2.

(a) A : 1

B : 6

C : -15

(b) $54 - 30\ln(2)$

3. $\int 2x\sqrt{x+2} = 2\left(\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2}\right) + c$

$$\int_0^2 2x\sqrt{x+2} = 2\left[\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2}\right]_0^2$$

$$= 2\left(\frac{32}{15} + \frac{16\sqrt{2}}{15}\right)$$

$$= \frac{32}{15}\left(2+\sqrt{2}\right)$$

4. $\int_k^{3k} \frac{2}{(3x-k)}\,dx$ is independent of k

$= 2\int_k^{3k} \frac{1}{(3x-k)}$; let $3x-k=t$, $\frac{dt}{dx}=3$; $dx = \frac{dt}{3}$

t: $2k$ to $8k$.

$$= \frac{2}{3}\int_{2k}^{8k} \frac{1}{t}\,dt = \frac{2}{3}\left[\ln(t)\right]_{2k}^{8k}$$

$$= \frac{2}{3}\left[\ln 8k - 2k\right]$$

$= \frac{2}{3}\ln\left(\frac{8k}{2k}\right) = \frac{2}{3}\ln(4)$ which is independent of k

(b) $\displaystyle\int_k^{2k} \frac{2}{(2x-k)^2}$ is inversely proportional to k

$$2\int_k^{2k} \frac{1}{(2x-k)^2} \quad ; \quad t = 2x-k \, , \quad \frac{dx}{dt} = 2, \quad dx = \frac{dt}{2}$$

from k to $3k$

$$= \int_k^{3k} \frac{dt}{t^2} = \int_k^{3k} t^{-2}\, dt$$

$$= \left[t^{-1} \right]_k^{3k}$$

$$= \left[-\frac{1}{t} \right]_k^{3k} = -\frac{1}{3k} + \frac{1}{k} = \frac{2}{3k}$$

$$\int_k^{2k} \frac{2}{(2x-k)^2}\, dx = \frac{2}{3k}$$

Thus $\displaystyle\int_k^{2k} \frac{2}{(2x-k)^2}\, dx$ is inversely proportional to k

5.

$$y = xe^{-2x}$$

(a)

$$\frac{dy}{dx} = x \cdot -2e^{-2x} + e^{-2x}$$

$$= -2xe^{-2x} + e^{-2x}$$

$$-2xe^{-2x} + e^{-2x} = 0$$

$$-2x\cancel{e^{-2x}} = -\cancel{e^{-2x}}$$

$$-2x = -1$$

$$\boxed{a = \frac{1}{2}}$$

$$b = \frac{1}{2} e^{-2 \times 1/2} = \frac{1}{2} e^{-1} = \frac{1}{2e}$$

$$\boxed{b = \frac{1}{2e}}$$

(b) Area $R = \dfrac{-e + 3}{4e}$

(7) (a) $V = \frac{4}{3}\pi r^3$

$$\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = -k_1$$

$$\frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt} = -k_1$$

$$4\pi r^2 \frac{dr}{dt} = -k_1$$

$$\frac{dr}{dt} = \frac{-k_1}{4\pi r^2} \quad ; \quad \frac{k_1}{4\pi} = k$$

Thus $\frac{dr}{dt} = \frac{-k}{r^2}$

(b) Solution : $r(t) = \sqrt[3]{-3kt + c_1}$

$$r(0) = 40 = \sqrt[3]{c_1} \quad ; \quad c_1 = 64000$$

$$r(t) = \sqrt[3]{-11200t + 64000}$$

(c) $0 \le t \le \frac{40}{7}$

Scanned with CamScanner

(8)
$$\int_0^{\frac{\pi}{2}} 60 \sin t \ \cos^2 t \ dt$$

$$u = \cos t \ ; \ \frac{du}{dx} = -\sin x, \quad dx = \frac{du}{-\sin t}$$

$$-\sin 0 = 0, \ \sin\left(\frac{\pi}{2}\right) = 1$$

$$= -60 \int_0^1 \sin t \cdot \frac{du}{\sin t} \cdot u^2$$

$$= -60 \int_0^1 u^2 du \quad = -60 \left[\frac{u^3}{3}\right]_0^1$$

$$= -60 \left[\frac{1}{3}\right]$$

$$= 20$$