

1.

- c - scalar
- $|a|$ - scalar
- $\hat{a}$ - vector
- $a \cdot b$ - scalar
- $a \times b$ - vector
- $c(a \times b)$ - vector
- $(a \cdot b)a$ - vector
- $(a \times b) \cdot \hat{a}$ - scalar

2 $a = 3i + 4j + 12k$, $b = 2i - j + 2k$, $c = -2i + j - 2k$

- $a - b = (3 - 2)i + (4 + 1)j + (12 - 2)k$
 $= i + 5j + 10k$

- $b + c = (2 - 2)i + (-1 + 1)j + (2 - 2)k$
 $= 0$

- $|a| = \sqrt{3^2 + 4^2 + 12^2} = \sqrt{9 + 16 + 144}$
 $= 13$

- $\hat{a} = \frac{1}{|a|} \cdot a = \frac{1}{13} (3i + 4j + 12k) = \frac{3}{13}i + \frac{4}{13}j + \frac{12}{13}k$

- $\hat{b} = \frac{1}{|b|} b$

$$|b| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4 + 1 + 4} = 3$$

$$= \frac{1}{3} (2i - j + 2k) = \frac{2}{3}i - \frac{1}{3}j + \frac{2}{3}k$$

$$\begin{aligned} \bullet a \cdot c &= 3x-2 + 4x1 + 12x-2 \\ &= -6 + 4 - 24 \\ &= -26 \end{aligned}$$

$$\bullet a \times b = \begin{vmatrix} i & j & k \\ 3 & 4 & 12 \\ 2 & -1 & 2 \end{vmatrix}$$

$$= (8+12)i - (6-24)j + (-3-8)k$$

$$= 20i + 18j - 11k$$

$$\bullet (a \times b) \cdot \hat{c}$$

$$\hat{c} = \frac{1}{|c|} \cdot c$$

$$|c| = \sqrt{(-2)^2 + 1^2 + (-2)^2} = \sqrt{4+1+4} = 3$$

$$\hat{c} = \frac{1}{3} (-2i + j - 2k) = -\frac{2}{3}i + \frac{1}{3}j - \frac{2}{3}k$$

$$a \times b = 20i + 18j - 11k$$

$$= (20 \times -\frac{2}{3}) + (18 \times \frac{1}{3}) + (-11 \times -\frac{2}{3})$$

$$= 0$$

$$3. \bullet \vec{BD} = \vec{BO} + \vec{OD}$$

$$= -4b + 4a + 8a$$

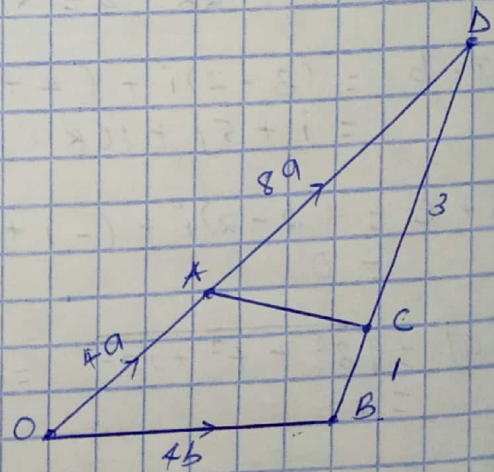
$$= -4b + 12a$$

$$\bullet \vec{AC} = \vec{AD} + \vec{DC}$$

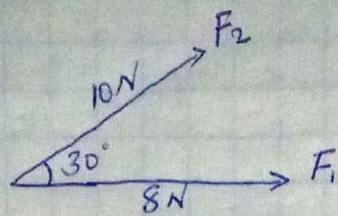
$$\vec{DC} = -\frac{3}{4}(-4b + 12a) = 3b - 9a$$

$$\vec{AC} = 8a + 3b - 9a$$

$$= -a + 3b$$



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$$F_1 = 8 \cos 0^\circ i + 8 \sin 0^\circ j \\ = 8i$$

$$F_2 = 10 \cos 30^\circ i + 10 \sin 30^\circ j \\ = 5\sqrt{3}i + 5j$$

$$F_R = 8i + 5\sqrt{3}i + 5j$$

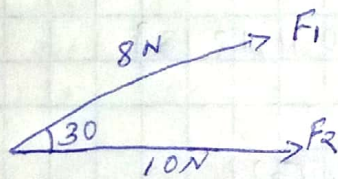
$$= (8 + 5\sqrt{3})i + 5j$$

$$\text{Magnitude} = \sqrt{(8 + 5\sqrt{3})^2 + 5^2} = 17.39$$

$$\text{Angle} \Rightarrow \tan \theta = \frac{5}{8 + 5\sqrt{3}}$$

$$\theta = \tan^{-1}(0.30012)$$

$$= 16.71^\circ$$



$$F_1 = 8 \cos 30^\circ i + 8 \sin 30^\circ j$$

$$= 4\sqrt{3}i + 4j$$

$$F_2 = 10 \cos 0^\circ i + 10 \sin 0^\circ j$$

$$= 10i$$

$$F_R = 4\sqrt{3}i + 4j + 10i$$

$$= (4\sqrt{3} + 10)i + 4j$$

$$\text{Angle} = \frac{4}{4\sqrt{3} + 10} = \tan \theta$$

$$\theta = \tan^{-1}(0.2363)$$

$$= 13.29^\circ$$

$$5. F_R = F_1 + F_2 + F_3$$

$$F_1 = 5(\cos 30^\circ i + \sin 30^\circ j) = \frac{5}{2}\sqrt{3}i + \frac{5}{2}j$$

$$F_2 = 10(\cos 60^\circ i + \sin 60^\circ j) = 5i + 5\sqrt{3}j$$

$$F_3 = 20(\cos 45^\circ i + \sin 45^\circ j) = 10\sqrt{2}i + 10\sqrt{2}j$$

$$FR = \left(\frac{5\sqrt{3}}{2} + 5 - 10\sqrt{2} \right) i + \left(\frac{5}{2} + 5\sqrt{3} - 10\sqrt{2} \right) j$$

$$\begin{aligned} \text{Magnitude} &= \sqrt{\left(\frac{5\sqrt{3}}{2} + 5 - 10\sqrt{2} \right)^2 + \left(\frac{5}{2} + 5\sqrt{3} - 10\sqrt{2} \right)^2} \\ &= 5.66 \end{aligned}$$

$$\text{Angle, } \tan \theta = 0.29303$$

$$\begin{aligned} \theta &= \tan^{-1}(0.29303) \\ &= 16.33^\circ \end{aligned}$$

$$6. \vec{AB} = \begin{pmatrix} -1 \\ 1 \\ 15 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \\ 14 \end{pmatrix} = \begin{pmatrix} -8 \\ -4 \\ 1 \end{pmatrix} = -8i - 4j + k$$

$$7. a = 6i + 2j, \quad b = 6i + 3j - 2k$$

$$\begin{aligned} a \times b &= \begin{vmatrix} i & j & k \\ 6 & 2 & 0 \\ 6 & 3 & -2 \end{vmatrix} = (-4-0)j - (-12-0)j + (18-12)k \\ &= -4j + 12j + 6k \end{aligned}$$

$$\|a \times b\| = \sqrt{(-4)^2 + (12)^2 + (6)^2}$$

$$= 14$$

$$\text{Unit vector} = \frac{1}{14} (-4i + 12j + 6k)$$

$$= -\frac{2}{7}i + \frac{6}{7}j + \frac{3}{7}k$$

$$= \frac{(-2i + 6j + 3k)}{7}$$

$$8. (r_1 \times F_1) = \begin{vmatrix} i & j & k \\ 1 & x & 1 \\ 2 & 1 & -3 \end{vmatrix} = (-3x-1)i - (-3-2)j + (1-2x)k$$

$$= (-3x-1)i + 5j + (1-2x)k$$

$$(r_2 \times F_2) = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 3 & -4 & 6 \end{vmatrix} = (-6+8)i - (6-6)j + (-4+3)k$$

$$= 2i - k$$

$$(r_3 \times F_3) = \begin{vmatrix} i & j & k \\ 1 & y & 2 \\ 1 & 10 & 7 \end{vmatrix} = (7y-20)i - (7-2)j + (10-y)k$$

$$= (7y-20)i - 5j + (10-y)k$$

$$((-3x-1) + 2 + (7y-20))i + (5-5)j + ((1-2x) - 1 + (10-y))k = 0$$

$$(-3x+7y-19)i + (-2x-y+10)k = 0$$

$$\begin{aligned} -3x+7y-19 &= 0 & -3x+7y &= 19 \\ -2x-y+10 &= 0 & -2x-y &= -10 & y &= -2x+10 \end{aligned}$$

$$-3x+7(-2x+10) = 19$$

$$-3x-14x+70 = 19$$

$$-17x+70 = 19$$

$$-17x = -51$$

$$x = 3$$

$$y = -2(3)+10 = 4$$

$$x=3, y=4$$

$$9. W = F \cdot D \quad F = (2, -7, 12)$$

$$D = (7, -11, 37) - (2, 3, 7) = (5, -14, 30)$$

$$W = (2, -7, 12) \cdot (5, -14, 30)$$

$$= 2 \times 5 + (-7) \times (-14) + 12 \times 30 = 468 \text{ J}$$

$$10. F = (5, -1, -1)$$

$$\vec{AB} = (2, -3, 7) - (3, 1, 8) = (-1, -4, -1)$$

$$M = \vec{AB} \times \vec{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -4 & -1 \\ 5 & -1 & -1 \end{vmatrix} = (4-1)\mathbf{i} - (1+5)\mathbf{j} + (1+20)\mathbf{k} \\ = 3\mathbf{i} - 6\mathbf{j} + 21\mathbf{k}$$

$$\text{Magnitude} = \sqrt{3^2 + (-6)^2 + (21)^2}$$

$$= 9\sqrt{6}$$