Question 1

$f(x) = \sin(4x + \pi/2)$, $f'(\pi/4)$

$f'(x) = 4\cos(4x + \pi/2)$

$f'(\pi/4) = 4\cos(\pi + \pi/2)$

$= 4\cos(3/2\,\pi) = 0$

Question 2

$f(x) = 3\cos(4x - \pi/2)$ $f'(\pi/4)$

Chain rule:

$f'(x) = -12\sin(4x - \pi/2)$

$f'(\pi/4) = -12 \cdot \sin(\pi/2) = -12$

Question 3

$f(x) = \cos^2(x)$, $f'(\pi/4)$

$f(x) = [\cos(x)]^2$ $f'(x) = 2(-\sin(x))\cos(x)$

$= -\sin(2x)$

$= -\sin(\pi/2) = -\frac{\sqrt{2}}{2} \cdot 1$

Question 4

$f(x) = \sin(x^2 + 1)$

$f'(x) = \cos(x^2 + 1) \times 2x = 2x\cos(x^2 + 1)$

$f'(1) = 2\cos(2)$

$= -0.832$

Question 5

$f(x) = \sin x + 2\cos x - x$

$f'(x) = \cos(x) - 2\sin x - 1$

$f'(\pi/2) = \cos(\pi/2) - 2\sin(\pi/2) - 1$

$0 - 2 - 1$

$= -3$

Question 6

$f(x) = 2\sin(x)\cos(x)$

$f'(x) = 2\sin(x)(-\sin x) + 2\cos(x)(\cos x)$

$= -2(\sin^2(x) - \cos^2(x)$

$f'(\pi/2) = -2(\sin^2(\pi/2) - \cos^2(\pi/2))$

$= -2(1 - 0) = -2$

Question 7

$f(x) = 3\tan(2x)$

$f'(x) = 3\sec^2(2x) \cdot 2$

$= 6\sec^2(2x)$

$f'(\pi/2) = 6\sec^2(\pi)$

$= 6 \cdot 1$

$= 6$

Question 8

$$f(x) = \frac{\sin x}{\cos(x)}$$

$$\tan(x) = \frac{\sin x}{\cos x} \; ; \; \frac{d}{dx} \tan(x) = \sec^2 x$$

$$f'(\pi/4) = \sec^2(\pi/4) = 2$$

Question 9

$$y = \sin 2x - \cos 2x \quad @ \; x = \pi/2$$

$$\frac{d}{dx}(\sin 2x - \cos 2x) = (2\cos(2x) + 2\sin 2x)$$

$$= 2(\cos(2x) + \sin(2x))$$

$$f'(\pi/2) = 2(-1 + 0)$$

$$= -2$$

Question 10

$$y = \frac{1}{9} \sin 3x$$

$$f'(x) = \frac{3}{9} \cos 3x$$

$$f''(x) = -\frac{9}{9} \sin(3x)$$

$$= -\sin(3x)$$

$$y''(\pi/6) = -\sin(3 \times \pi/6) = -\sin(\pi/2)$$

$$= -1$$

Application

Question 11

$y = \frac{1}{4} \cos 2x$

$y' = -\frac{2}{4} \sin 2x$

$y'' = -\frac{4}{4} \cos(2x) = -\cos(2x)$

$-\cos(\pi) = 1$

Question 12

$y = \sin(3x^2 + 5)$

$y' = \cos(3x^2+5) \cdot 6x$

$= 6x \cos(3x^2+5)$

$y'(1) = 6 \cos(8)$

$= -6.873$

Question 13

$y = \sin^2 x + \cos^2 x \quad ; \; y'(\pi)$

$y' = 2\cos(x)\sin(x) + -2\cos(x)\sin(x)$

$= 0$

Question 14

$f(x) = (2 \sin x \cos x)^2 + 3x$

$f'(x) \Rightarrow \quad 2u \cdot u' + 3$

$= 2u \cdot (2\cos(x)\sin(x)) + 3 \quad$ where $u = 2 \sin x \cos x$

$= -\cancel{8 \sin^2 x \cos^2 x + 3} - 8(\cos(x) + \sin^3(x)) + 8\cos^3(x)\sin x$

$f''(x) = 8(\sin^4(x) - 6\cos^2(x)\sin^2(x) + \cos^4(x))$

$f''(\pi/2) = 8(\sin^4(\pi/2) - 6\cos^2(\pi/2)\sin(\pi/2) + \cos^4(\pi/2)$

$= 8(1 - 0 + 0)$

$= 8$

Question 15

$f(x) = \tan(x) + 7x$

$f'(x) = \sec^2(x) + 7$

$f''(x) = 2\sec^2 x \tan x \quad ; \quad d/dx \sec^2(x) = \sec^2 x \tan x$

$f''(\pi/3) = 2\sec^2(\pi/3)\tan(\pi/3)$

$= 8\sqrt{3}$

Question 16

$f(x) = 2\cos^2(x) + x^3 + x^2 - 1$

$f'(x) = -2\sin(2x) + 3x^2 + 2x$

$f''(x) = -4\cos(2x) + 6x + 2$

$f'''(x) = +8\sin(2x) + 6 \Rightarrow 8\sin(2x) + 6$

$f'''(3\pi/4) = 8(-1) + 6$

$= -2$

Question 17

$f(x) = \cos nx$

$f'(x) = -n\sin(nx)$

$f''(x) = -(n^2)\cos(nx)$

$f'''(x) = n^3\sin(nx)$

$f''''(x) = n^4\cos(nx)$

$f^{(4)}(2\pi/n) = n^4\cos\left(n \cdot \frac{2\pi}{n}\right) = n^4\cos(2\pi)$

$= n^4$

Question 18

$f(x) = \sin(nx)$

$f'(x) = n\cos(nx)$

$f''(x) = -n^2\sin(nx)$

$f'''(x) = -n^3\cos(nx)$

$f''''(x) = n^4\sin(nx)$

$f^{(5)}(x) = n^5\cos(nx)$

$f^{(5)}(2\pi/n) = n^5\cos(n \cdot 2\pi/n) = n^5$

Question 19

$f(x) = +\sin(nx) + \cos(nx)$; $f^{(3)}(\pi/n)$

$f'(x) + n\cos(nx) - n\sin(nx)$

$f''(x) = -n^2 \sin(nx) - n^2 \cos(nx)$

$f'''(x) = n^3 \cos(nx) + n^3 \sin(nx)$

$f'''(\pi/n) = -n^3 \cos(\pi) + n^3 \sin(\pi)$

$-1(-n^3) + 0$

$= n^3$

Question 20

$f(x) = \cos^2(nx) - \sin^2(nx) \Rightarrow [\cos(nx)]^2 - [\sin(nx)]^2$

$f'(x) = -2\cos(nx)\sin(nx)n - 2\sin(nx)\cos nx \cdot n$

$f''(x) = -2[\cos^2 nx + (-\sin^2 nx)]n^2 - 2[-\sin^2 nx + \cos^2 nx]n^2$

$f'''(x) = 4n^2 \sin^2(nx) - 4n^2 \cos^2(nx)$

~~$f'''(x) = -4 \frac{d}{dx} \sin nx \cdot \sin nx$~~

$f'''(x) = 4n^2 (2n\sin(nx)\cos nx \cdot \frac{d}{dx}(x) - 2\cos nx - \sin nx \cdot \frac{d}{dx} nx)$

$= 4n^2 (2\sin(nx)\, n\cos n(x) + 2n\cos nx \sin nx)$

$f'''(\pi/4n) = 4n^2 (2\sin(\pi/4)\, n\cos(\pi/4) + 2n\cos \pi/4 \sin \pi/4)$

$= 16n^3 (1/2)$

$= 8n^3$