Exam 3 Part 2 : Integration

MATH 161 SECTION 03, FALL 2021

INSTRUCTIONS:

Complete ONE of the following exercises.
(The rubric for scoring is below.)

- You may use external sources to help (Textbook, Youtube, each other), but you MAY NOT use a service/person to give you solutions.
- Your work must be professionally presented :
  - Write your name on each sheet of paper that you turn in.
  - Do NOT include more than one problem on a single sheet of paper.
  - Your work must be INCREDIBLY neat (consider typing your responses and/or rewriting each question). This include writing, in words, which steps you are taking prior doing the technical step.
  - Scratch work and/or Messy work will not be graded.
  - Explanations must be thorough and complete.
  - You must follow all rules of standard english spelling and grammar.
- I should be able to read your solution from top to bottom (no arrows directing attention in other directions) and understand the problem, exactly what you've done as well as your understanding of the solution.

Scoring Rubric

Item	Subitem	Total
Mathematics	Analytical	5
	Qualitative	5
	Numerical	5
	Visuals (Graphs/Tables)	5
Presentation	Format	5
	Professionalism	5
Writing	Exposition/Content	5
	Organization/Clarity	5
Total		**40**

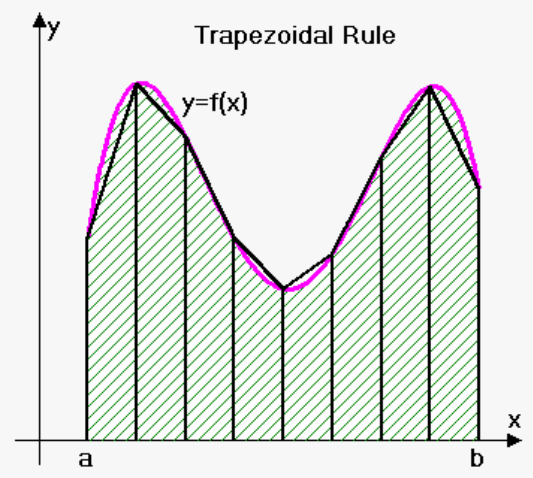
The Trapezoid Rule

In class, we hinted at using trapezoids, rather than rectangles to approximate the area under the curve of a function. In this exercise, you'll need to use the Trapezoid Rule to help you to better understand concepts in numerical integration.

To start :

- Watch the following video to illustrate the main idea of the Trapezoid Rule : `https://www.khanacademy.org/math/ap-calculus-ab/ab-integration-new/ab-6-2/v/trapezoidal-approximation-of-area-under-curve`
- Check your understanding of the Trapezoid Rule by going through the example : `https://www.khanacademy.org/math/ap-calculus-ab/ab-integration-new/ab-6-2/a/understanding-the-trapezoid-rule`

To give you another visual, below is a diagram showing what we're going for when approximating using trapezoids



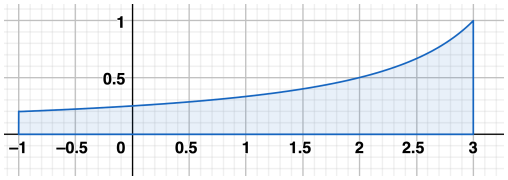
Using the same notation that we discussed in class, to approximate the area under the curve of $f(x)$ on the interval [a,b] using trapezoids, we compute :

$$T_N = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \cdots + 2f(x_{N-1}) + f(x_N)\right]$$

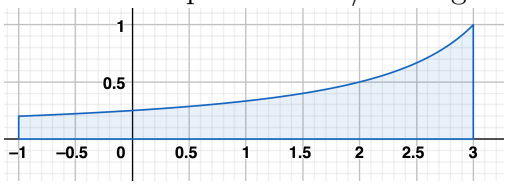
**** Please note that the coefficient of $f(x_0)$ and $f(x_N)$ is 1. While the coefficients of all the other $f(x_i)$ for $i = 1...N-1$ is 2.****

Problem 1. *[Math without Application]* Consider the function $\frac{1}{4-x}$. Use $n = 4$ intervals to approximate the area bounded between the curve and the horizontal axis over the interval [-1,3] in the following ways:

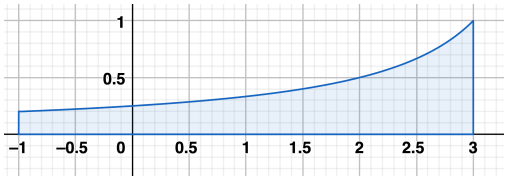
a) Using the rectangle method and the left end points of the intervals. On top of the given graph of the area, sketch a picture representing your approximation/rectangles.



b) Using the rectangle method and the right end points of the intervals. On top of the given graph of the area, sketch a picture representing your approximation/rectangles.



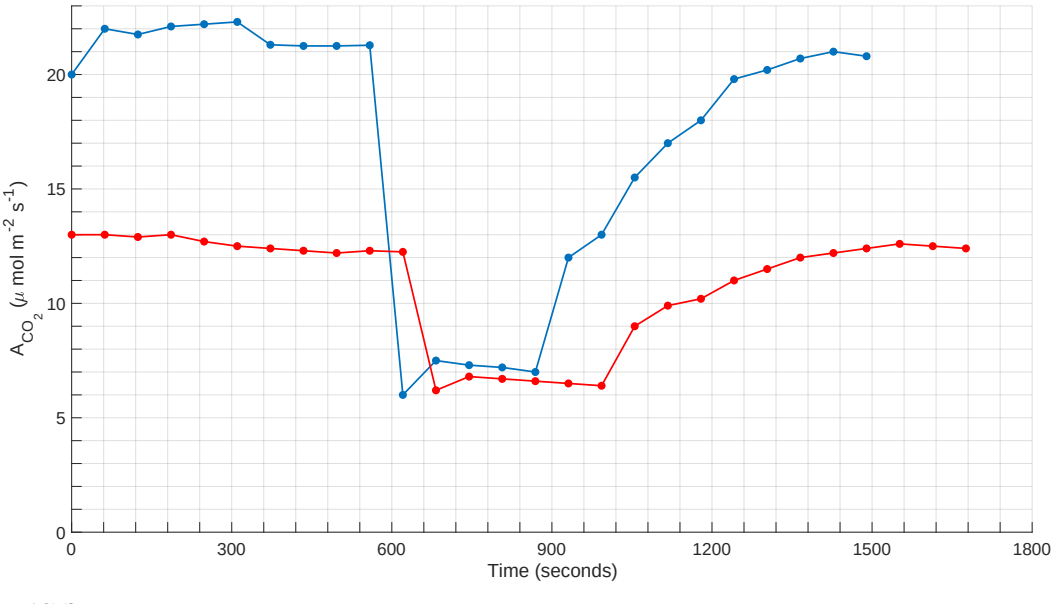
c) Using the trapezoidal method. On top of the given graph of the area, sketch a picture representing your approximation/trapezoids.



d) Given that the exact value of the area is $A \approx 1.6094379$, which of the methods above gives you the best approximation?

e) Write a summary discussion the three methods that we've used to approximate area (right, left, trapezoid). Include descriptions, definitions, the results of your example above. Describe which method you think is more accurate and detail why you think it is more accurate.

Problem 2. *[Biology/Chemistry]* Fay and Knapp measured the amount of CO_2 assimilated by the upper leaves of soybean plants over a 30-minute period when the plant was grown under normal and the under drought conditions. During the 30-minute experiment, the soybean plant is initially in full sunlight. After 8 to 10 minutes, the plant is shaded for approximately 8 to 10 minutes, after which it is returned to full sunlight. Below are the graphs showing the assimilation rate of CO_2 in μmol per m^2 of leaf area per second over the full experiment for the soybean plant under normal conditions (shown in blue) and drought conditions (shown in red). The assimilation rate was sampled roughly at 1-minute inter-



a) Use a combination of rectangles and trapezoids to estimate the area under each curve for the first 25 minutes of the experiment.

b) What does the total area represent?

c) When the areas under each curve are compared, what conclusions can be drawn from the two experiments?

Problem 3. *[Physics/Engineering/Computer Science]* You are stationed at a NASA terrestrial observation platform in the Rocky Mountains. Your station gathers data from space, including radio waves and other phenomena. A new solar phenomenon is creating a certain type of x-ray that your station is observing. You have two pieces of equipment that are capable of observing this phenomenon and measuring the intensity of these waves. The goal is to accurately measure this phenomenon over time and sum up the intensity of these waves over time to determine the cumulative exposure. This means integration.

The first piece of equipment costs over $100,000 per second to run, but generates a nearly true-to-life reading of the phenomenon over time, allowing very precise sampling over time and precise integration. The second piece of equipment costs virtually nothing to run, but only takes two samples per second. Due to budget constraints, we prefer the latter.

Note: The S.I. units of x-ray intensity are joules (J). The integration is over time, measured in seconds, so the result of these computations is joules-seconds (Js).

In order to test and calibrate these instruments, they are being run together to measure the phenomenon. Both machines are now properly calibrated, but you have been asked to consult on how to interpret the data from the second, cheaper machine.

Both systems have analyzed an eight-second sample of this phenomenon. According to the operators assigned to the more expensive system, the total ought to be $16.677Js$. Your goal is to determine how accurate, in this test case, certain types of area-approximation will be using the data. Our hope is to not use the first machine very often.

The data collected by your team are as follows:

Time (s)	Reading (J)
0.0	3.0000000000
0.5	2.9122081864
1.0	1.9193049302
1.5	1.9303242086
2.0	2.2033677913
2.5	1.3877110431
3.0	0.8094542399
3.5	2.1814496707
4.0	3.8332122053
4.5	3.0070391485
5.0	0.6962909762
5.5	0.2976127359
6.0	2.1237437902
6.5	3.2159820066
7.0	2.4428780954
7.5	1.7769832000
8.0	2.1362756906

Using this data, estimate the intensity using all the rules we have talked about so far: (a) left-endpoint rectangles, (b) right-endpoint rectangles, and (c) trapezoids. I ***HIGHLY*** suggest that you use a technical tool to help you with your calculation (Microsoft Excel, Matlab, Python, etc) as this will make the calculations automated and much easier to repeat. Assess which of these methods is more accurate. Consider that we may want accuracy up to $0.01Js$ relative to the true value.

Prepare your report for me, Dr. Wilson, your NASA supervisor. Do your best to answer the question thoroughly and completely. Please include all your work, justify your calculations, and explain your reasoning.