(5) Section 2.1

11. $f(x) = 2x+3$: Double, then add 3.

15. Matching Diagram of $f x = \sqrt{x-1}$

x	$f(x)$
1	0
2	1
5	2
10	3
17	4

17. Evaluating functions.

$f(x) = x^2-6$: $f(-3), f(3), f(0), f(1/2)$

soln.

$f(-3) = (-3)^2 - 6 = 3.$

$f(3) = (3)^2 - 6 = 3.$

$f(0) = (0)^2 - 6 = -6$

$f(1/2) = \left(\frac{1}{2}\right)^2 - 6 = 5\frac{3}{4}$ or 5.75.

(23) $f(x) = x^2+2x$. $f(0), f(3), f(-3), f(a), f(-x)\ f\left(\frac{1}{a}\right)$

soln.

$f(0) = (0)^2 + 2(0) = 0.$

$f(3) = (3)^2 + 2(3) = 9+6 = 15.$

$f(-3) = (-3)^2 + 2(-3) = 9-6 = 3$

$f(a) = (a)^2 + 2(a) = a^2+2a.$

$f(-x) = (-x)^2 + 2(-x) = x^2-2x.$

(27) $K(x) = -x^2 - 2x + 3$. $\{K(0), K(2), K(-2), K(\sqrt{2}), K(a+2), K(-x), K(x^2)\}$

$K(0) = -(0)^2 - 2(0) + 3 = 3$

$K(2) = -(2)^2 - 2(2) + 3 = -4 - 4 + 3 = -5$

$K(-2) = -(-2)^2 - 2(-2) + 3 = -4 + 4 + 3 = 3$

$K(\sqrt{2}) = -(\sqrt{2})^2 - 2(\sqrt{2}) + 3 = -2 - 2\sqrt{2} + 3 = 1 - 2\sqrt{2}.$

$K(a+2) = -(a+2)^2 - 2(a+2) + 3$
$= -(a^2+4a+4) - 2a - 4 + 3$
$= -a^2 - 4a - 4 - 2a - 4 + 3.$
$= a^2 - 6a - 5.$

$K(-x) = -(-x)^2 - 2(-x) + 3.$
$= -x^2 + 2x + 3.$

$K(x^2) = -(x^2)^2 - 2(x^2) + 3$
$= -x^4 - 2x^2 + 3.$

Piecewise Defined functions.

31. $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0. \end{cases}$ $\{f(-2), f(-1), f(0), f(1), f(2)\}$

soln.

$f(-2) = (-2)^2 = 4$
$f(-1) = (-1)^2 = 1$
$f(0) = 0 + 1 = 1$
$f(1) = 1 + 1 = 2.$
$f(2) = 2 + 1 = 3.$

Evaluating functions.

35 $f(x) = x^2 + 1$: $f(x+2)$, $f(x) + f(2)$

$f(x+2) = (x+2)^2 + 1 = (x^2 + 4x + 4) + 1$ $\cancel{x}$
$= x^2 + 4x + 5.$

$f(x) + f(2) = (x^2+1) + (2^2+1)$
$= x^2 + 1 + 5$
$= x^2 + 6.$

Net change.

39. $f(x) = 3x - 2$; from 1 to 5.
$= f(5) - f(1)$
$= \{3(5)-2\} - \{3(1)-2\}$
$= (15-2) - (3-2)$
$= 13 - 1 = 12$

(4) 3

Difference quotient

43 $f(a) = 5-2x$

$f(a) = 5-2a$, $f(a+h) = 5-2(a+h) = 5-2a-2h$.

$\therefore \dfrac{f(a+h)-f(a)}{h} = \dfrac{5-2a-2h-5+2a}{h} = \dfrac{-2h}{h} = -2$

47 $f(x) = \dfrac{x}{x+1}$

$f(a) = \dfrac{a}{a+1}$, $f(a+h) = \dfrac{a+h}{a+h+1}$

$\dfrac{f(a+h) - f(a)}{h} = \dfrac{\frac{a+h}{a+h+1} - \frac{a}{a+1}}{h} = \dfrac{\frac{(a+h)(a+1) - a(a+h+1)}{(a+h+1)(a+1)}}{h}$

$= \dfrac{\frac{a^2+a+ah+h-a^2-ah-a}{(a+h+1)(a+1)}}{h} = \dfrac{\frac{h}{(a+h+1)(a+1)}}{h}$

$= \dfrac{1}{(x+1)^2}$

Domain and range

51. $f(x) = 3x$

Domain : $x \in \mathbb{R}$

Range: $f(x) \in \mathbb{R}$

Domain.

55 $f(x) = \dfrac{1}{x-3}$ Domain: $x \neq 3$.

59. $f(t) = \sqrt{t+1}$ Domain $t \geq -1$

63 $f(x) = \sqrt{1-2x}$ Domain: $x \leq \frac{1}{2}$.

67 $g(x) = \sqrt[4]{x^2-6x}$ Domain $x \in (-\infty, 0] \cup [6, +\infty)$

71 $f(x) = \frac{(x+1)^2}{\sqrt{2x-1}}$ Domain: $x \in (\frac{1}{2}, +\infty)$

Section 2.3.

Values of a Function.

7. (a) $h(-2) = 1$, $h(0) = -1$, $h(3) = 4$.

b) Domain: $-3 \le x \le 4$

Range: $-1 \le h(x) \le 4$

(C) $x = -3, 2, 4$

d) $x = -3, -2, -1, 0, 1, 2, 4$.

e) $h(-3) = 3$, $h(3) = 4$.

$h(3) - h(-3) = 4 - 3 = \underline{\underline{1}}$

9. (a) $f(0) = 3$ $g(0) = 0.5$

$\therefore f(0) > g(0)$ ie, $f(0)$ is larger.

(b) $f(-3) = -1$ $g(-3) = 2$.

$\therefore g(-3) > f(-3)$ ie $g(-3)$ is larger.

c) where $f(x) = g(x)$ is: $x = -2, 2$.

d) where $f(x) \le g(x)$ is: $x = \cancel{-2, -1, 0, 1, 2}$ $-4, -3, -2, 2, 3$ or $\cancel{-2 \le x}$

$(-4 \le x \le -2) \cup (2 \le x \le 3)$

e) where $f(x) > g(x)$ is: $x = -1, 0, 1$ or $(-1 \le x \le 1)$

2. Increasing and decreasing.

31. (a) Domain: $(-1 \le x \le 4)$
Range: $(-1 \le f(x) \le 3)$
(b) Intervals of increasing: $(-1 \le x \le 1) \cup (2 < x < 4)$
Interval of decreasing: $(1 < x < 2)$

33. (a) Domain: $(-3 \le x \le 3)$
Range: $(2 \le x \le 4)$

b) Intervals of increasing: $(2 < x < -1) \cup (1 < x < 2]$ or ~~(2, 1), (1, 2)~~
Intervals of decreasing: $(-3 \le x < 2) \cup (-1 < x < 1) \cup (2 < x \le 3$
~~or (-3, 2), (-1, 1), (2, 3)~~

<u>Local maximum and minimum values.</u>

43. (a) local maximum: $x = 2$.
local minimum: $x = -2, 2$.
(b) Intervals of Increasing: $(2, 0)$ and $(2, +\infty)$
Intervals of decreasing: $(-\infty, -2)$ and $(0, 2)$

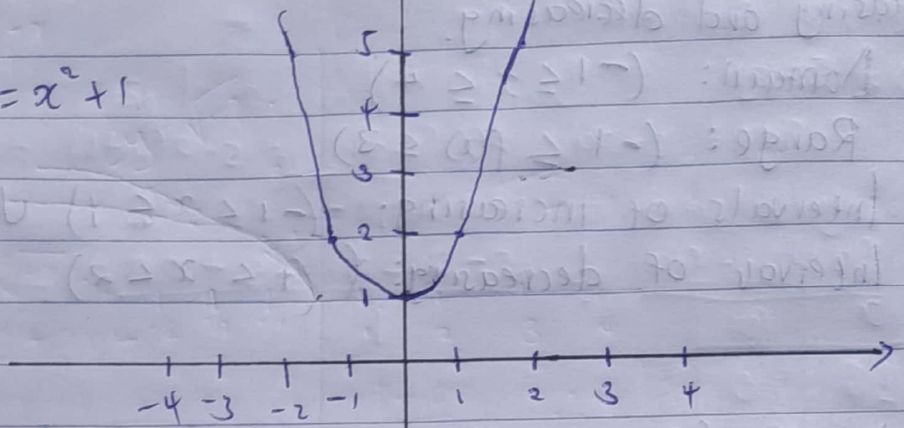
45. a) local maximum: $x = 0, 3$
local Minimum: $x = -2, 1$
b) Intervals of Increasing: ~~x~~ = $(-2, 0)$ and $(1, 3)$
b) Intervals of decreasing: $(-\infty, -2), (0, 1)\ (3, +\infty)$

Section 2.6.

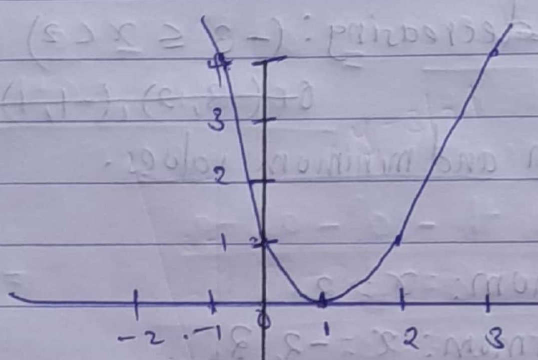
23. Graphing transformations.

(c)

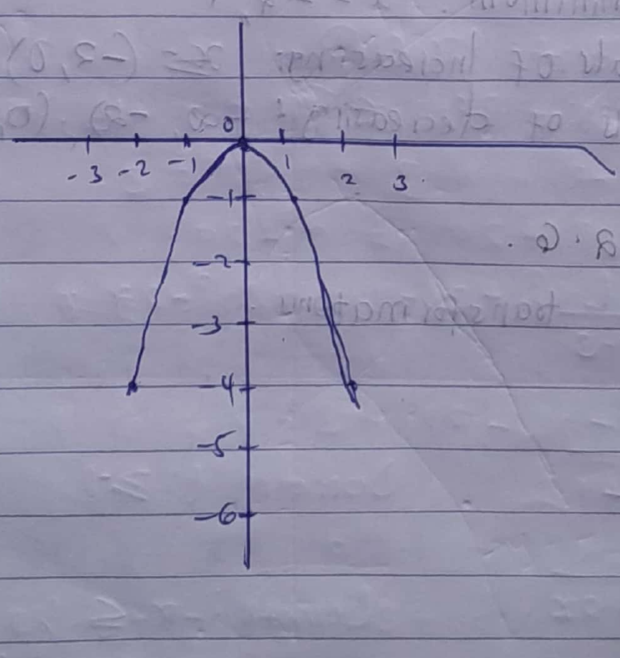
(a) $y = g(x) = x^2 + 1$



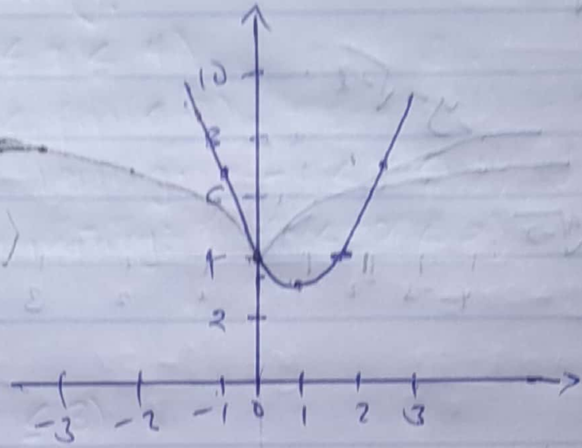
(b) $g(x) = (x-1)^2$



(c) $g(x) = -x^2$



(d) $g(x) = (x-1)^2 + 3$.

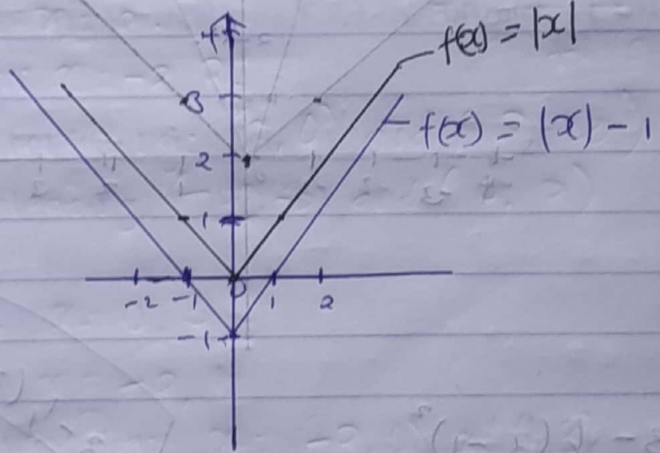


Identify transformation:

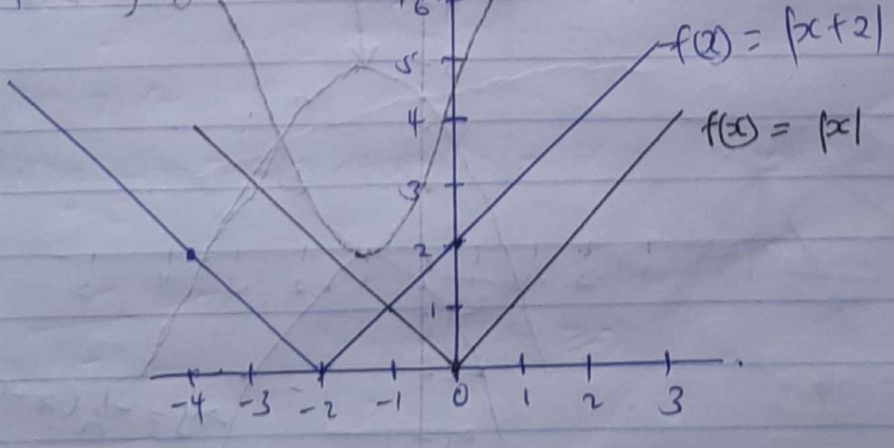
25. $y = |x| - 1$ Correct answer: Graph 1

~~34~~. 3 from (29-52): standard ~~fun~~ function (in black) transformation: (in blue)

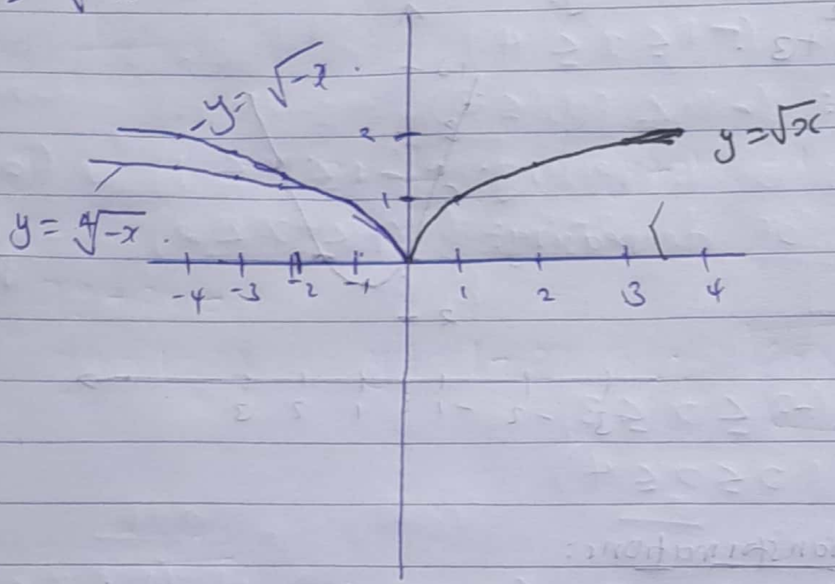
31 $f(x) = |x| - 1$



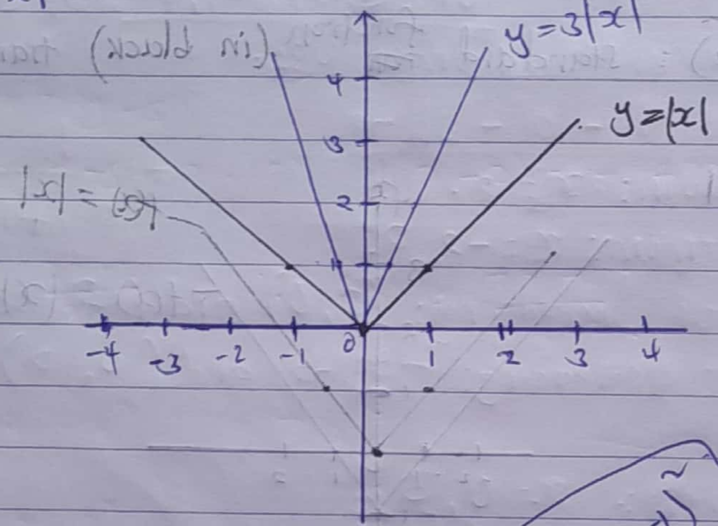
35. $f(x) = |x + 2|$



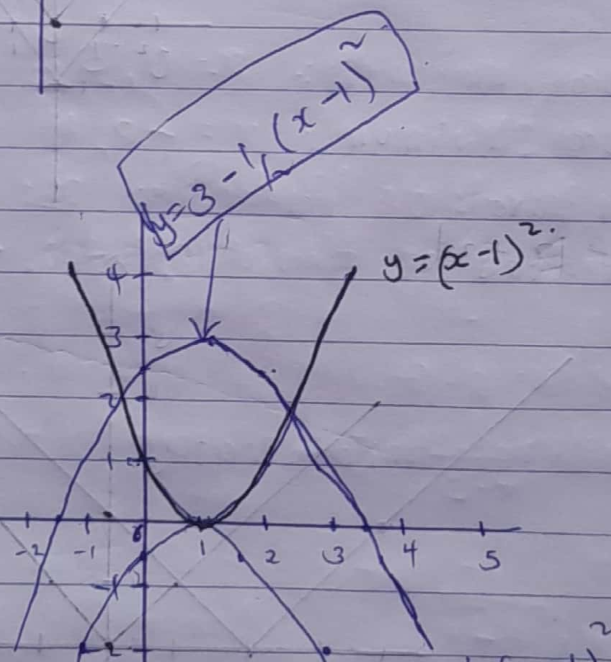
(39) $y = \sqrt[4]{-x}$



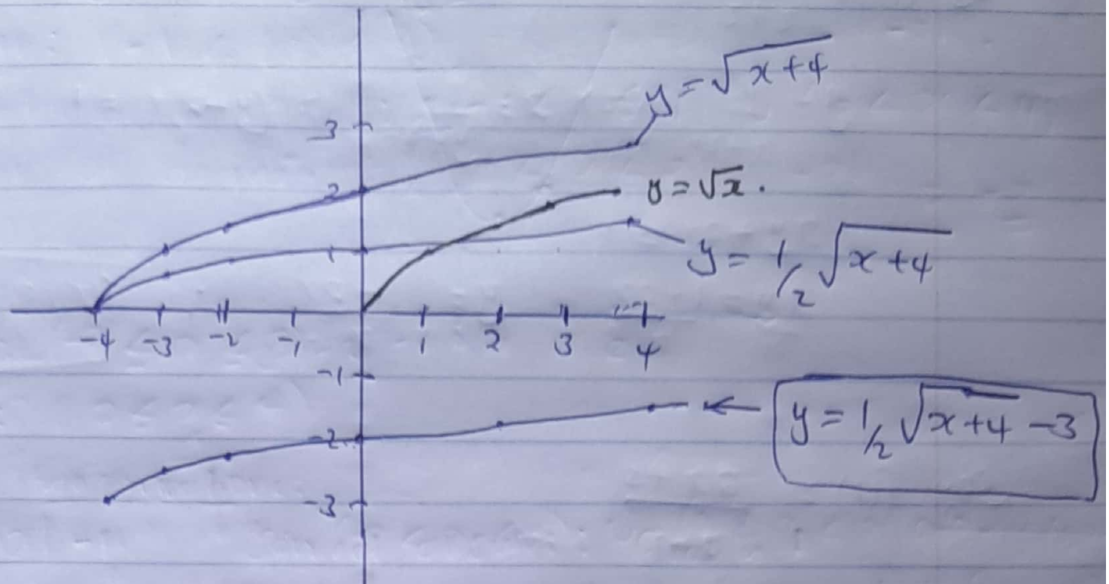
43. $y = 3|x|$



(47) $y = 3 - \frac{1}{2}(x-1)^2$



51. $y = \frac{1}{2}\sqrt{x+4} - 3$.



Finding Equations for transformations.

55. $y = \sqrt{x+2}$.

59. $fx = \sqrt[4]{x}$: Reflect in y-axis and shift upward 1 unit.

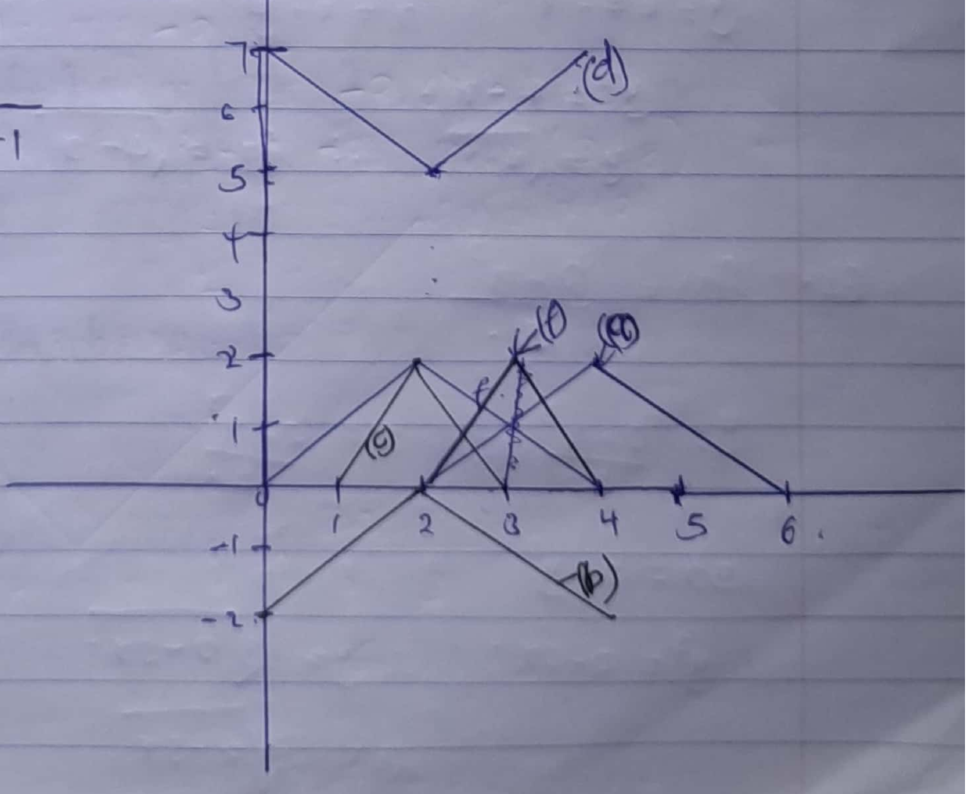
$$y = \sqrt[4]{-x} + 1$$

Finding formulas for transformation

63. $g(x) = (x-1)^2$.

(67) $g(x) = -\sqrt{x+1}$

70. (a) $y = f(x-2)$
(b) $y = f(x) - 2$.
c) $y = 2(f(x))$
d) $y = -f(x) + 3$.
e) $y = f(-x)$
f) $y = \frac{1}{2}(x-1)$



Section 2.8

Inverse Function property.

37. $f(x) = x-6 \quad g(x) = x+6.$

$g(f x) = g(x-6) = (x-6)+6 = x.$

$f(g(x)) = f(x+6) = (x+6)-6 = x$

Hence $f(g(x)) = g(f(x))$

41. $f(x) = \frac{1}{x}. \quad g(x) = \frac{1}{x}.$

$f((g x)) = f\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)} = x.$

$g(f(x)) = g\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)} = x.$

hence $f(g(x)) = g(f(x))$

45. $f(x) = \frac{1}{x-1}, \quad g(x) = \frac{1}{x}+1$

$f(g(x)) = f\left(\frac{1}{x}+1\right) = \frac{1}{\left(\frac{1}{x}+1\right)-1} = \frac{1}{\frac{1}{x}} = x.$

$g(f(x)) = g\left(\frac{1}{x-1}\right) = \frac{1}{\frac{1}{x-1}}+1 = x-1+1 = x.$

hence $f(g(x)) = g(f(x))$

Find Inverse of the function.

49. $f(x) = 3x+5.$

$y = 3x+5.$

$y-5 = 3x$

$x = \frac{y-5}{3} \qquad \therefore y = \frac{x-5}{3}$

$f(x) = \frac{x-5}{3}.$

53. $f(x) = \frac{1}{x+2}$

$y = \frac{1}{x+2} \Rightarrow x = \frac{1}{y+2} \Rightarrow (y+2)x = 1$

$xy + 2x = 1$

$\frac{xy}{x} = \frac{1-2x}{x} \quad y = \frac{1-2x}{x} \quad \therefore \boxed{f(x) = \frac{1-2x}{x}}$

57. $f(x) = \frac{2x+5}{x-7}$

$y = \frac{2x+5}{x-7} \quad x = \frac{2y+5}{y-7} \quad x(y-7) = 2y+5$

$xy - 7x = 2y - 5$

$xy - 2y = -5 + 7x \Rightarrow y(x-2) = -5 + 7x$

$y = \frac{7x-5}{x-2} \quad \therefore f(x) = \frac{7x-5}{x-2}$

61. $f(x) = 4 - x^2, \; x \geq 0$

$y = 4 - x^2 \quad x = 4 - y^2$

$y^2 = 4 - x$

$y = \sqrt{4-x} \quad \therefore f(x) = \sqrt{4-x}$ for $x \leq 4$.

65. $f(x) = \frac{2-x^3}{5}$

$y = \frac{2-x^3}{5} \Rightarrow x = \frac{2-y^3}{5}$

$5x = 2 - y^3$

$y^3 = 2 - 5x \quad \therefore y = \sqrt[3]{2-5x}$

$f(x) = \sqrt[3]{2-5x}$

69. $f(x) = 2 + \sqrt[3]{x}$.

$y = 2 + \sqrt[3]{x}$.

$x = 2 + 3\sqrt{y}$.

$(x-2) = \sqrt[3]{y}$

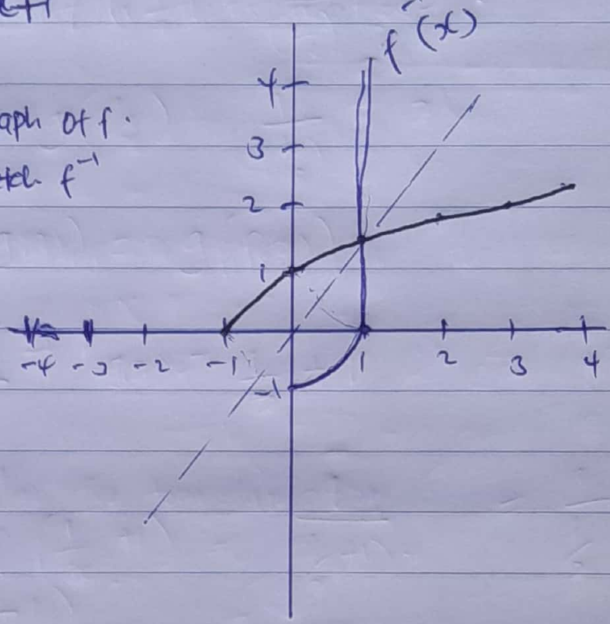
$y = (x-2)^3$ $\therefore$ $f(x) = (x-2)^3$

Graph of Inverse function.

73. $f(x) = \sqrt{x+1}$

(a) sketch the graph of f.

b) use graph to sketch f^{-1}



c) Find f^{-1}

$f(x) = \sqrt{x+1}$

$y = \sqrt{x+1}$

$x = \sqrt{y+1}$

$x^2 = y+1$

$y = x^2 - 1$

$f(x) = x^2 - 1$ for $x \geq 0$