

Assignment 4

This assignment, based on the content of Unit 4 of the *Study Guide*, is worth 3% of your grade. We recommend that you hand it in after you complete Unit 4. You must show all of your work in order to obtain full marks. For your convenience, each exercise mentions the section of the *Study Guide* that corresponds to the given problem.

Note: Each of the questions below is of equal weight, and each will be marked out of twenty (20) points. To gain full marks, you must show all your work.

(Section 4.1)

20 points

1. a. Let $\vec{u} = (1, 2, -3, 5, 0)$, $\vec{v} = (0, 4, -1, 1, 2)$, and $\vec{w} = (7, 1, -4, -2, 3)$. Find the components of
 - i. $\vec{v} + \vec{w}$
 - ii. $3(2\vec{u} - \vec{v})$
 - iii. $(3\vec{u} - \vec{v}) - (2\vec{u} + 4\vec{w})$
 - iv. $\frac{1}{2}(\vec{w} - 5\vec{v} + 2\vec{u}) + \vec{v}$
- b. Find scalars c_1 , c_2 , and c_3 for which the equation is satisfied,
$$c_1(-1, 0, 2) + c_2(2, 2, -2) + c_3(1, -2, 1) = (-6, 12, 4)$$

(Section 4.2)

20 points

2. a. Find the Euclidean distance between $\vec{u}$ and $\vec{v}$ and the cosine of the angle between those vectors. State whether that angle is acute, obtuse, or 90° .
 - i. $\vec{u} = (1, 2, -3, 0)$, $\vec{v} = (5, 1, 2, -2)$
 - ii. $\vec{u} = (0, 1, 1, 1, 2)$, $\vec{v} = (2, 1, 0, -1, 3)$
- b. Verify that the Cauchy-Schwarz inequality holds.
 - i. $\vec{u} = (4, 1, 1)$, $\vec{v} = (1, 2, 3)$
 - ii. $\vec{u} = (1, 2, 1, 2, 3)$, $\vec{v} = (0, 1, 1, 5, -2)$

(Section 4.3)

20 points

3. a. Find the distance between the given parallel planes
$$2x - y + z = 1; \quad 2x - y + z = -1.$$
- b.
 - i. Show that $\vec{v} = (a, b)$ and $\vec{w} = (-b, a)$ are orthogonal vectors.
 - ii. Use the result in part i. to find two vectors that are orthogonal to $\vec{v} = (2, -3)$.
 - iii. Find two unit vectors that are orthogonal to $\vec{v} = (-3, 4)$.

(Section 4.4)

20 points

4. a. Find the general solution to the linear system and confirm that the row vectors of the coefficient matrix are orthogonal to the solution vectors.

$$x_1 + 3x_2 - 4x_3 = 0$$

$$x_1 + 2x_2 + 3x_3 = 0$$

- b. i. Find a homogeneous linear system of two equations in three unknowns whose solution space consists of those vectors in $\mathbb{R}^3$ that are orthogonal to $\vec{a} = (-3, 2, -1)$ and $\vec{b} = (0, -2, -2)$.
- ii. What kind of geometric object is the solution space?
- iii. Find a general solution of the system obtained in part i., and confirm that Theorem 3.4.3 of the textbook holds.

(Section 4.5)

20 points

5. a. Find the area of the triangle in 3-space that has vertices

$$P(1, -1, 2), Q(0, 3, 4) \text{ and } R(6, 1, 8).$$

- b. Given $\vec{u} = (-1, 2, 4)$, $\vec{v} = (3, 4, -2)$, $\vec{w} = (-1, 2, 5)$, compute the scalar triple product $\vec{u} \cdot (\vec{v} \times \vec{w})$
- c. Given $\vec{u} \cdot (\vec{v} \times \vec{w}) = 3$, find the following.
- i. $\vec{v} \cdot (\vec{u} \times \vec{w})$
- ii. $(\vec{u} \times \vec{w}) \cdot \vec{v}$
- iii. $\vec{v} \cdot (\vec{w} \times \vec{w})$