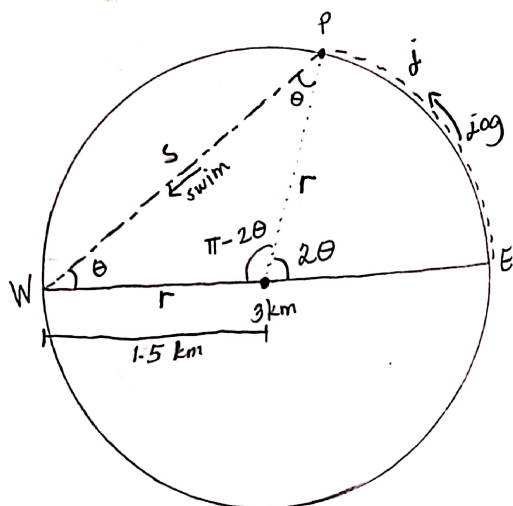


4.)



$$\Rightarrow \sin \theta = \frac{1}{8}$$

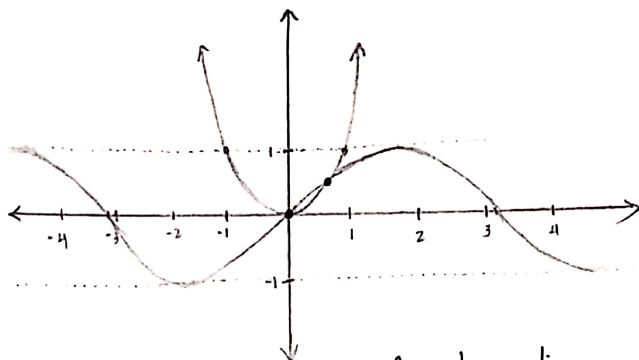
$$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{8}\right) \approx 0.125 \text{ rad}$$

Hence,

$$j = 3(0.125) = 0.375$$

Therefore, Peter should jog 0.375 km in order to arrive at point W in the least amount of time.

5) a. Sketch the graphs of the curves $y = \sin x$ and $y = x^2$ showing their points of intersection.



There are two points of intersection.

b. Use the IVT to identify an interval where the equation $\sin x - x^2 = 0$ has a nonzero solution.

IVT says that if $(a, f(a))$ lies above the x -axis and $(b, f(b))$ lies below the x -axis, there must exist a point $(c, f(c))$ such that $a < c < b$ where the graph of f crosses the x -axis. In other words, we are assured that there exists c such that $f(c) = 0$.

Since we want a nonzero solution c , we must have $a > 0$. Now, consider

$$a = 0.5 \quad \& \quad b = 1$$

$$\text{Then, } f(a) = \sin(0.5) - (0.5)^2 \approx 0.25$$

$$f(b) = \sin(1) - (1)^2 \approx -0.16$$

Clearly, $(0.5, 0.25)$ lies above x -axis & $(1, -0.16)$ lies below x -axis.

Therefore, by IVT there exist c such that $0.5 < c < 1$ (that is, c is nonzero) such that $f(c) = 0$.

SOLUTION:

Let j be the length/distance Peter needs to jog.
 s be the distance he needs to swim.
 θ be the angle between the x -axis and the line PW . (θ in radians)

• length of j :

Using the formula for arc length, we have

$$j = r(2\theta) = (1.5)(2\theta) = 3\theta$$

• length of s :

Using Law of Cosines, we have

$$r^2 = s^2 + r^2 - 2(s)(r)\cos\theta$$

$$\Rightarrow -s^2 = -r^2 + r^2 - 2sr\cos\theta$$

$$\Rightarrow s^2 = 2sr\cos\theta$$

$$\Rightarrow s = 2r\cos\theta$$

$$= 2(1.5)\cos\theta$$

$$= 3\cos\theta$$

Thus, given his swimming & jogging speeds, the total time it takes for him to get to point W is

$$T = \frac{j}{24} + \frac{s}{3} = \frac{3\theta}{24} + \frac{3\cos\theta}{3}$$

$$T = \frac{1}{8}\theta + \cos\theta$$

Now, we have expressed T as a function of θ . To get the minimum for T , we just need to equate its derivative to 0.

$$\frac{dT}{d\theta} = \frac{1}{8} - \sin\theta = 0$$

This means that on the interval $(0.5, 1)$, the equation $\sin x - x^2 = 0$ has a nonzero solution.

c. Use Newton's method to approximate the non-zero solution of the equation $\sin x - x^2 = 0$

sol:

$$\text{Let } f(x) = \sin x - x^2.$$

Since we have the interval $(0.5, 1)$ from the result in (b), we take $x_1 = 0.5$ as the initial estimate.

Now,

$$f'(x) = \cos x - 2x.$$

Then, using Newton's method,

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &= 0.5 - \frac{\sin(0.5) - (0.5)^2}{\cos(0.5) - 2(0.5)} \end{aligned}$$

$$\approx 2.37412465$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \approx 1.47027962$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} \approx 1.05947584$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} \approx 0.90582679$$

$$x_6 = x_5 - \frac{f(x_5)}{f'(x_5)} \approx 0.87771188$$

$$x_7 = x_6 - \frac{f(x_6)}{f'(x_6)} \approx 0.87672742$$

$$x_8 = x_7 - \frac{f(x_7)}{f'(x_7)} \approx 0.87672622$$

You can now consider x_8 as a good approximation (5 decimal places did not change within our last two iterations).