Question 1

$y = \frac{1}{5} \tan(9x-5)$

$\frac{d}{dx}\left(\frac{1}{5}\tan(9x-5)\right) \Rightarrow \frac{9}{5}\sec^2(9x-5)$; Applying chain rule

$\frac{d}{dx}\left(\frac{9}{5}\sec^2(9x-5)\right) \Rightarrow$ Applying chain rule:

$$= \frac{9}{5} \times 9\left(2\sec^2(9x-5)\tan(9x-5)\right)$$

$$= \frac{162}{5}\sec^2(9x-5)\tan(9x-5)$$

Question 2

$S = t^3 - 15t^2 + 48t$

$\frac{ds}{dt} = 2t^2 - 30t + 48$

$$\int_0^3 (3t^2 - 30t + 48)dt = \left[t^3 - 15t^2 + 48t\right]_0^3$$

$$= 3^3 - 15(9) + 48(3)$$

$$= 36\,m$$

Question 3

$x^3 + 3x^2y + y^3 = 8$

$\frac{d}{dx} x^3 + 3\frac{d}{dx}(yx^2) + \frac{d}{dx}(y^3) = 0$ ie $\frac{d}{dx}(8) = 0$

$= 3x^2 + 3\left[y' x^2 + y \times 2x\right] + 3y^2 \frac{dy}{dx} = 0$

$\Rightarrow 3x^2 + 3\frac{dy}{dx} x^2 + 6xy + 3y^2 \frac{dy}{dx} = 0$

$3x^2 \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = -3x^2 - 6xy$

$\frac{dy}{dx} = \frac{-3x^2 - 6xy}{3x^2 + 3y^2}$

$= \frac{-x^2 - 2xy}{x^2 + y^2}$

Question 4

$\frac{d}{dx}\left(\sqrt{4x+2}\right)$

$\sqrt{4x+2} = (4x+2)^{1/2}$

Applying chain rule:

$\frac{dy}{dx} = 1/2\,(4x+2)^{-1/2} \times 4$

$= \frac{2}{\sqrt{4x+2}}$

Question 5

$y = \cos x$ at $x = \frac{\pi}{2}$

$$\frac{dy}{dx} = -\sin(x)$$

$$m = -\sin(\pi/2) = -1$$

$$y - 0 = -1\left(x - \pi/2\right)$$

$$y = -x + \pi/2$$

Question 6

$$y = (2x+9)^5$$

$y = u^5$, $u = 2x+9$, $\frac{dy}{dx} = 5u^4 \times u'$

$$= 5(2x+9)^4 \times 2$$

$$= 10(2x+9)^4$$

Question 7

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$2 = 2\pi \times 131 \times r'$$

$$r' = \frac{3}{2\pi \times 131} = 0.0036 \text{ mm/s}$$

Question 8

$h(t) = t^3 - 25t - 4, \quad (5, -4)$

$h' = 3t^2 - 25$

$m = 3(25) - 25$

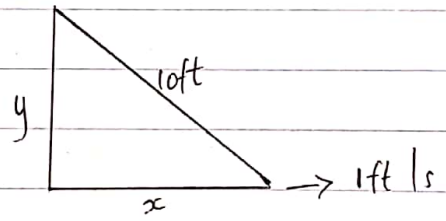
$= 50$

$y - (-4) = 50(t - 5)$

$y + 4 = 50t - 250$

$y = 50t - 254$

Question 9



$\therefore l^2 = x^2 + y^2$

$2l\frac{dl}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

but $\frac{dl}{dt} = 0 \quad ; \quad 0 = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

Also, $\frac{dx}{dt} = 1$

$0 = 2x + 2y\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{x}{y}$ where $x = 2$

$= \frac{2}{\sqrt{10^2 - 2^2}}$

$= 0.204$ ft/sec

Question 10

$y = \cos(\sqrt{8t+11})$

$u = \sqrt{8t+11} = (8t+11)^{1/2}$

$u' = +\frac{1}{2}(8t+11)^{-1/2} \cdot 8$

$= \frac{4}{\sqrt{8t+11}}$

$\frac{dy}{dt} = -\sin(\sqrt{8t+11}) \frac{4}{\sqrt{8t+11}}$

$= \frac{-4\sin(\sqrt{8t+11})}{\sqrt{8t+11}}$

Question 11

$S = 256 - 16t^2$

$256 = 16t^2$

$t^2 = 16$

$t = 4$ secs

$\frac{ds}{dt} = -16 \times 2t$

$V(4) = -16 \times 2 \times 4$

$= -128 \text{ m/s}$

$= 4 \text{ sec}, -128 \text{ m/s}$

Question 12

$$\tan y = e^x \ln 6x$$

$$\frac{d}{dx}\tan y = \frac{d}{dx}(e^x + \ln 6x) \rightarrow e^x + \frac{1}{x}$$

$$\Rightarrow \sec^2 y \frac{dy}{dx} = e^x \ln 6x + \frac{e^x}{x}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}\left(e^x \ln 6x + \frac{e^x}{x}\right)$$

$$\sec^2 y \frac{dy}{dx} = e^x + \frac{e^x}{x}$$

$$\frac{dy}{dx} = \left(e^x + \frac{1}{x}\right)\frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \frac{xe^x + 1}{\sec^2(y)}$$

Question 13

$$xy^2 = 4 \quad , \frac{dx}{dt} = -5$$

$\frac{dy}{dt}$ when $x = 4,\ y = 1$

$$y^2 \frac{dx}{dt} + 2y \frac{dy}{dt} x = 0$$

$$-5 + 2 \frac{dy}{dt} \times 4 = 0$$

$$-5 + 8 \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{5}{8}$$

Question 14

$\frac{dt}{dx}$ if $t = \frac{x}{8x-3}$

Applying Quotient rule: $\frac{(8x-3) - x(8)}{(8x-3)^2}$

$= \frac{8x-3-8x}{(8x-3)^2}$

$= \frac{-3}{(8x-3)^2}$

Question 15

$S = \sqrt{2+2t}$

$\frac{ds}{dt} = V = \frac{d}{dt}\left((2+2t)^{1/2}\right)$

$= \frac{1}{2}(2t+2)^{-1/2} \times 2$

$V = \frac{1}{\sqrt{2t+2}}$

$V(1) = \frac{1}{\sqrt{2+2}} = \frac{1}{2}$ m/s

Question 16

$f(x) = 7xe^x - 7e^x$

Applying produd rule $f'(x) = 7(e^x + xe^x) - 7e^x$

$= 7e^x + 7xe^x - 7e^x$

$= 7xe^x$

Question 17

$s = t^3 - 27t^2 + 240t$

$\frac{ds}{dt} = 3t^2 - 54t + 240$

$3t^2 - 54t + 240 = 0$

solving, $t = 10$, $t = 8$

$\frac{dv}{dt} = 6t - 54$

$a(8) = -6 \text{ m/sec}^2$

$a(10) = 6 \text{ m/sec}^2$

Question 18

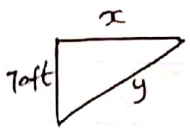
$q = \sqrt{11r - r^3} = (11r - r^3)^{1/2}$

Applying chain rule; $u = 11r - r^3$

$\frac{dy}{dx} = \frac{1}{2}(11r - r^3)^{-1/2}(11 - 3r^2)$

$= 11 - 3r^2$

Question 19



$x^2 = 70^2 + y^2$

$y^2 = 70^2 + x^2$

$\frac{dx}{dt} = 25 \text{ ft/s}$

$\frac{d}{dt}(x^2 + 70^2) = \frac{d}{dt}(y^2)$

$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$

$x^2 + 70^2 = 250^2$

$x = 240$

$x\frac{dx}{dt} = y\frac{dy}{dt}$

$240(25) = 250\frac{dy}{dt}$

$\frac{dy}{dt} = 24 \text{ ft/sec}$

Q20

$V = t^2 - 9t + 8$

Body is moving backwards when velocity is negative

$t^2 - 9t + 8 < 0$

Boundaries: $(-\infty, 1), (1, 8), (8, \infty)$

$t^2 - 9t + 8 = 0$

$t = 1, 8$

Testing for intervals, $V < 0$ when $1 < t < 8$

Question 21

$y = 6^{12x} \Rightarrow \ln(y) = 12x \ln 6.$ but $y = 6^{12x}$

$\frac{d}{dx}\ln(y) = \frac{d}{dx}(12x\ln(6))$

$= \frac{1}{y}\frac{dy}{dx} = 12\ln 6$

$\frac{dy}{dx} = y(12\ln(6))$

Thus:

$\frac{dy}{dx} = 6^{12x}\,12\ln(6)$

Question 22

$y = t^{2-e}$ Independent variable: t

$$\frac{dy}{dx} = (2-e)\,t^{2-e-1}$$

$$= (2-e)\,t^{1-e}$$

Question 23

$$y = \frac{1}{11x^2} + \frac{1}{5x} \Rightarrow \frac{1}{11}x^{-2} + \frac{1}{5}x^{-1}$$

$$\frac{dy}{dx} = \frac{-2}{11}x^{-3} + \frac{-1}{5}x^{-2}$$

$$\frac{d^2y}{dx^2} = \frac{+6x^{-4}}{11} + \frac{2}{5}x^{-3}$$

$$= \frac{6}{11x^4} + \frac{2}{5x^3}$$

Question 24

$S = 150 - 16t^2$

$$\frac{ds}{dt} = -32t \Rightarrow -32t$$

$$\frac{dv}{dt} = -32$$

$150 = 16t^2$

$t^2 = 9.375$

$t = 3.06s$

$V(3.062) = -32 \times 3.062$

$= -97.98\ ft/s$

Question 25

$y = x^6 \cos x - 12x \sin x - 12 \cos x$

Apply product rule:

$\frac{d}{dx} x^6 \cos x = 6x^5 \cos x + (-\sin x) x^6$

$\frac{d}{dx} 12x \sin x = 12 \sin x + ((\cos x) 12x$

$\frac{dy}{dx} = 6x^5 \cos x - x^6 \sin x - 12 \sin x - 12x \cos x + 12 \sin x$

$= 6x^5 \cos x - x^6 \sin x - 12x \cos x$

Question 26

$y = 5x^2 e^{3x}$

$\frac{dy}{dx} = \frac{d}{dx} 5x^2 \times e^{3x} + \frac{d}{dx} e^{3x} \times 5x^2$

$= 10x (e^{3x}) + 3e^{3x} \times 5x^2$

$= 10xe^{3x} + 15x^2 e^{3x} = 7 5xe^{3x}(2 + 3x^2)$

Question 27

$f(x) = 7x^2 + 5x - 2$

$f'(x) = 14x + 5$

$14x + 5 = 0$

$x = \frac{-5}{14}$

$f\left(\frac{-5}{14}\right) = \frac{-81}{28}$

$= \left(\frac{-5}{14}, \frac{-81}{28}\right)$

Question 28

$\sin y = 6x + 6y$

$\frac{d}{dx} \sin y = \frac{d}{dx} 6x + \frac{d}{dx} 6y$

$\cos y \frac{dy}{dx} = 6 + 6\frac{dy}{dx}$

$\cos y \frac{dy}{dx} - 6\frac{dy}{dx} = 6$

$\frac{dy}{dx} = \frac{6}{\cos(y) - 6}$

Question 29

$f(x) = x^5 \log_8 x$

$f'(x) = \frac{d}{dx} x^5 \times \log_8 x + \frac{d}{dx} \log_8 x \times x^5$

$= 5x^4 \log_8 x + \frac{d}{dx} \log_8 x \times x^5$

$y = \log_8 x$

$8^y = x$

$8 \ln y = \ln x$

$\frac{\ln(8)}{y} \frac{dy}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = \frac{y}{\ln(8)x} = \frac{1}{x \ln(8)}$

$= 5x^4 \log_8 x + \frac{x^5}{x \ln(8)}$

Quartion 30

$y = 4e^{-9x}$

$\frac{dy}{dx} = 4 \times -9\, e^{-9x}$

$= -36\, e^{-9x}$

Question 31

$f(x) = 8x^3 - 15x^2 - 3, \quad x \geq 1.5$

$f'(x) = 24x^2 - 30x$

$f'(1185) = \frac{1}{f'(x)} = \frac{1}{24(36) - 30(6)}$

$= 1/684$

Quartion 32

$y = \cos(e^{x^9})$

$\frac{d}{dx} e^{x^9} = 9x^8 e^{x^9}$

$\frac{d}{dx} \cos(e^{x^9}) = -9x^8 e^{x^9} \sin(e^{x^9})$

Question 33

$y = \ln(x+4)^2 = (\ln(x+4))^2$

$u = \ln(x+4)$

Applying chain rule:

$\frac{dy}{dx} = \frac{2}{x+4}(\ln(x+4))$

$\frac{d}{dx}(\ln(x+4)^2) = \frac{2(x+4)}{(x+4)^2}$

$= \frac{2}{x+4}$

Question 34

$f(x) = \frac{1}{x^7+2} = (x^7+2)^{-1}$

Applying chain rule: $-1(x^7+2)^{-2} \times 7x^6$

$= \frac{-7x^6}{(x^7+2)^2}$

Question 35

$$y = \sqrt[4]{\frac{x(x-3)}{x^4+8}}$$

$$\ln y = \frac{1}{4} \ln \left(\frac{x^2-3x}{x^4+8} \right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{4} \left(\frac{\frac{x^4+8(2x-3) - x^2-3x(4x^3)}{(x^4+8)^2}}{\frac{(x^2-3x)}{x^4+8}} \right)$$

$$\downarrow$$

$$\frac{1}{4} \frac{\left[x^4+8(2x-3) \cdot (x^2-3x)\,4x \right](x^4+8)}{x^2-3x}$$

$$\frac{dy}{dx} = \frac{1}{4} \sqrt[4]{\frac{x(x-3)}{x^4+8}} \left[\frac{\left[(x^4+8)(2x-3) - (x^2-3x)4x \right] x^4+8}{x^2-3x} \right]$$

Question 36

$$f(x) = (x^2+3)^{1/2}, \quad x = 1$$

when $x = 1$, $y = 2$

$$f'(x) = \frac{1}{2}(x^2+3)^{-1/2} \times 2x = \frac{x}{\sqrt{x^2+3}}$$

$$m = \frac{1}{2}$$

$$y - 2 = \frac{1}{2}(x-1)$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

Question 37

$y = \cos x \quad @ \quad \pi/2$

$y = \cos \pi/2 = 0$

$\frac{dy}{dx} = -\sin x$

$m = -\sin \pi/2 = -1$

$y - 0 = -1(x - \pi/2)$

$y = -x + \pi/2$

Question 39

$f(x) = (2x-2)(5x^3 - x^2 + 1)$

$\frac{d}{dx} 2x - 2 = 2$

$\frac{d}{dx} 5x^3 - x^2 + 1 = 15x^2 - 2x$

$f'(x) = \frac{d}{dx}(2x-2) \times 5x^3 - x^2 + 1 + \frac{d}{dx} 5x^3 - x^2 + 1 \times 2x - 2$

$= 2(5x^3 - x^2 + 1) + (2x-2)(15x^2 - 2x)$

$= 10x^3 - 2x^2 + 2 + 30x^3 - 4x^2 - 30x^2 + 4x$

$= 40x^2 - 36x^2 + 4x + 2$