

2. In our first week we are learning about limits. Let's consider the function $f(x) = \frac{x^2 - 1}{x + 1}$.

If I want to evaluate $f(-1)$ what value do I get? If I want to evaluate $\lim_{x \rightarrow -1} f(x)$ what value do I get? Why do I get different answers?

Soln

First, let's start by evaluating $f(-1)$ of the function $f(x) = \frac{x^2 - 1}{x + 1}$.

In this case, we put (-1) where x appears in the function $f(x) = \frac{x^2 - 1}{x + 1}$.

Putting -1 in the function we get:

$$\frac{(-1)^2 - 1}{-1 + 1} = \frac{0}{0} = 0.$$

The value of the denominator is zero and the value of the numerator is also zero. Therefore, the overall answer is zero.

This means that the curve $\frac{x^2 - 1}{x + 1}$ is 0,

where the value of x is -1 . It cuts the ~~y-axis~~ x -axis at the point where $x = -1$ i.e. $(-1, 0)$.

Next, let's evaluate the $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1}$.

Since the numerator is a difference of two squares, it can be re-written as:

$$x^2 - 1 = (x + 1)(x - 1).$$

Therefore;

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1} = \frac{(x + 1)(x - 1)}{x + 1}$$

~~$x(x+1)$~~ appears both in the numerator and denominator.
They can cancel out.

We are left with:

$$\lim_{x \rightarrow -1} (x - 1)$$

Now we replace x with -1 to get:

$$-1 - 1 = -2$$

Therefore,

$$\lim_{x \rightarrow -1} \frac{x^2 - 1}{x + 1} = -2$$

The two answers are different because ^{the first} one describes the value of the function at the point where $x = -1$.
Whereas $\lim_{x \rightarrow -1}$ means that: "As you approach -1 along the x -axis, on the function $\frac{x^2 - 1}{x + 1}$, the value of y is getting closer to -2 ."