

AMERICAN UNIVERSITY OF THE MIDDLE EAST
MA265 – Linear Algebra, Online Spring 2, 2020
Final Assessment

Student Name:

Student ID:

Q1

$$A = \begin{bmatrix} 6 & -1 \\ 50 & -9 \end{bmatrix}$$

$$\begin{aligned} \text{a) } A - \lambda I_2 &= \begin{pmatrix} 6 & -1 \\ 50 & -9 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 6-\lambda & -1 \\ 50 & -9-\lambda \end{pmatrix} \end{aligned}$$

$$p(\lambda) = \det(A - \lambda I_2) = \begin{vmatrix} 6-\lambda & -1 \\ 50 & -9-\lambda \end{vmatrix}$$

$$= (6-\lambda)(-9-\lambda) - (-50)$$

$$= -54 - 6\lambda + 9\lambda + \lambda^2 + 50$$

$$p(\lambda) = \lambda^2 + 3\lambda - 4 \Rightarrow \text{Characteristic polynomial}$$

$$\text{b) put } p(\lambda) = 0 \Rightarrow \lambda^2 + 3\lambda - 4 = 0$$

$$(\lambda + 4)(\lambda - 1) = 0$$

$$\lambda_1 = 1$$

$$\lambda_2 = -4$$

These are the eigen values to find eigen vectors
associated with eigen values, we have to solve $\Rightarrow$

the Non trivial homogenous system

$$(A - \lambda I_2) V = 0 \iff \left(\begin{array}{cc|c} 6-\lambda & -1 & 0 \\ 50 & -9-\lambda & 0 \end{array} \right)$$

for $\lambda_1 = 1$:

$$\left(\begin{array}{cc|c} 5 & -1 & 0 \\ 50 & -10 & 0 \end{array} \right) \xrightarrow{R_1/5} R_1$$

$$\left(\begin{array}{cc|c} 1 & -1/5 & 0 \\ 50 & -10 & 0 \end{array} \right) \xrightarrow{-50R_1 + R_2} R_2$$

$$\left(\begin{array}{cc|c} 1 & -1/5 & 0 \\ 0 & 0 & 0 \end{array} \right) \text{ Non trivial solution}$$

$$V_1 - \frac{1}{5} V_2 = 0$$

If we take $V_2 = t$, $t \in \mathbb{R} \Rightarrow V_1 = \frac{t}{5}$

$\therefore V = \begin{pmatrix} \frac{t}{5} \\ t \end{pmatrix} = \begin{pmatrix} 1/5 \\ 1 \end{pmatrix} t$ is an eigen vector with
 $\lambda_1 = 1$

Vector with $\lambda_1 \neq \lambda_2$

for $\lambda = -4$:

$$\left(\begin{array}{cc|c} 10 & -1 & 0 \\ 50 & -5 & 0 \end{array} \right) R_1/10 \rightarrow R_1$$

$$\left(\begin{array}{cc|c} 1 & -1/10 & 0 \\ 50 & -5 & 0 \end{array} \right) -50R_1 + R_2 \rightarrow R_2 \left(\begin{array}{cc|c} 1 & -1/10 & 0 \\ 0 & 0 & 0 \end{array} \right)$$

Non trivial solution

$$v_1 - \frac{1}{10}v_2 = 0$$

If we take $v_2 = t$, $t \in \mathbb{R} \Rightarrow v_1 = \frac{1}{10}t$

$$\therefore V = \begin{pmatrix} \frac{1}{10}t \\ t \end{pmatrix} = \begin{pmatrix} \frac{1}{10} \\ 1 \end{pmatrix} t \text{ is eigen vector with } \lambda_2 = -4$$

c If λ is an eigen value of A then the eigen value for $-8A$ is -8λ we know that;

$$(AV_A = \lambda_A V_A) * -8$$

$$\begin{array}{ccc} -8A V_A & = & -8\lambda_A V_A \\ \downarrow & & \downarrow \\ B \cdot V_B & = & \lambda_B V_B \end{array}$$

So the eigen value will be

$$\boxed{\lambda_B = -8\lambda_A}$$

Q2

$$A = \begin{pmatrix} 6 & 0 & 0 \\ 3 & -1 & -2 \\ -7 & 0 & 5 \end{pmatrix}$$

$$a) \quad A - \lambda I_3 = \begin{pmatrix} 6 & 0 & 0 \\ 3 & -1 & -2 \\ -7 & 0 & 5 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 6-\lambda & 0 & 0 \\ 3 & -1-\lambda & -2 \\ -7 & 0 & 5-\lambda \end{pmatrix}$$

$$p(\lambda) = \det [A - \lambda I_3] = \begin{vmatrix} 6-\lambda & 0 & 0 \\ 3 & -1-\lambda & -2 \\ -7 & 0 & 5-\lambda \end{vmatrix}$$

$$= (6-\lambda) \begin{vmatrix} -1-\lambda & -2 \\ 0 & 5-\lambda \end{vmatrix} = (6-\lambda) [(-1-\lambda)(5-\lambda) - 0]$$

$$p(\lambda) = (6-\lambda)(-1-\lambda)(5-\lambda)$$

Characteristic polynomial

$$\text{Put } p(\lambda) = 0 \Rightarrow (6-\lambda)(-1-\lambda)(5-\lambda) = 0$$

$$\lambda_1 = 6$$

$$\lambda_2 = 5$$

$$\lambda_3 = -1$$

These are the eigen values

Q₂ b) to find eigen vectors associated with eigen values, we solve the Non trivial homogenous system.

$$(A - \lambda I_3)V = 0 \iff \left(\begin{array}{ccc|c} 6-\lambda & 0 & 0 & 0 \\ 3 & -1-\lambda & -2 & 0 \\ -7 & 0 & 5-\lambda & 0 \end{array} \right)$$

For $\lambda_1 = 6$:

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 3 & -7 & -2 & 0 \\ -7 & 0 & -1 & 0 \end{array} \right) R_1 \leftrightarrow R_3$$

$$\left(\begin{array}{ccc|c} -7 & 0 & -1 & 0 \\ 3 & -7 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) R_1 / -7 \rightarrow R_1$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1/7 & 0 \\ 3 & -7 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) -3R_1 + R_2 \rightarrow R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1/7 & 0 \\ 0 & -7 & -17/7 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) -1/7 R_2 \rightarrow R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1/7 & 0 \\ 0 & 1 & 17/49 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \text{Non trivial solution}$$

$$V_1 + 1/7 V_3 = 0$$

$$V_2 + 17/49 V_3 = 0 \implies$$

If we take $V_3 = t$, $t \in \mathbb{R}$

$$\therefore V_1 = -\frac{1}{7}t, V_2 = -\frac{17}{49}t$$

$$\therefore V = \begin{bmatrix} -\frac{1}{7}t \\ -\frac{17}{49}t \\ t \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} \\ -\frac{17}{49} \\ 1 \end{bmatrix} t \text{ is eigen vector}$$

For $\lambda_2 = 5$:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 3 & -6 & -2 & 0 \\ -7 & 0 & 0 & 0 \end{array} \right) \begin{array}{l} -3R_1 + R_2 \rightarrow R_2 \\ 7R_1 + R_3 \rightarrow R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & -6 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) R_2 / -6 \rightarrow R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \text{ Non trivial solution}$$

$$V_1 = 0, V_2 + \frac{1}{3}V_3 = 0$$

$$\text{take } V_3 = t, t \in \mathbb{R} \Rightarrow V_2 = -\frac{1}{3}t$$

$$\therefore V = \begin{pmatrix} 0 \\ -\frac{1}{3}t \\ t \end{pmatrix} = t \begin{pmatrix} 0 \\ -\frac{1}{3} \\ 1 \end{pmatrix} \text{ is eigen vector}$$

$$\text{for } \lambda_3 = -1 : \left(\begin{array}{ccc|c} 7 & 0 & 0 & 0 \\ 3 & 0 & -2 & 0 \\ -7 & 0 & 6 & 0 \end{array} \right) R_1/7 \longrightarrow R_1$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 3 & 0 & -2 & 0 \\ -7 & 0 & 6 & 0 \end{array} \right) \begin{array}{l} -3R_1 + R_2 \longrightarrow R_2 \\ 7R_1 + R_3 \longrightarrow R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 6 & 0 \end{array} \right) R_2/-2 \longrightarrow R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 6 & 0 \end{array} \right) -6R_2 + R_3 \longrightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \text{Non trivial solution}$$

$$v_1 = 0, v_3 = 0, v_2 = t, t \in \mathbb{R}$$

$$\therefore V = \begin{pmatrix} 0 \\ t \\ 0 \end{pmatrix} = t \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ is eigen vector}$$

Q₂ (c) Yes, the set of the eigen vectors of A is a basis for the vector space $\mathbb{R}^3$ because three distinct eigen values associated three eigen vectors are linearly independent.

$$\det(p) = \begin{vmatrix} -\frac{1}{7} & 0 & 0 \\ -\frac{17}{49} & -\frac{1}{3} & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$\det(p) = -\frac{1}{7} \begin{vmatrix} -\frac{1}{3} & 1 \\ 1 & 0 \end{vmatrix} = -\frac{1}{7} (0 - 1) = \frac{1}{7} \neq 0$$

$\therefore$ the set of eigen vectors $\left\{ \left(-\frac{1}{7}, -\frac{17}{49}, 1\right), \right.$
 $\left. \left(0, -\frac{1}{3}, 1\right), \left(0, 1, 0\right) \right\}$ is
a basis for $\mathbb{R}^3$

Q3 a) $c_1 p_1(t) + c_2 p_2(t) = 0_v$

$$c_1(-t^2 - 7) + c_2(-2t^2 + 2t - 7) = (0, 0, 0)$$

$$\begin{pmatrix} c_1 & c_2 \\ -1 & -2 \\ 0 & 2 \\ -7 & -7 \end{pmatrix} \xrightarrow{-R_1} R_1 \begin{pmatrix} 1 & 2 \\ 0 & 2 \\ -7 & -7 \end{pmatrix} \xrightarrow{7R_1 + R_3} R_3$$

$$\begin{pmatrix} 1 & 2 \\ 0 & 2 \\ 0 & 7 \end{pmatrix} \xrightarrow{R_2/2} R_2 \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 7 \end{pmatrix} \xrightarrow{-7R_2 + R_3} R_3$$

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \text{this system has trivial solution with} \\ c_1 = c_2 = 0$$

$\therefore$ the set $S = \{p_1(t), p_2(t)\}$ is ~~not~~
linearly independent in P_2

b) $c_1 p_1(t) + c_2 p_2(t) = p_2$

$$c_1(-t^2 - 7) + c_2(-2t^2 + 2t - 7) = at^2 + bt + c$$

$$\begin{pmatrix} c_1 & c_2 \\ -1 & -2 \\ 0 & 2 \\ -7 & -7 \end{pmatrix} \xrightarrow{-R_1} R_1 \begin{pmatrix} 1 & 2 \\ 0 & 2 \\ -7 & -7 \end{pmatrix} \xrightarrow{7R_1 + R_3} R_3$$

$$\left(\begin{array}{cc|c} 1 & 2 & -a \\ 0 & 2 & b \\ 0 & 7 & -7a+c \end{array} \right) \xrightarrow{R_2/2} R_2 \left(\begin{array}{cc|c} 1 & 2 & -a \\ 0 & 1 & b/2 \\ 0 & 7 & -7a+c \end{array} \right)$$

$$\left(\begin{array}{cc|c} 1 & 2 & -a \\ 0 & 1 & b/2 \\ 0 & 7 & -7a+c \end{array} \right) \xrightarrow{-7R_2+R_3} R_3$$

$$\left(\begin{array}{cc|c} 1 & 2 & -a \\ 0 & 1 & b/2 \\ 0 & 0 & -7/2 b - 7a + c \end{array} \right)$$

It follows that the system is consistent if and only

if a, b, c satisfy $-7/2 b - 7a + c = 0$

That means Not every $V = (a, b, c)$ in P_2 belongs

to the span.

Hence the set $S = \{P_1(t), P_2(t)\}$ do not span

the vector space P_2

Q3 c) Yes it is possible, to find $P_3(t)$ in P_2 such that

$\{P_1(t), P_2(t), P_3(t)\}$ is a basis for the vector space

P_2 .

We must add e_1, e_2, e_3 and 100 K for
Linearly independent vectors.

$$\begin{array}{ccccc} p_1 & p_2 & e_1 & e_2 & e_3 \\ \left(\begin{array}{ccccc} -1 & -2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ -7 & -7 & 0 & 0 & 1 \end{array} \right) & -R_1 \longrightarrow & R_1 \end{array}$$

$$\left(\begin{array}{ccccc} 1 & 2 & -1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ -7 & -7 & 0 & 0 & 1 \end{array} \right) 7R_1 + R_3 \longrightarrow R_3$$

$$\left(\begin{array}{ccccc} 1 & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 7 & -7 & 0 & 1 \end{array} \right) -7R_2 + R_3 \longrightarrow R_3$$

$$\left(\begin{array}{ccccc} 1 & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & -7 & -\frac{7}{2} & 1 \end{array} \right) R_3 / -7 \longrightarrow R_3$$

$$\left(\begin{array}{ccccc} 1 & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{7} \end{array} \right)$$

$\therefore$ the set of $\{p_1(t), p_2(t), e_1\}$ is a basis for

P_2

We must add $p_3(t) = t^2$

$\therefore S = \{-t^2 - 7, -2t^2 + 2t - 7, t^2\}$ is a
basis for P_2

Q4

$$L: \mathbb{R}^2 \rightarrow \mathbb{R}^3, \quad L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} 7u_2 - u_1 \\ u_1 - u_2 \\ -3u_2 \end{bmatrix}$$

a) let $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$, $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$L(u+v) = L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right) = L\left(\begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 7(u_2 + v_2) - (u_1 + v_1) \\ (u_1 + v_1) - (u_2 + v_2) \\ -3(u_2 + v_2) \end{bmatrix}$$

$$= \begin{bmatrix} 7u_2 + 7v_2 - u_1 - v_1 \\ u_1 + v_1 - u_2 - v_2 \\ -3u_2 - 3v_2 \end{bmatrix}$$

$$= \begin{bmatrix} (7u_2 - u_1) + (7v_2 - v_1) \\ (u_1 - u_2) + (v_1 - v_2) \\ -3u_2 - 3v_2 \end{bmatrix}$$

$$= \begin{bmatrix} 7u_2 - u_1 \\ u_1 - u_2 \\ -3u_2 \end{bmatrix} + \begin{bmatrix} 7v_2 - v_1 \\ v_1 - v_2 \\ -3v_2 \end{bmatrix}$$

$$= \mathcal{L} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \mathcal{L} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\mathcal{L}(u+v) = \mathcal{L}(u) + \mathcal{L}(v) \quad \text{--- [1]}$$

$$k \in \mathbb{R}, \mathcal{L}(ku) = \mathcal{L}\left(k \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\right)$$

$$= \mathcal{L} \begin{pmatrix} [ku_1] \\ [ku_2] \end{pmatrix} = \begin{bmatrix} 7ku_2 - ku_1 \\ ku_1 - ku_2 \\ -3ku_2 \end{bmatrix}$$

$$= \begin{bmatrix} k(7u_2 - u_1) \\ k(u_1 - u_2) \\ -3ku_2 \end{bmatrix}$$

$$= k \begin{bmatrix} 7u_2 - u_1 \\ u_1 - u_2 \\ -3u_2 \end{bmatrix}$$

$$\mathcal{L}(ku) = k \mathcal{L} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = k \mathcal{L}(u) \quad \text{--- [2]}$$

from 1, 2

$\therefore \mathcal{L}$ is a linear transformation.

Q4 b) the standard basis for $\mathbb{R}^2$ is $e_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $e_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\mathcal{L}(e_1) = \mathcal{L} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 7(0) - 1 \\ 1 - 0 \\ -3(0) \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\mathcal{L}(e_2) = \mathcal{L} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 7(1) - 0 \\ 0 - 1 \\ -3(1) \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \\ -3 \end{pmatrix}$$

~~Q4~~ ~~A/4~~

$$\therefore A = \begin{pmatrix} \mathcal{L}(e_1) & \mathcal{L}(e_2) \\ -1 & 7 \\ 1 & -1 \\ 0 & -3 \end{pmatrix}$$

$$\mathcal{L} \begin{pmatrix} -4 \\ 3 \end{pmatrix} = A \begin{pmatrix} -4 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 & 7 \\ 1 & -1 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 + 21 \\ -4 + -3 \\ 0 + -9 \end{pmatrix} = \begin{pmatrix} 25 \\ -7 \\ -9 \end{pmatrix}$$

Q4 c) NO.

$$\text{If } T: \mathbb{R}^2 \rightarrow \mathbb{R}^3, T \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ u \end{pmatrix}$$

$$\text{Let } u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$T(u+v) = T \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \end{pmatrix} = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u \end{pmatrix}$$

$$T(u) + T(v) = \begin{pmatrix} u_1 \\ u_2 \\ u \end{pmatrix} + \begin{pmatrix} v_1 \\ v_2 \\ v \end{pmatrix} = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u + v \end{pmatrix}$$

$$\therefore T(u+v) \neq T(u) + T(v) \Rightarrow \text{Not linear}$$

Q5

$$A = \begin{pmatrix} 2 & -2 \\ -4 & 1 \end{pmatrix}$$

$$a) A - \lambda I_2 = \begin{pmatrix} 2 & -2 \\ -4 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2-\lambda & -2 \\ -4 & 1-\lambda \end{pmatrix}$$

$$p(\lambda) = \det(A - \lambda I_2) = \begin{vmatrix} 2-\lambda & -2 \\ -4 & 1-\lambda \end{vmatrix}$$

$$= (2-\lambda)(1-\lambda) - 8$$

$$= 2 - 2\lambda - \lambda + \lambda^2 - 8$$

$$p(\lambda) = \lambda^2 - 3\lambda - 6 \text{ is the characteristic}$$

polynomial

$$b) B = A^2 - 3A - 6I_2$$

=

Q5 b)

$$B = A^2 - 3A - 6I_2$$

$$= \begin{pmatrix} 2 & -2 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 2 & -2 \\ -4 & 1 \end{pmatrix} - 3 \begin{pmatrix} 2 & -2 \\ -4 & 1 \end{pmatrix} - 6 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4+8 & -4+-2 \\ -8-4 & 8+1 \end{pmatrix} - \begin{pmatrix} 6 & -6 \\ -12 & 3 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & -6 \\ -12 & 9 \end{pmatrix} - \begin{pmatrix} 6 & -6 \\ -12 & 3 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O_2$$

5 c) From part (b)

$$P(A) = 0$$

$$A^2 - 3A - 6I_2 = 0$$

$$A^2 - 3A - 6I_2 + 6I_2 = 0 + 6I_2$$

$$\therefore A^2 - 3A = 6I_2$$

Q5 d) Yes it is possible.

$$A^2 - 3A = 6I_2 \quad (\text{Multiple by } \frac{1}{6} \text{ both sides})$$

$$\frac{1}{6} (A^2 - 3A) = I_2$$

$$\frac{1}{6} (A (A - 3I_2)) = I_2$$

$$A \left(\frac{1}{6} (A - 3I_2) \right) = I_2$$

$\downarrow$
B

$$A \cdot B = I_2 \iff B = A^{-1}$$

$$\therefore A^{-1} = \frac{1}{6} (A - 3I_2), \quad \text{hint } \boxed{A \cdot A = I_1}$$