

Assignment 5

This assignment, based on the content of Unit 5 of the *Study Guide*, is worth 3% of your grade. We recommend that you hand it in after you complete Unit 5. You must show all of your work in order to obtain full marks. For your convenience, each exercise mentions the section of the *Study Guide* that corresponds to the given problem.

Note: Each of the questions below is of equal weight, and each will be marked out of twenty (20) points. To gain full marks, you must show all your work.

(Section 5.1)

20 points

1. a. Let V be the set of real-valued functions that are defined at each x in the interval $(-\infty, \infty)$. If $\vec{f} = f(x)$ and $\vec{g} = g(x)$ are two functions in V and if k is any scalar, then define the operations of addition and scalar multiplication by

$$(\vec{f} + \vec{g}) = f(x) + g(x)$$

$$k\vec{f} = kf(x)$$

Verify the Vector Space Axioms for the given set of vectors.

- b. Let V consist of the form $\vec{u} = (u_1, u_2, \dots, u_n, \dots)$ in which $u_1, u_2, \dots, u_n, \dots$ is an infinite sequence of real numbers. Define two infinite sequences to be *equal* if their corresponding components are equal, and define addition and scalar multiplication componentwise by

$$\vec{u} + \vec{v} = (u_1, u_2, \dots, u_n, \dots) + (v_1, v_2, \dots, v_n, \dots)$$

$$= (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n, \dots)$$

$$k\vec{u} = (ku_1, ku_2, \dots, ku_n, \dots).$$

Prove that the given set is a vector space.

(Section 5.2)

20 points

2. a. In each part express the vector as a linear combination of $\vec{p}_1 = 2 + x + 4x^2$, $\vec{p}_2 = 1 - x + 3x^2$, and $\vec{p}_3 = 3 + 2x + 5x^2$.
- $-9 - 7x - 15x^2$
 - $6 + 11x + 6x^2$
 - 0
 - $7 + 8x + 9x^2$
- b. Suppose that $\vec{v}_1 = (2, 1, 0, 3)$, $\vec{v}_2 = (3, -1, 5, 2)$, and $\vec{v}_3 = (-1, 0, 2, 1)$. Which of the following vectors are in $\text{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$?
- $(2, 3, -7, 3)$
 - $(0, 0, 0, 0)$
 - $(1, 1, 1, 1)$
 - $(-4, 6, -13, 4)$

(Section 5.3)

- 20 points**
3. a. i. Show that the vectors $\vec{v}_1 = (1, 2, 3, 4)$, $\vec{v}_2 = (0, 1, 0, -1)$, and $\vec{v}_3 = (1, 3, 3, 3)$ form a linearly dependent set in $\mathbb{R}^n$.
- ii. Express each vector in part i. as a linear combination of the other two.
- b. Prove that if $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is a linearly independent set of vectors, then so are $\{\vec{v}_1, \vec{v}_3\}$, and $\{\vec{v}_2\}$.

(Section 5.4)

- 20 points**
4. a. i. Use matrix multiplication to find the construction of (a, b) with factor $k = 1/\alpha$, where $\alpha > 1$.
- ii. Use matrix multiplication to find the construction of (a, b) with factor $k = \alpha$, where $\alpha > 1$.
- b. Find the standard matrix for the reflection of $\mathbb{R}^2$ about the line that makes an angle of $\pi/4$ ($= 45^\circ$) with the positive x -axis, and then use that matrix to find the reflection of $(1, 2)$ onto this line.

(Section 5.5)

- 20 points**
5. a. Find the matrix for a shear in the x -direction that transforms the triangle with vertices $(0, 0)$, $(2, 1)$, and $(3, 0)$ into a right triangle with the right angle at the origin.
- b. Draw a figure that shows the image of the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0.5, 1)$ under a shear by a factor of 2 in the x -direction.