Exam 3 Part 1 : Differentiation

Math 161 Section 03, Fall 2021

INSTRUCTIONS:

Complete 8 of the following problems (each completed problem is worth 8 points)

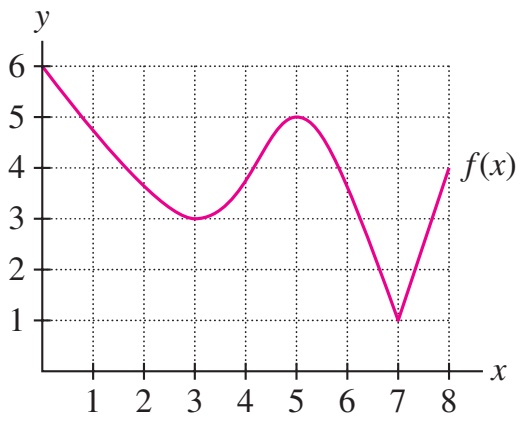
- You may use external sources to help (Textbook, Youtube, each other), but you MAY NOT use a service/person to give you solutions.
- Your work must be professionally presented :
  - Write your name on each sheet of paper that you turn in.
  - Do NOT include more than one problem on a single sheet of paper.
  - Your work must be INCREDIBLY neat (consider typing and/or rewriting each question). This include writing, in words, which steps you are taking prior doing the technical step.
  - Scratch work and/or messy work will not be graded.
  - Explanations must be thorough and complete.
- I should be able to read your solution from top to bottom (no arrows directing attention in other directions) and understand the problem, exactly what you've done as well as your understanding of the solution.

Problem 1. Find all the critical points for each function, f. Classify the critical points using the first derivative test and draw the sign chart for f'.

a) $f(x) = 2x^3 + 3x^2 - 12x$

b) $f(x) = (x^2 - 4)^2$

Problem 2. Answer the questions about the following figure



(a) How many critical points does $f(x)$ have on $[0, 8]$?

(b) What is the maximum value of $f(x)$ on [0, 8]?

(c) What are the local maximum values of $f(x)$?

(d) Find a closed interval on which both the minimum and maximum values of $f(x)$ occur at critical points.

Problem 3. The following will lead to you producing a graph of the following function

$$f(x) = x^3 - 12x^2 + 45x$$

a) Find the critical points and apply the Second Derivative Test (or state that it fails) of $f(x) = x^3 - 12x^2 + 45x$.

b) Determine the extreme values of $f(x) = x^3 - 12x^2 + 45x$ (This is the same function as part a.) on the interval [1,4].

c) Find the inflection points of $f(x) = x^3 - 12x^2 + 45x$ (Again, this is the same function as part a.)

d) On what intervals is $f(x)$ concave up? On what intervals is $f(x)$ concave down?

e) Finally consider $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ and use the previous steps to sketch a graph of

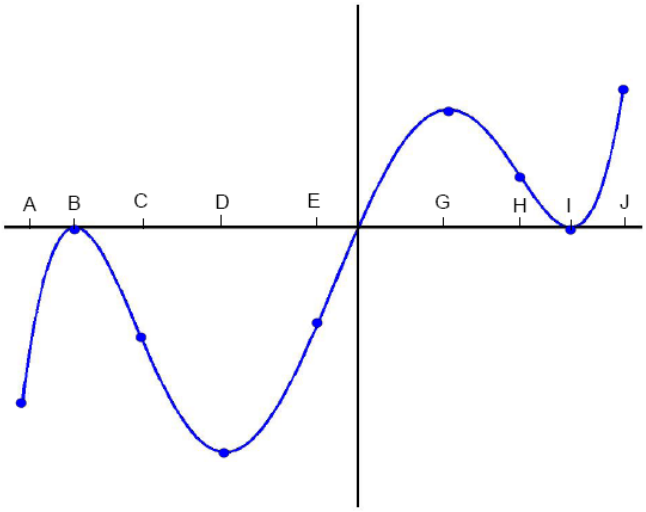
$$f(x) = x^3 - 12x^2 + 45x$$

Problem 4. Sketch a graph of the function $f(x) = \dfrac{2x^2 - 1}{x^2 + 1}$

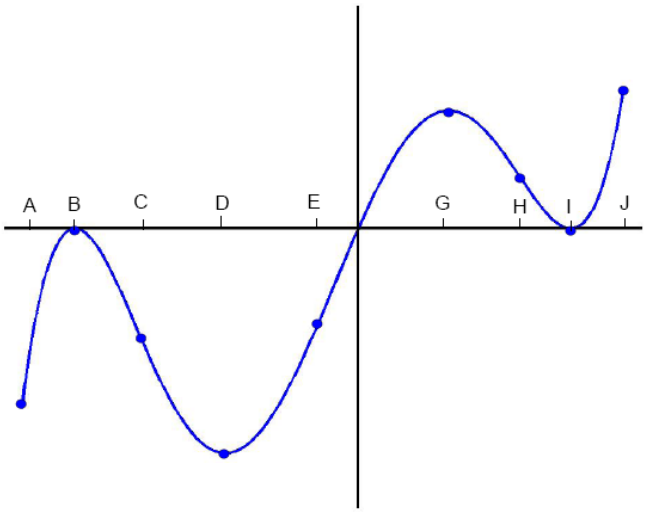
Indicate the asymptotes, local extrema, and points of inflection if they exist. Be sure to show all of your work and to neatly label each step.

Problem 5.

a) The following is the graph of $*f'(x)*$ (the derivative of f). Where do the **critical points** of $f(x)$ occur, and on which intervals is $f(x)$ increasing? **Explain your reasoning.**



b) The following is the graph of $*f'(x)*$ (the derivative of f). Where do the **points of inflection** of $f(x)$ occur, and on which intervals is $f(x)$ concave up? **Explain your reasoning.**



Problem 6. Sketch the graph of a function such that all of the following are satisfied

i) $f' < 0$ for $x < -2$

ii) $f' > 0$ for $x > -2$

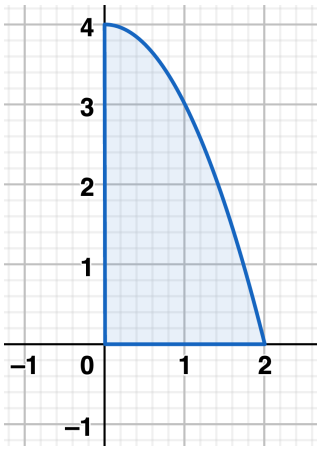
iii) $f'' > 0$ for $x < 0$

iv) $f'' < 0$ for $x > 0$

Problem 7. The function $f(x) = 3x^5 - 30x^4 + ax^3 + bx^2 + 60x + 1$ has inflection points at $x = 2$ and $x = 3$. Find a and b.

Problem 8. Find two positive real numbers such that the sum of the first number squared and the second number is 48 and their product is maximum.

Problem 9. Find the rectangle with the largest area that can be placed inside the region bounded between the curve $y = 4 - x^2$ and the horizontal axis $y = 0$.



Problem 10. For infants less than 9 months old, the relationship between the rate of growth, R (in pounds/month), and the present weight, W (in pounds), is approximated by

$$R = cW(21 - W)$$

for some constant c. At what weight is the growth rate the largest?

Problem 11. In a simple model of territory, a single animal defends a circular territory of radius x (miles. The following assumptions are made:

- The energy, L, spent per day in looking for food and defending the territory is directly proportional to the area of the region.
- The energy, G, gained per day is directly proportional to the radius, x.

a) Write an equation relating L to the radius, x. Use the fact that $L = 3000$ calories when $x = 1$ to determine the proportionality constant.

b) Write an equation relating G to the radius x. Use the fact that $G = 3500$ calories when $x = 1$ to determine the proportionality constant.

c) Find the territorial size that will result in the maximum benefit to the animal, that is, maximize the net energy, E where $E = G - L$.