

Please show **all** your work! Answers without supporting work will not be given credit.
Write answers in spaces provided.

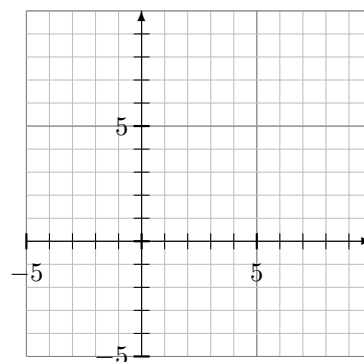
Name: _____

1. Find the maximum of the function $f = 9x + 10y$ subject to

(10 pts)

$$\begin{aligned}x + 2y &\leq 8 \\ 3x + 2y &\leq 12 \\ x &\geq 0 \\ y &\geq 0\end{aligned}$$

- (a) Sketch the feasible region



- (b) List the corners of the feasible region

Answer: _____

- (c) Find the maximum of the function

Answer: _____

2. Set up a system of inequalities and the objective function for the following optimization problem. Then write the simplex matrix for the problem.

A company manufactures backyard swings of two different sizes. The larger set requires 5 hours of labor to complete, the smaller set requires 2 hours. There are 700 hours of labor available each week. The packaging department can package at most 185 swings each week. If the profit is \$100 on each larger swing and \$50 on each smaller swing set, how many of each type should be produced to yield the maximum profit?

Let x be the number of larger swings and y be the number of smaller swings.

The inequalities:

(5 pts)

The objective function:

(3 pts)

The simplex matrix:

(7 pts)

3. Find the derivatives of the following functions (do not simplify):

(a) $n(x) = 5x^4 - 2x^2 + 3x - \frac{1}{x^2} + \sqrt{x} + 7$ (5 pts)

Answer: _____

(b) $v(x) = \frac{x^2 + 1}{2x - 1}$ (5 pts)

Answer: _____

(c) $g(x) = \sqrt{x^2 + 3x + 2}$ (5 pts)

Answer: _____

(d) $h(x) = e^{3x^2-5}$

(5 pts)

Answer:_____

(e) $r(t) = \ln(2t^3 - 5t)$

(5 pts)

Answer:_____

(f) $w(x) = x^2 \ln x$

(5 pts)

Answer:_____

4. Find the points of relative maxima and minima of the function

$$y = \frac{x^3(x-5)^2}{27}, \text{ with } y' = \frac{5x^2(x-3)(x-5)}{27}$$

- (a) Find the critical points.

(5 pts)

Answer: _____

- (b) Make a sign-chart (sign diagram) for y'

(5 pts)

- (c) Find the maxima and minima.

(5 pts)

Maxima: _____

Minima: _____

Some Useful Formulas

Quadratic Functions and Equations

- Standard Form: $f(x) = ax^2 + bx + c$
- Vertex Form: $f(x) = a(x - h)^2 + k$
- Vertex: $(h, k) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
- x -intercepts (Quadratic Formula): $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Matrices

- Inverse of a 2×2 matrix: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Properties of Exponents

- $a^{x+y} = a^x a^y$
- $a^{x-y} = \frac{a^x}{a^y}$
- $a^{xy} = (a^x)^y$

Properties of Logarithms

- $\log_a(xy) = \log_a(x) + \log_a(y)$
- $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$
- $\log_a(x^r) = r \log_a(x)$
- $\log_a(1) = 0$ and $\log_a(a) = 1$

Financial Formulas

- **Simple interest:** $I = Prt$ where I is the interest, P is the principal, r is the annual interest rate and t is the time in years.
- **Future value of investment or loan:** $S = P + I$.
- **Compound interest:** $S = P(1+i)^n$ where S is the future value of investment or loan, i is the interest rate per compounding period, and n is the number of compounding periods.
- **Continuously compounded interest:** $S = Pe^{rt}$ where p , r and t as as described above.
- **Future value of an ordinary annuity:** $S = R \cdot \left[\frac{(1+i)^n - 1}{i} \right]$.
- **Future value of an annuity due:** $S = R \cdot \left[\frac{(1+i)^n - 1}{i} \right] (1+i)$.
- **Present value of an ordinary annuity:** $A = R \cdot \left[\frac{1 - (1+i)^{-n}}{i} \right]$.
- **Present value of an annuity due:** $A = R \cdot \left[\frac{1 - (1+i)^{-n}}{i} \right] (1+i)$.

- **Present value of a deferred annuity:** $A = R \cdot \left[\frac{1 - (1+i)^{-n}}{i} \right] (1+i)^{-k}$ where the annuity is deferred for k periods.

Derivative formulas

- **Constant rule:** $(c)' = 0$.
- **Power rule:** $(x^n)' = nx^{n-1}$.
- **Coefficient rule:** $(cf)' = c(f)'$.
- **Sum and difference rule:** $(f \pm g)' = f' \pm g'$.
- **Product rule:** $(f \cdot g)' = f' \cdot g + f \cdot g'$.
- **Quotient rule:** $\left(\frac{f}{g} \right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$.
- **Chain rule:** $(f(g(x)))' = f'(g(x)) \cdot g'(x)$.
- **Exponential function:** $(e^x)' = e^x$.
- **Logarithmic function:** $(\ln x)' = \frac{1}{x}$.