

MAT1080 Worksheet #3

Name: _____

Complete this worksheet and scan it to a SINGLE PDF document before submitting it in D2L.

Name the file WS3LastnameFirstname.pdf

Independent Events: $P(A \text{ and } B) = P(A) \cdot P(B)$ (equation 1)**Dependent Events:** $P(A \text{ and } B) = P(A) \cdot P(B \text{ given } A) = P(A) \cdot P(B | A)$ (equation 2)**Non-overlapping Events:** $P(A \text{ or } B) = P(A) + P(B)$ (equation 3)**Overlapping Events:** $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ (equation 4)**Complementary Probabilities:** $P(A) = 1 - P(\text{not } A) = 1 - P(\bar{A})$ **At Least Once Rule:** $P(\text{at least one event } A \text{ in } n \text{ trials}) = 1 - P(\text{no events } A \text{ in } n \text{ trials})$ **Bayes Rule:**
$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(\bar{A}) \cdot P(B | \bar{A})}$$
Examples of Conditional Probabilities, Marginal Totals, and Bayes Theorem

Book Example 4, Section 4-3

	Positive Test Result (positive)	Negative Test Result (negative)
Cancer C	8 (true positive)	2 (false negative)
No Cancer $\bar{C}$	99 (false positive)	891 (true negative)

The table is summarized and the marginal totals calculated below. From the marginal totals can be calculated any desired conditional probability.

	positive	negative	total	
C	8	2	10	$P(\text{positive} C) = 8/10 = .8$
$\bar{C}$	99	891	990	$P(\text{positive} \bar{C}) = 99/990 = .1$
total	107	893	1000	
	$P(C \text{positive}) = 8/107 = .0748$			

$$P(\text{positive}) = 107/1000 = .107$$

$$P(C) = 10/1000 = .01$$

$$P(\bar{C}) = 990/1000 = .99$$

What is the odds of having cancer if tested positive, $P(C|\text{positive})$?

They are NOT equal to $P(\text{positive}|C) = 8/10 = 80\%$

We can see from our table calculations that $P(C | \text{positive}) = .0748$

If you didn't have a nice table to directly calculate $P(C | \text{positive})$, but only the published statistics that $P(C) = .01$, $P(\text{positive}|C) = .8$, $P(\text{positive}|\bar{C}) = .1$, then you could use Bayes' Theorem, which becomes:

$$P(C | \text{positive}) = \frac{P(C) \cdot P(\text{positive} | C)}{P(C) \cdot P(\text{positive} | C) + P(\bar{C}) \cdot P(\text{positive} | \bar{C})} = \frac{0.01 \cdot 0.80}{0.01 \cdot 0.80 + 0.99 \cdot 0.10} = 0.0748$$

To justify the above equation, and Bayes' Theorem in general:

$$P(C | \text{positive}) = \frac{P(\text{positive and } C)}{P(\text{positive})} \quad (\text{from equation \#2})$$

$$P(\text{positive}) = P(\text{positive and } C) \text{ or } P(\text{positive and } \bar{C}) \quad (\text{either } C \text{ or } \bar{C} \text{ must happen})$$

$$P(C | \text{positive}) = \frac{P(\text{positive and } C)}{P(\text{positive and } C) \text{ or } P(\text{positive and } \bar{C})}$$

$$P(C | \text{positive}) = \frac{P(C) \cdot P(\text{positive} | C)}{P(C) \cdot P(\text{positive} | C) + P(\bar{C}) \cdot P(\text{positive} | \bar{C})} \quad (\text{from equation \#2 and equation \#3})$$

In problems 1 & 2, state whether the events are dependent or independent.

Then find the probability of the event.

1. The next three births at the hospital are girls.

2. Randomly drawing first one red MM (and eating it) and then a second red MM (and eating it) from a bag that contains 8 red MMs out of 20 MMs total.

In problems 3 & 4, state whether the events are overlapping or non-overlapping.

Then find the probability of the event

3. Drawing either a red 6 or a red 7 from a deck of 52 cards.

4. Drawing either a jack or a spade from a deck of 52 cards.

5. What is the probability of rolling a 2 at least once when you roll a die three times?

6. Refer to the table which summarizes the results of testing for a certain disease.

	Positive Test Result	Negative Test Result
Subject has the disease	111	4
Subject does not have the disease	11	172

a. In the table, calculate all 5 marginal totals.

b. If a result is negative, what is the probability that it is a false negative (test indicates the person does not have the disease when in fact they do)?

HINT: This is a conditional probability and will involve a marginal total.

c. If a result is positive, what is the probability that it is a false positive (test indicates the person does have the disease when in fact they don't)?

HINT: This is a conditional probability and will involve a marginal total.

7. Suppose subjects with a particular disease test positive 98% of the time, and subjects without the disease test positive 4% of the time. 10% of the subjects have the disease. Given that a subject tests positive, what is the probability that they have the disease?