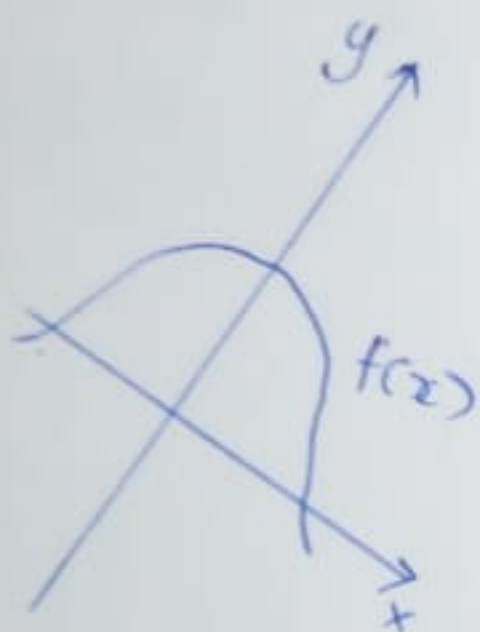


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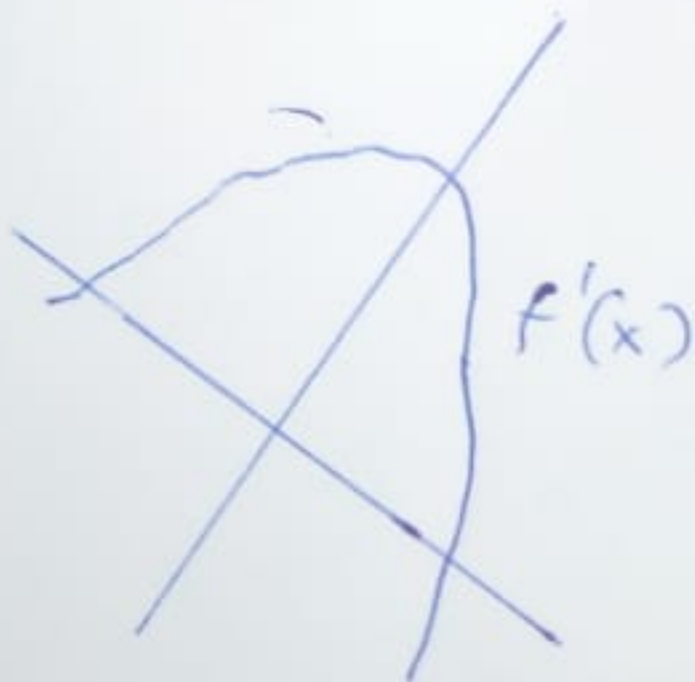


Q1

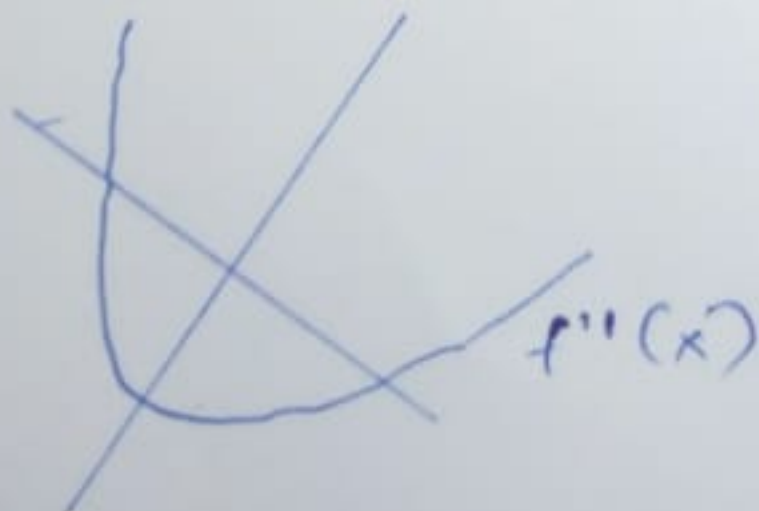
Q2



Q3



Q4



$$P(t) = P_0(1.02)^t$$

$$P_0 = 1$$

$$P(t) = (1.02)^t$$

$$P(t) = e^{\ln(1.02)t}$$

$$P'(t) = \ln(1.02) \times e^{\ln(1.02)t}$$

$$P'(t) = \ln(1.02) \times e$$

$P'(t)$ - rate of price increase (how fast prices are rising)

When $t = 10$

$$P'(t) = P'(10)$$

$$= \ln(1.02) \times e^{\ln(1.02)(10)}$$

$$= 0.0198 \times 1.12189942 = 0.024139292 \text{ cents}$$

converting to cents from dollars

$$\$ 0.024139292$$

$$= (0.024139292 \times 100) \text{ cents}.$$

$$= 2.4139292$$

$\therefore$ Prices are rising at $2.41^{\text{c.d.p.}}$ cents per year.

Question 8

$$f(x) = (x^2 - \sqrt{x}) 3^x$$

Product rule

$$\frac{d}{dx} f(x) = (x^2 - \sqrt{x}) \frac{d}{dx} 3^x + \frac{d}{dx} (x^2 - \sqrt{x}) 3^x$$

Derivative of 3^x

$$\text{Let } y = 3^x$$

$$\ln y = \ln 3^x = x \ln 3$$

$$\frac{d}{dx} \ln y = \frac{d}{dx} x \ln 3$$

$$= \ln 3$$

Using implicit differentiation w.r.t x

$$\frac{1}{y} y' = \ln 3$$

$$y' = y \ln 3$$

$$y' = 3^x \ln 3$$

$$\therefore \frac{d}{dx} f(x) = \cancel{x^2} (x^2 - \sqrt{x}) 3^x \ln 3 + \left(2x - \frac{1}{2} x^{-1/2} \right) 3^x$$

Question 4.

$$g(t) = \frac{4}{3+\sqrt{t}}$$

$$\frac{d}{dt} g(t) = \frac{d}{dt} 4(3+t^{1/2})^{-1}$$

$$= 4 \frac{d}{dt} (3+t^{1/2})^{-1}$$

Using chain rule

$$= 4(-1)(3+t^{1/2})^{-2} (\frac{1}{2} t^{-1/2})$$

$$= \frac{-4}{(3+\sqrt{t})^2} \cdot \frac{1}{2\sqrt{t}}$$

$$= \frac{-2}{\sqrt{t} (3+\sqrt{t})^2}$$

$$= \frac{-2}{\sqrt{t} (9+3\sqrt{t}+t)}$$

$$= \frac{-2}{9\sqrt{t}+3t+t^{3/2}}$$

$$\begin{aligned} \Rightarrow (3+\sqrt{t})^2 &= (3+\sqrt{t})(3+\sqrt{t}) \\ &= 9+3\sqrt{t}+3\sqrt{t}+t \\ &= 9+3\sqrt{t}+t \end{aligned}$$