QUAN 203 - 2021

Tutorial 9: try, scan and upload these exercises to Blackboard. Those submitted by the notified deadline will be scored and contribute towards the Tutorial Assignment component of your final mark. Video answers will be provided after the due date.

1. Show that the maximiser of $L(\theta)$ is also the maximiser of $\ln L(\theta)$.

2. Let X have density function proportional to $x^p(1-x)$, and let $X_1, \ldots, X_n$ be a random sample (iid) from this random variable.

   (a) Show that the constant of proportionality is $(p+1)(p+2)$

   (b) Show that $E(X) = \dfrac{p+1}{p+3}$ and use this to derive the method of moments estimator of p.

   (c) Show that the log-likelihood is $\ln L(p) = n\ln(p+1) + n\ln(p+2) + p\sum \ln x_i + \sum \ln(1-x_i)$ and hence show that the maximum likelihood estimator of p solves a quadratic equation with coefficients functions of $\frac{1}{n}\sum \ln X_i$.

3. Suppose $X_1, \ldots, X_n$ are iid random variables each with probability density function

$$f(x) = \theta x^{\theta-1} \qquad 0 < x < 1, \theta > 0.$$

   (a) Show that $\ln L(\theta) = n\ln\theta + (\theta-1)\sum \ln x_i$, and hence find the maximum likelihood estimator for θ.

   (b) Show that $E(X) = \dfrac{\theta}{\theta+1}$, and hence find the method of moments estimator for θ.

   (c) Are the ML and MM estimators the same?

4. Suppose $X_1, \ldots, X_n$ are iid gamma random variables with probability density function

$$f(x) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x} \qquad x > 0, \lambda > 0, r > 0.$$

   (a) Find the maximum likelihood estimator for λ and show that the MLE for r solves the equation

$$\ln r - \frac{\Gamma(r)'}{\Gamma(r)} = \ln \bar{X} - \frac{1}{n}\sum_{i=1}^{n} \ln x_i$$

   where $\Gamma(r)' = \frac{d}{dr}\Gamma(r)$.

   (b) Noting that the gamma random variable has mgf

$$M(t) = \left(\frac{\lambda}{\lambda - t}\right)^r$$

   derive the mean and variance of X. Hence, derive the method of moments estimators for r and λ. Will these equal the MLEs?