

ASSIGNMENT 5

1.) a) For vector addition:

- Closure: Let $\vec{f}, \vec{g} \in V$.
 $\vec{f} + \vec{g} = f(x) + g(x) \in V$, since the sum of two real-valued fcn is also a real-valued fcn.

- Commutativity & Associativity follow directly from the commutativity & associativity of addition of fcn.

- Additive identity: Note that $\vec{0} = 0(x) \in V$, the function that maps any $x \in \mathbb{R}$ to 0. Now,
 $\vec{f} + \vec{0} = f(x) + 0(x) = f(x) + 0 = f(x)$
 $\Rightarrow \vec{0} \in V$ is the identity

- Additive inverse: Note that $-\vec{f} = -f(x) \in V$. Now,
 $\vec{f} + (-\vec{f}) = f(x) - f(x) = 0$
 $\Rightarrow -\vec{f}$ is the additive inverse of $\vec{f}$ in V .

For Scalar Multiplication:

- Closure: Let $c \in \mathbb{R}$, $\vec{f} \in V$.
 $c \cdot \vec{f} = cf(x) \in V$, by property of scalar mult in fcn.

- Associativity: Let $c, d \in \mathbb{R}$.

$$c \cdot (d \cdot \vec{f}) = c \cdot (df(x)) \\ = (cd)f(x) = (cd) \cdot \vec{f}$$

- Distributive laws follow directly from the distributivity of functions.

- Unitary Law: Note that $1 \in \mathbb{R}$. Now,
 $1 \cdot \vec{f} = 1 \cdot f(x) = f(x) = f(x) \cdot 1 = \vec{f} \cdot 1 = \vec{f}$

b.) For addition:

- Closure: Let $\vec{u}, \vec{v} \in V$. Then

$$\vec{u} + \vec{v} = (u_1, u_2, \dots, u_n, \dots) + (v_1, v_2, \dots, v_n, \dots) \\ = (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n, \dots) \in V$$

since sum of real number is a real number

- Commutativity & Associativity follow from the assoc & comm of addition of real numbers

- Additive identity: Define $\vec{0} = (0, 0, \dots, 0, \dots) \in V$.
Then $\vec{u} + \vec{0} = (u_1 + 0, u_2 + 0, \dots, u_n + 0, \dots)$
 $= (u_1, u_2, \dots, u_n, \dots) = \vec{u}$
therefore $\vec{0} \in V$ is the identity.

- Additive inverse: Note that $-\vec{u} = (-u_1, -u_2, \dots, -u_n, \dots) \in V$ for any $\vec{u} \in V$. Now,

$$\vec{u} + (-\vec{u}) = (u_1 + (-u_1), u_2 + (-u_2), \dots, u_n + (-u_n), \dots) \\ = (0, 0, \dots, 0, \dots) = \vec{0}$$

therefore there exists an additive inverse $-\vec{u} \in V$ for $\vec{u} \in V$.

For scalar mult:

- Closure: Let $c \in \mathbb{R}$.

$c\vec{u} = (cu_1, cu_2, \dots, cu_n, \dots) \in V$, since $\mathbb{R}$ is closed under multiplication

- Associativity in V follows from the associativity of multiplication in $\mathbb{R}$.

- Distributivity in V follows from the distributivity of multiplication over addition in $\mathbb{R}$.

- Unitary Law: Define $\vec{1} = (1, 1, \dots, 1, \dots) \in V$. Now,
 $\vec{1} \cdot \vec{u} = (1 \cdot u_1, 1 \cdot u_2, \dots, 1 \cdot u_n, \dots)$
 $= (u_1, u_2, u_3, \dots, u_n, \dots)$
 $= \vec{u}$

Similarly, $\vec{u} \cdot \vec{1} = \vec{u}$.

Therefore, V is a vector space.

2.) a.)

$$(i) -9 - 7x - 15x^2$$

$$= a_1 \vec{p}_1 + a_2 \vec{p}_2 + a_3 \vec{p}_3$$

$$= a_1(2 + x + 4x^2) + a_2(1 - x + 3x^2) + a_3(3 + 2x + 5x^2)$$

$$= 2a_1 + a_2 + 3a_3 + (a_1 - a_2 + 2a_3)x + (4a_1 + 3a_2 + 5a_3)x^2$$

$$\Rightarrow \begin{cases} 2a_1 + a_2 + 3a_3 = -9 \\ a_1 - a_2 + 2a_3 = -7 \\ 4a_1 + 3a_2 + 5a_3 = -15 \end{cases}$$