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American University Of The Middle East

CH. 13
Electric Field

CH. 13 – Electric Field: Study Guide

➢ **Concepts, Definitions, and Formulas:**

- Coulomb Force Law (Statement + Formula): Sec. 13.2, P.514
- Definition of Electric Field (Statement + Formula): Sec. 13.2, P.517
- Electric Field of a point a charge (Statement + Formula): Sec. 13.4, P.519
- The superposition principles (Statement + Formula): Sec. 13.5, P. 523
- Electric field at location on the dipole axis (Formula): Sec. 13.6, P. 525
- Electric field at location on the perpendicular axis (Formula): Sec. 13.6, P. 527
- Electric dipole moment (Formula): Sec. 13.6, P. 531
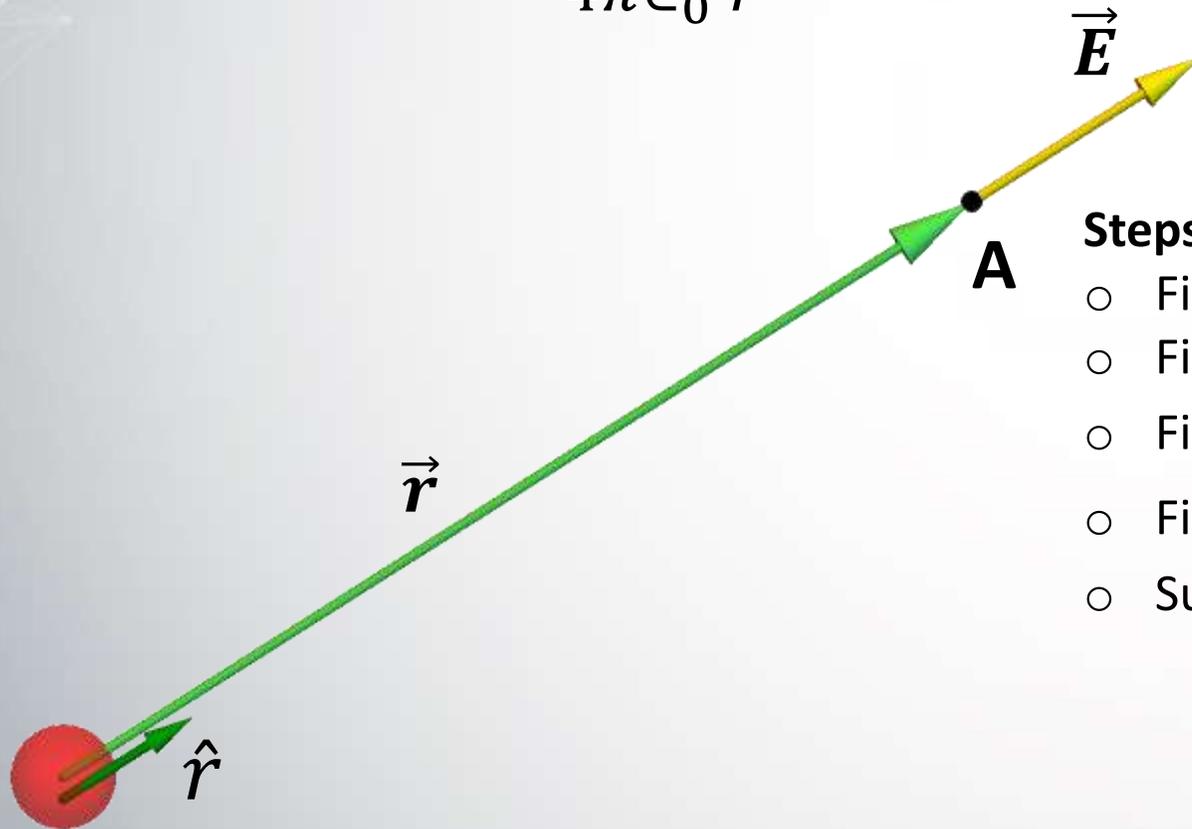
➢ **Problem Solving:**

- Examples – From the Textbook (4): Electric Force and Electric Field, Field of a particle, electric Field and Force Due to Two Charges, and Dipole and Charged Ball.
- Checkpoints – From the Textbook (4): 3, 6, 8 and 14
- Problems (6):
  - Problem 48: P. 542
  - Problem 57: P. 543
  - Problem 59: P. 543
- Extra Examples and Problems – From the Slides.

- Coulomb's law (REMINDER):

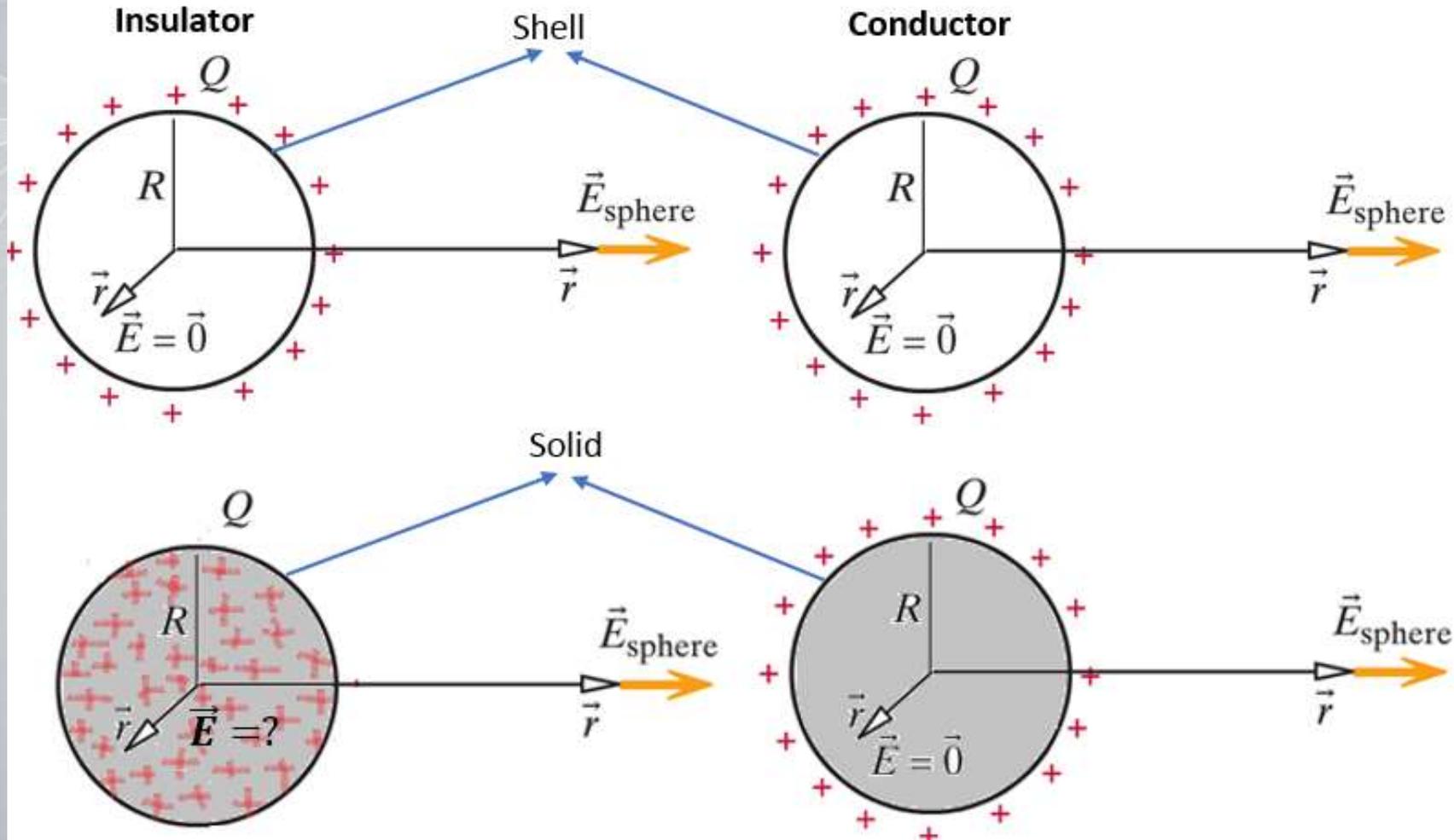$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Steps in applying Coulomb's law:
o Find $\vec{r}_s$ and $\vec{r}_O$ from the given.
o Find $\vec{r} = \vec{r}_O - \vec{r}_s$.
o Find $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.
o Find $\hat{r} = \frac{\vec{r}}{r}$.
o Substitute in Coulomb's law.

Insulator

Shell

Conductor

Q

R

$\vec{r}$

$\vec{E} = \vec{0}$

$\vec{E}_{\text{sphere}}$

$\vec{r}$

Q

R

$\vec{r}$

$\vec{E} = \vec{0}$

$\vec{E}_{\text{sphere}}$

$\vec{r}$

Solid

Q

R

$\vec{r}$

$\vec{E} = ?$

$\vec{E}_{\text{sphere}}$

$\vec{r}$

Q

R

$\vec{r}$

$\vec{E} = \vec{0}$

$\vec{E}_{\text{sphere}}$

$\vec{r}$

$$\vec{E}_{\text{sphere}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \quad \text{for } r > R \text{ (outside the sphere)}$$

$$\vec{E}_{\text{sphere}} = \vec{0} \quad \text{for } r < R \text{ (inside the sphere)}$$

THE SUPERPOSITION PRINCIPLE

The net electric field at a location in space is the vector sum of the individual electric fields contributed by all charged particles located elsewhere.
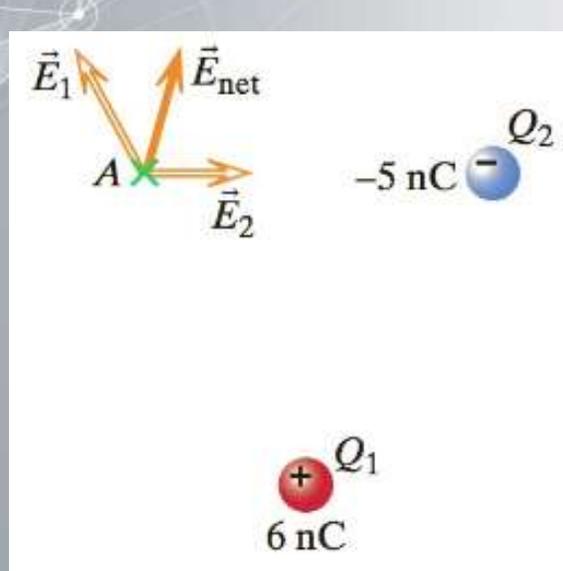
The electric field contributed by a charged particle is unaffected by the presence of other charged particles.
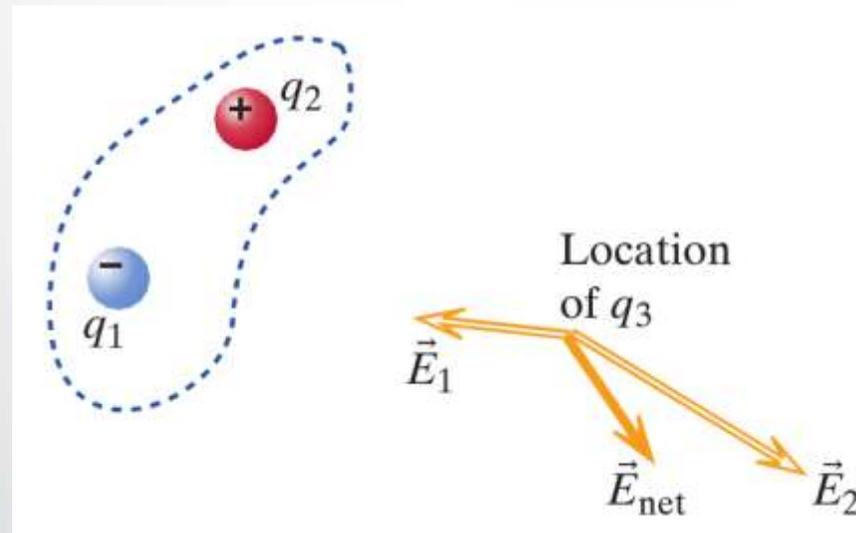
EXAMPLE

Electric Field and Force Due to Two Charges

A small object with charge $Q_1 = 6\,nC$ is located at the origin. A second small object with charge $Q_2 = -5\,nC$ is located at $\langle 0.05, 0.08, 0 \rangle$ m. What is the net electric field at location A $\langle -0.04, 0.08, 0 \rangle$ m due to Q_1 and Q_2? If a small object with a charge of $Q_3 = -3\,nC$ were placed at location A, what would be the force on this object?

Detailed solution available
in the tutoring notes

••**P47** At a particular moment, three charged particles are located as shown in Figure 13.67. $Q_1 = -4\,\mu C$, $Q_2 = +3\,\mu C$, and $Q_3 = -2\,\mu C$. Your answers to the following questions should be vectors. (Recall that $1\,\mu C = 1 \times 10^{-6}$ C.) **(a)** Find the electric field at the location of Q_3, due to Q_1. **(b)** Find the electric field at the location of Q_3, due to Q_2. **(c)** Find the net electric field at the location of Q_3. **(d)** Find the net force on Q_3. **(e)** Find the electric field at location A due to Q_1. **(f)** Find the electric field at location A due to Q_2. **(g)** Find the electric field at location A due to Q_3. **(h)** What is the net electric field at location A? **(i)** If a particle with charge -3 nC were placed at location A, what would be the force on this particle?
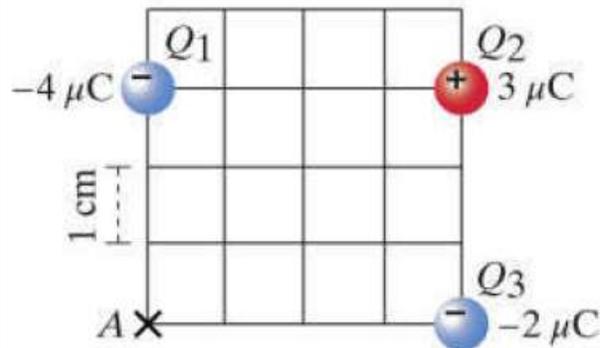

Figure 13.67

Detailed solution available in the tutoring notes

Example P. 523 (Slide 6): Electric Field and Force Due to Two Charges

A small object with charge $Q_1 = 6 \, nC$ is located at the origin. A second small object with charge $Q_2 = -5 \, nC$ is located at $\langle 0.05, 0.08, 0 \rangle \, m$. What is the net electric field at location A $\langle -0.04, 0.08, 0 \rangle \, m$ due to Q_1 and Q_2? If a small object with a charge of $Q_3 = -3 \, nC$ were placed at location A, what would be the force on this object?

Solution:

Two charges, Q_1 and Q_2 are nearby location A. Each one of these two charges will create an electric field at point A: Q_1 will create $\vec{E}_1$ and Q_2 will create $\vec{E}_2$.

The net electric field at point A will be the sum of the two electric fields $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$

Recall that $1nC = 10^{-9} \, C$

$\underline{\vec{E}_1:}$ the electric field created by Q_1

The observation point is point A, so $\vec{r}_o = \vec{r}_A = \langle -0.04, 0.08, 0 \rangle \, m$

The source is the charge Q_1, which is here located the origin, so $\vec{r}_S = \langle 0, 0, 0 \rangle \, m$

- $\vec{r} = \vec{r}_o - \vec{r}_S = \langle -0.04, 0.08, 0 \rangle - \langle 0, 0, 0 \rangle = \langle -0.04, 0.08, 0 \rangle \, m$

- $r = |\vec{r}| = \sqrt{(-0.04)^2 + 0.08^2 + 0^2} = 0.089 \, m$

- $\hat{r} = \dfrac{\vec{r}}{r} = \dfrac{\langle -0.04, 0.08, 0 \rangle}{0.089} = \langle -0.45, 0.89, 0 \rangle$

- $\vec{E}_1 = \dfrac{1}{4\pi\epsilon_0} \dfrac{Q_1}{r^2} \hat{r} = (9 * 10^9) * \dfrac{6*10^{-9}}{0.089^2} * \langle -0.45, 0.89, 0 \rangle = \langle -3.07 * 10^3, 6.07 * 10^3, 0 \rangle \, (N/C)$

The net electric field at point A will be the sum of the two electric fields $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2$

$\vec{E}_2$: the electric field created by Q_2

The observation point is also point A, so $\vec{r}_o = \vec{r}_A = \langle -0.04, 0.08, 0 \rangle\ m$

The source is the charge Q_2, which is here $\vec{r}_S = \langle 0.05, 0.08, 0 \rangle\ m$

- $\vec{r} = \vec{r}_o - \vec{r}_S = \langle -0.04, 0.08, 0 \rangle - \langle 0.05, 0.08, 0 \rangle = \langle -0.09, 0, 0 \rangle\ m$
- $r = |\vec{r}| = \sqrt{(-0.09)^2 + 0^2 + 0^2} = 0.09\ m$
- $\hat{r} = \dfrac{\vec{r}}{r} = \dfrac{\langle -0.09, 0, 0 \rangle}{0.09} = \langle -1, 0, 0 \rangle$

- $\vec{E}_2 = \dfrac{1}{4\pi\epsilon_0} \dfrac{Q_2}{r^2} \hat{r} = (9 * 10^9) * \dfrac{(-5*10^{-9})}{0.09^2} * \langle -1, 0, 0 \rangle = \langle 5.55 * 10^3, 0, 0 \rangle$ (N/C)

$\vec{E}_{net}$: the net electric field at location A

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = \langle -3.07 * 10^3, 6.07 * 10^3, 0 \rangle + \langle 5.55 * 10^3, 0, 0 \rangle$$
$$= \langle 2.48 * 10^3, 6.07 * 10^3, 0 \rangle\ \text{(N/C)}$$

Q_3 is placed at location A, where the net electric field is $\vec{E}_{net}$, therefore an electric force $\vec{F}_{net}$ will be applied on the charge Q_3.

$$\vec{F}_{net} = Q_3 * \vec{E}_{net} = (-3 * 10^{-9}) * \langle 2.48 * 10^3, 6.07 * 10^3, 0 \rangle$$
$$= \langle -7.44 * 10^{-6}, -1.82 * 10^{-5}, 0 \rangle\ \text{(N)}$$

At a particular moment, three charged particles are located as shown in Figure 13.67. $Q_1 = -4 \ \mu C$, $Q_2 = +3 \ \mu C$, and $Q_3 = -2 \ \mu C$. Your answers to each part of this problem should be vectors (Recall that $1\mu C = 1 \times 10^{-6} \ C$). (a) Find the electric field at the location of Q_3, due to Q_1. (b) Find the electric field at the location of Q_3, due to Q_2. (c) Find the net electric field at the location of Q_3. (d) Find the net force on Q_3. (e) Find the electric field at location A, due to Q_1. (f) Find the electric field at location A, due to Q_2. (g) Find the electric field at location A, due to Q_3. (h) What is the net electric field at location A? (i) If a particle of charge $Q_4 = -3 \ nC$ were placed at location A, what would be the force on this particle?

Solution:

The locations of the three particles and the point A are not given. Therefore, we will have to find these locations graphically before solving. On figure 13.67 each square represent 1 cm, recall that $1 \ cm = 10^{-2} \ m$.

For simplicity let us consider that point A as the origin: $\vec{r}_A = \langle 0, 0, 0 \rangle \ m$.

The locations of the three particles are obtained by counting the number of the horizontal and vertical squares, starting from point A (the considered origin).

Location of charge Q_1: $\vec{r}_1 = \langle 0, 0.03, 0 \rangle \ m$

Location of charge Q_2: $\vec{r}_2 = \langle 0.04, 0.03, 0 \rangle \ m$

Location of charge Q_3: $\vec{r}_3 = \langle 0.04, 0, 0 \rangle \ m$

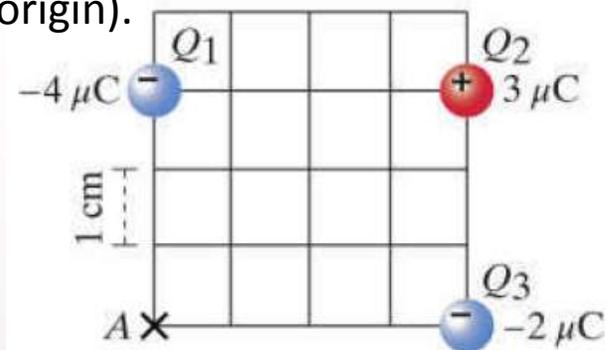*Figure 13.67*

Solution (Cont.):

a) The electric field at the location of Q_3, due to Q_1.

The observation point is the location of Q_3, so $\vec{r}_o = \vec{r}_3 = \langle 0.04, 0, 0 \rangle\ m$

The source is the charge Q_1, so $\vec{r}_S = \vec{r}_1 = \langle 0, 0.03, 0 \rangle\ m$

- $\vec{r} = \vec{r}_o - \vec{r}_S = \langle 0.04, 0, 0 \rangle - \langle 0, 0.03, 0 \rangle = \langle 0.04, -0.03, 0 \rangle\ m$
- $r = |\vec{r}| = \sqrt{0.04^2 + (-0.03)^2 + 0^2} = 0.05\ m$
- $\hat{r} = \dfrac{\vec{r}}{r} = \dfrac{\langle 0.04, -0.03, 0 \rangle}{0.05} = \langle 0.8, -0.6, 0 \rangle$
- $\vec{E}_{by\ Q1\ on\ \vec{r}_3} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_1}{r^2}\hat{r} = (9 * 10^9) * \dfrac{(-4*10^{-6})}{0.05^2} * \langle 0.8, -0.6, 0 \rangle$
  $= \langle -1.15 * 10^7, 8.64 * 10^6, 0 \rangle$ (N/C)

b) The electric field at the location of Q_3, due to Q_2.

The observation point is the location of Q_3, so $\vec{r}_o = \vec{r}_3 = \langle 0.04, 0, 0 \rangle\ m$

The source is the charge Q_2, so $\vec{r}_S = \vec{r}_2 = \langle 0.04, 0.03, 0 \rangle\ m$

- $\vec{r} = \vec{r}_o - \vec{r}_S = \langle 0.04, 0, 0 \rangle - \langle 0.04, 0.03, 0 \rangle = \langle 0, -0.03, 0 \rangle\ m$
- $r = |\vec{r}| = \sqrt{0^2 + (-0.03)^2 + 0^2} = 0.03\ m$
- $\hat{r} = \dfrac{\vec{r}}{r} = \dfrac{\langle 0, -0.03, 0 \rangle}{0.03} = \langle 0, -1, 0 \rangle$
- $\vec{E}_{by\ Q2\ on\ \vec{r}_3} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_2}{r^2}\hat{r} = (9 * 10^9) * \dfrac{(3*10^{-6})}{0.03^2} * \langle 0, -1, 0 \rangle$
  $= \langle 0, -3 * 10^7, 0 \rangle$ (N/C)

c) The net electric field at the location of Q_3.

Two charges are nearby the location of Q_3; Q_1 creating $\vec{E}_{by\ Q1\ on\ \vec{r}_3}$ and Q_2 creating $\vec{E}_{by\ Q2\ on\ \vec{r}_3}$.

Therefore, $\vec{E}_{net} = \vec{E}_{by\ Q1\ on\ \vec{r}_3} + \vec{E}_{by\ Q2\ on\ \vec{r}_3} = \langle -1.15 * 10^7, 8.64 * 10^6, 0 \rangle + \langle 0, -3 * 10^7, 0 \rangle$
$$= \langle -1.15 * 10^7, -2.14 * 10^7, 0 \rangle \ \text{(N/C)}$$

d) The net force on Q_3.

Charge Q_3 is located at $\vec{r}_3$, where the net electric field is $\vec{E}_{net}$ (found in question c). An electric force will be applied on Q_3: $\vec{F}_{net} = Q_3 * \vec{E}_{net} = (-2 * 10^{-6}) * \langle -1.15 * 10^7, -2.14 * 10^7, 0 \rangle$
$$= \langle 23, 42.8, 0 \rangle \ \text{(N)}$$

e) The electric field at location A, due to Q_1

The observation point is A, so $\vec{r}_o = \vec{r}_A = \langle 0, 0, 0 \rangle\ m$

The source is the charge Q_1, so $\vec{r}_S = \vec{r}_1 = \langle 0, 0.03, 0 \rangle\ m$

- $\vec{r} = \vec{r}_o - \vec{r}_S = \langle 0, 0, 0 \rangle - \langle 0, 0.03, 0 \rangle = \langle 0, -0.03, 0 \rangle\ m$
- $r = |\vec{r}| = \sqrt{0^2 + (-0.03)^2 + 0^2} = 0.03\ m$
- $\hat{r} = \dfrac{\vec{r}}{r} = \dfrac{\langle 0, -0.03, 0 \rangle}{0.03} = \langle 0, -1, 0 \rangle$
- $\vec{E}_{by\ Q1\ on\ A} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_1}{r^2}\hat{r} = (9 * 10^9) * \dfrac{(-4*10^{-6})}{0.03^2} * \langle 0, -1, 0 \rangle$
$$= \langle 0, 4 * 10^7, 0 \rangle \ \text{(N/C)}$$

f) The electric field at location A, due to Q_2

The observation point is A, so $\vec{r}_o = \vec{r}_A = \langle 0, 0, 0 \rangle \, m$

The source is the charge Q_1, so $\vec{r}_S = \vec{r}_2 = \langle 0.04, 0.03, 0 \rangle m$

- $\vec{r} = \vec{r}_o - \vec{r}_S = \langle 0, 0, 0 \rangle - \langle 0.04, 0.03, 0 \rangle = \langle -0.04, -0.03, 0 \rangle \, m$
- $r = |\vec{r}| = \sqrt{(-0.04)^2 + (-0.03)^2 + 0^2} = 0.05 \, m$
- $\hat{r} = \dfrac{\vec{r}}{r} = \dfrac{\langle -0.04, -0.03, 0 \rangle}{0.05} = \langle -0.8, -0.6, 0 \rangle$
- $\vec{E}_{by \, Q2 \, on \, A} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_2}{r^2}\hat{r} = (9*10^9) * \dfrac{(3*10^{-6})}{0.05^2} * \langle -0.8, -0.6, 0 \rangle$

$= \langle -8.64 * 10^6, -6.48 * 10^6, 0 \rangle \, (N/C)$

g) The electric field at location A, due to Q_3

The observation point is A, so $\vec{r}_o = \vec{r}_A = \langle 0, 0, 0 \rangle \, m$

The source is the charge Q_1, so $\vec{r}_S = \vec{r}_3 = \langle 0.04, 0, 0 \rangle m$

- $\vec{r} = \vec{r}_o - \vec{r}_S = \langle 0, 0, 0 \rangle - \langle 0.04, 0, 0 \rangle = \langle -0.04, 0, 0 \rangle \, m$
- $r = |\vec{r}| = \sqrt{(-0.04)^2 + 0^2 + 0^2} = 0.04 \, m$
- $\hat{r} = \dfrac{\vec{r}}{r} = \dfrac{\langle -0.04, 0, 0 \rangle}{0.04} = \langle -1, 0, 0 \rangle$
- $\vec{E}_{by \, Q3 \, on \, A} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_3}{r^2}\hat{r} = (9*10^9) * \dfrac{(-2*10^{-6})}{0.05^2} * \langle -1, 0, 0 \rangle$

$= \langle 7.2 * 10^6, 0, 0 \rangle \, (N/C)$

h) The net electric field at the location of Q_3.

Three charges are nearby the location A; Q_1 creating $\vec{E}_{by\ Q1\ on\ A}$, Q_2 creating $\vec{E}_{by\ Q2\ on\ A}$ and Q_3 creating $\vec{E}_{by\ Q3\ on\ A}$.

Therefore, $\vec{E}_{net} = \vec{E}_{by\ Q1\ on\ A} + \vec{E}_{by\ Q2\ on\ A} + \vec{E}_{by\ Q3\ on\ A}$
$$= \langle 0, 4 * 10^7, 0 \rangle + \langle -8.64 * 10^6, -6.48 * 10^6, 0 \rangle + \langle 7.2 * 10^6, 0, 0 \rangle$$
$$= \langle -1.44 * 10^6, 3.352 * 10^7, 0 \rangle \text{ (N/C)}$$

i) The net force on Q_4.

Charge Q_4 is located at A, where the net electric field is $\vec{E}_{net}$ (found in question h). An electric force will be applied on Q_4: $\vec{F}_{net} = Q_4 * \vec{E}_{net} = (-3 * 10^{-9}) * \langle -1.44 * 10^6, 3.352 * 10^7, 0 \rangle$
$$= \langle 4.32 * 10^{-3}, -0.1, 0 \rangle \text{ (N)}$$