

Linear equations and linear systems

1. The following adjacency matrix represents direct flights between four cities.

a) Draw a graph corresponding to the matrix.

b) Find the sum of the entries on the main diagonal of matrix A_G^2 . What does this number represent?

$$A_G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

2. i) Find the general solution (the set of all solutions) for each of the following linear equations.

a) $\frac{3x}{4} = 6$

b) $x + 3y = 5$

c) $5 - 2x + 3y + z = 0$

ii) For each one of equations (b) and (c) above, give any 2 particular solutions by assigning arbitrary values to the parameter(s).

3. For each linear system, state the number of equations and the number of unknowns (variables).

a) $\begin{cases} x + y = 0 \\ 2x - 4y = 5 \end{cases}$

b) $\begin{cases} 2x + 3y = 5 \\ 5z = 0 \\ y + 2z = -1 \end{cases}$

c) $\begin{cases} x_1 + x_2 = 5 \\ x_2 - x_3 = 0 \\ x_3 + x_4 = 2 \end{cases}$

ii) Write the augmented matrix of each of the above systems.

4. For each of the following matrices of a linear system, state whether it is in reduced row-echelon form (rref), row-echelon form (ref), or neither. If it is neither, give a reason why (i.e. which rule is not satisfied).

a) $\left[\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{array} \right]$

b) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array} \right]$

c) $\left[\begin{array}{cccc|c} 1 & 0 & 7 & -5 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$

d) $\left[\begin{array}{cccc|c} 1 & 0 & 5 & 0 & 2 \\ 0 & 1 & 2 & 0 & 4 \\ 0 & 0 & 0 & 1 & 7 \end{array} \right]$

$$\text{e) } \left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 1 & 3 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{f) } \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\text{g) } \left[\begin{array}{cccc|c} 1 & 2 & -5 & 4 & 2 \\ 0 & 0 & 1 & 1 & 3 \end{array} \right]$$

5. The augmented matrix of a linear system has been reduced to the following REF or RREF through row reduction. Identify the free and the leading variables. Write the general solution of the system in vector form.

$$\text{a) } \left[\begin{array}{cccccc|c} 1 & 2 & 0 & 3 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{b) } \left[\begin{array}{ccccc|c} 1 & 2 & 1 & 3 & 1 & 1 \\ 0 & 1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{c) } \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{array} \right]$$

6. Use row reduction (Gauss-Jordan elimination or Gaussian elimination) to solve the following linear systems. Write the general solution in vector form.

$$\text{a) } \begin{array}{rcl} & y + z & = 1 \\ x + y & & = 1 \\ 2x - y + z & & = 1 \end{array}$$

$$\text{b) } \begin{array}{rcl} & y - 3z & = -5 \\ 2x - y - 3z & & = -3 \\ x - y & & = 1 \end{array}$$

$$\text{c) } \begin{array}{rcl} 3x + 2y & = & 4 \\ -6x - 4y & = & 0 \end{array}$$

$$\text{d) } \begin{array}{rcl} 2x + y + 3z & = & 0 \\ x + 2y & = & 0 \\ y + z & = & 0 \end{array}$$

$$\text{e) } \begin{array}{rcl} 3x_1 + x_2 + x_3 + x_4 & = & 0 \\ 5x_1 - x_2 + x_3 - x_4 & = & 0 \end{array}$$