{x^3 - x^2}$

$x^3 - x^2 = 0$

$x^3 = x^2$

$x = 1$ or 0

Vertical asymptotes: $x = 1$

$x = 0$

Question 7

$f(x) = x^3 - 3x^2 + x + 2$

$f'(x) = 3x^2 - 6x + 1$

$f''(x) = 6x - 6$

$3x^2 - 6x + 1 = 0$; $x =$

$6x - 6 = 0$

$x = 1$

$(-\infty, 1)$ $(1, \infty)$

$-$ $+$

Interval of concave up : $(1, \infty)$ or $x > 1$

Question 8

$$\lim_{x \to 1^+} \frac{x+1}{x^2-5x+4}$$

$$\frac{1+1}{1-5+4} = \frac{1}{0}$$

$= -\infty$ ie $\lim_{x \to 1^+} (x^2-5x+4)^{-1}$

Question 9

$$f(x) = x^2 + \frac{2}{\sqrt{x}} = x^2 + 2x^{-1/2}$$

$f'(x) = 2x + (2 \times -1/2)\, x^{-1/2}$ $\qquad f(x) = x^2 + 2x^{-1/2}$

$f''(x) = 2 + -1/2\, x^{-3/2}$ $\qquad f'(x) = 2x - x^{-3/2}$

$f''(x) = 2 + 3/2\, x^{-5/2}$

$= 2 + \frac{-1}{2(\sqrt{x})^3}$

$= 2 + \frac{3}{2x^{5/2}}$

$= 2 + \frac{3}{2x^2\sqrt{x}}$

Question 10

$$f(x) = a(x-h)^2 + k$$

$f'(x) = 2a(x-h)$

$f''(x) = 2a(x-h)^0$

$= 2a$

Question 11

$f(x) = x^3 + 3x^2 + x$

$f'(x) = 3x^2 + 6x + 1$

$f''(x) = 6x + 6$

$6x + 6 = 0$

$x = -1$

inflection point: $x = -1$

$m = 3(-1)^2 + 6(-1) + 1$

$= -2$

$y - 1 = -2(x + 1)$

$y = -2x - 1$

$(-\infty, -1)$ $(-1, \infty)$

$+$ $-$

Question 12

$f(x) = x(x^2 - 4) = x^3 - 4x$

$f'(x) = 3x^2 - 4$

$f''(x) = 6x$

$6x = 0$

$x = 0$

$(-\infty, 0)$ $(0, \infty)$

$-$ $+$

$m = 3(0) - 4$

$= -4$

Question 13

$f(x) = x(1-x^2) = x - x^3$

$f'(x) = 1 - 3x^2$

$m = 1 - 3(0)$

$= 1$

$f''(x) = -6x$

$-6x = 0$

$x = 0$

Question 20

$h(t) = 0.2t - 0.1t^2$

Max height is when $v = 0$

$v = \frac{d}{dt} 0.2t - 0.1t^2 = 0.2 - 0.2t$

$-0.2t + 0.2 = 0$

$t = 1$

Max height $= 0.2 - 0.1$

$= 0.1m$