

F20 MAT 101S Assignment E

Factoring Polynomial Functions (Content from Unit 4 in MAT 101)

Zeros	Factors	How do zeros & factors relate?
• x-values that make the function equal 0 • x-values of the x-intercepts	• two expressions that multiply together • any polynomial that divides evenly into $f(x)$ • of the form $(x - \text{"zero of polynomial"})$ • of the form $(x - c)$	• If 2 is a zero, then $(x - 2)$ is a factor. • If -1 is a zero, then $(x + 1)$ is a factor.

Example - Factor: $f(x) = 2x^3 - 5x^2 - x + 6$ ***Notice this is in function notation.**

Step 1: Find all possible zeros of the function. To narrow our search for zeros, we can use information from the function to figure out numbers that could possibly be a zero.

P is the **constant** in the function. In our case, P is 6.

Q is the **leading coefficient**. In our case, Q is 2.

List all factors of P: 1, 2, 3, 6. (What multiplied by what makes 6? Use all of those numbers.)

List all factors of Q: 1, 2. (What multiplied by what makes 2? Use all of those numbers.)

Use $\frac{P}{Q}$ to find all possible zeros. Remember the zero could possibly be negative or positive.

$$\frac{P}{Q} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2} = \pm \frac{1}{1}, \pm \frac{2}{1}, \pm \frac{3}{1}, \pm \frac{6}{1}, \pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{3}{2}, \pm \frac{6}{2}$$

$$\text{Simplify} = \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm 3 \quad \text{Omit Duplicates} = \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm 2, \pm 3, \pm 6$$

So, this list gives us a list of all POSSIBLE zeros of the function above.

How do we find the ACTUAL zeros?

Step 2: Use synthetic division to find a number that gives a ZERO remainder.

Let's start with one of the possible zeros in the list. It doesn't matter which one you pick to try first.

Use the process for synthetic division previously taught.

Let's try 1 as a possible zero.

$$\begin{array}{r|rrrr} 1 & 2 & -5 & -1 & 6 \\ & & 2 & -3 & -4 \\ \hline & 2 & -3 & -4 & 2 \end{array}$$

* So, our remainder was 2, which is not zero.

Therefore, 1 is not a zero of $f(x)$.

Let's try -1 as a possible zero.

$$\begin{array}{r|rrrr} -1 & 2 & -5 & -1 & 6 \\ & & -2 & 7 & -6 \\ \hline & 2 & -7 & 6 & 0 \end{array}$$

Bingo! Our remainder was zero.

Therefore, -1 is a zero of $f(x)$, and $(x + 1)$ is a factor of $f(x)$.

Step 3: Rewrite the equation with the new factor.

Let's rewrite $f(x)$ with our new factor. The format will be $(x - \text{"zero"})(\text{quotient})$.

$$f(x) = (x + 1)(2x^2 - 7x + 6)$$

Step 4: Using a method of factoring to factor the trinomial factor. Now that we have a semi-factored form, let's see what else we can do:

$$f(x) = (x + 1)(2x^2 - 7x + 6)$$

$$2x^2 - 7x + 6 \text{ can be factored even further into } (2x - 3)(x - 2).$$

*Use grouping to factor.

So, the final factored form is: $f(x) = (x + 1)(2x - 3)(x - 2)$.

If we set each of these factors = 0, we get the **zeros or solutions to $f(x) = 0$** : $-1, \frac{3}{2}, 2$.

1a. [2 pts] Show that 2 is a zero of $f(x) = 4x^3 - 12x^2 + 5x + 6$ using synthetic division.

$$\begin{array}{r|rrrr}
 2 & 4 & -12 & 5 & 6 \\
 & & 8 & -8 & -6 \\
 \hline
 & 4 & -4 & -3 & 0
 \end{array}$$

1b. [2 pts] Write $f(x)$ in the form $(x - c)(Q(x))$: $(x - 2)(4x^2 - 4x - 3)$

1c. [3 pts] Factor $Q(x)$ using any method you choose.

$$\begin{aligned}
 &(4x^2 - 4x - 3) \\
 &4x^2 - 6x + 2x - 3 \\
 &2x(2x + 1) - 3(2x + 1) \\
 &(2x - 3)(2x + 1)
 \end{aligned}$$

Factored form of $Q(x)$: $(2x - 3)(2x + 1)$

1d. [3 pts] Write the factored form of $f(x)$. $f(x) = (x - 2)(2x - 3)(2x + 1)$

Definition: A factor of an integer is any number that divides that integer evenly (zero remainder).

2a. [2pts] List the factors of 15: 1, 3, 5

2b. [2pts] List the factors of 20: 1, 2, 4, 5

2c. [2pts] List the factors of 24: 1, 2, 3, 4, 6, 8, 12

2d. [4 pts] If you wanted to create all the possible fractions you could given 1, 2, 5, and 10 as a list of numbers you must use for the numerator and 1, 2, and 4 as a list of numbers you must use for the denominator, what would be your list of fractions that would be created using these numbers.

I'll get you started: $\frac{1}{1}, \frac{2}{1}, \frac{5}{1}, \frac{10}{1}, \frac{1}{2}, \frac{2}{2}, \frac{5}{2}, \frac{10}{2}, \frac{1}{4}, \frac{2}{4}, \frac{5}{4}, \frac{10}{4}$

Then, simply each fraction. This would give:

$1, 2, 5, 10, \frac{1}{2}, \frac{5}{2}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2}, \frac{5}{2}$

3. Follow the step-by-step guide below to help you write the following function in factored form. Then, solve $f(x) = 0$.

$$f(x) = 2x^3 - 3x^2 - 3x + 2$$

[2 pts] Part A: Find all the possible zeros.

What is P? 2

What is Q? 2

What is $\frac{P}{Q}$? Be sure to simplify.

Write the list of all possible zeros: $-2, -1, -\frac{1}{2}, \frac{1}{2}, 1, 2$

[2 pts] Part B: Find one actual zero.

Use synthetic division to test the possible zeros. How do you know if it is an actual zero?

-1	2	-3	-3	2
		-2	5	-2
		2	-5	2
	2	-5	2	0

[2 pts] Part C: Rewrite the function by turning the actual zero into a factor and the coefficients into a trinomial factor.

Rewrite the function with the new factor: $f(x) = \underline{(x + 1)(2x^2 - 5x + 2)}$

[2 pts] Part D: Factor the trinomial factor (the quotient from above).

Factor the trinomial factor in the function above:

$$(2x^2 - 5x + 2)$$

$$(2x^2 - 4x - x + 2)$$

$$(2x(x - 2) - 1(x - 2))$$

$$(2x - 1)(x - 2)$$

[3 pts] Part E: Write the final factored form of the function. Remember: $f(x) = (x - \text{zero})(\text{factored quotient})$.

Final factored form: $f(x) = \underline{(x + 1)(2x - 1)(x - 2)}$

[3 pts] Part F: Set each of the binomial factors equal to zero and solve each for x to find the solutions to $f(x) = 0$.

What are the solutions to $f(x) = 0$? $x = -1, \frac{1}{2}, 2$

4. [15 pts] Write the following function in factored form. Then, solve $f(x) = 0$.

Write your final answers at the bottom.

$$f(x) = 2x^3 - 3x^2 - 39x + 20$$

P = 20, Factors = 1, 2, 4, 5

Q = 2, Factors = 1, 2

Possible zeros: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 4, \pm 5, \pm \frac{5}{2}, \pm 10, \pm 20$

-4	2	-3	-39	20
		-8	44	-20
	2	-11	5	0

$$f(x) = (x - 4)(2x^2 - 11x + 5)$$

$$2x^2 - 11x + 5$$

$$2x^2 - 10x - x + 5x$$

$$2x(x - 5) - (x - 5)$$

$$(2x - 5)(x - 5)$$

$$f(x) = (x + 4)(2x - 5)(x - 5)$$

Answers:

Write the list of all possible zeros: $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 4, \pm 5, \pm \frac{5}{2}, \pm 10, \pm 20$

Final factored form: $f(x) = (x + 4)(2x - 5)(x - 5)$

What are the solutions to $f(x) = 0$? $x = -4, \frac{5}{2}, 5$

5. [15 pts] Write the following function in factored form. Then, solve $f(x) = 0$.

Write your final answers at the bottom.

$$f(x) = 2x^3 - x^2 - 22x - 24$$

Answers:

Write the list of all possible zeros: _____

Final factored form: $f(x) =$ _____

What are the solutions to $f(x) = 0$? _____

6. Find the LCM of each set of terms.

a. [2 pts] 90 and 20

LCM: _____

b. [3 pts] $4x^2$, $2x$, & 11

LCM: _____

c. [3 pts] $x + 5$ and $x - 5$.

LCM: _____

Definition: The least common denominator (LCD) of two or more fractions is the least common multiple (LCM) of the denominators.

Example: Find the LCD of $\frac{2}{7}$ and $\frac{3}{5}$.

Solution: We are looking for the LCM of the two denominators: 7 & 5. So, the answer is **35**.

7. Find the LCD of each set of rational expressions.

a. [2 pts] $\frac{1}{2}$ and $\frac{7}{11}$

b. [3 pts] $\frac{x}{x+2}$ and $\frac{7}{x-2}$

c. [3 pts] $\frac{1}{x}$, $\frac{2}{x+4}$, and $\frac{1}{x^2+4x}$

Solving a **RATIONAL EQUATION**: Use this as a reference for the next two problems.

$$\frac{2}{x+4} + \frac{1}{x-4} = \frac{17}{x^2-16}$$

Step 1: Factor all denominators. $\frac{2}{x+4} + \frac{1}{x-4} = \frac{17}{(x+4)(x-4)}$

Step 2: Find the LCD. The LCD is $(x+4)(x-4)$.

Step 3: Multiply each term by a factor equaling 1 to get a common denominator.

$$\frac{2}{x+4} \cdot \frac{(x-4)}{(x-4)} + \frac{1}{x-4} \cdot \frac{(x+4)}{(x+4)} = \frac{17}{(x+4)(x-4)}$$

Step 4: Multiply each term by the LCD:

$$(x+4)(x-4) \left(\frac{2}{x+4} \cdot \frac{(x-4)}{(x-4)} \right) + (x+4)(x-4) \left(\frac{1}{x-4} \cdot \frac{(x+4)}{(x+4)} \right) = (x+4)(x-4) \left(\frac{17}{(x+4)(x-4)} \right)$$

Step 5: Cancel out common factor, eliminating the fractions.

$$\cancel{(x+4)}\cancel{(x-4)} \left(\frac{2}{\cancel{x+4}} \cdot \frac{\cancel{(x-4)}}{\cancel{(x-4)}} \right) + \cancel{(x+4)}\cancel{(x-4)} \left(\frac{1}{\cancel{x-4}} \cdot \frac{\cancel{(x+4)}}{\cancel{(x+4)}} \right) = \cancel{(x+4)}\cancel{(x-4)} \left(\frac{17}{\cancel{(x+4)}\cancel{(x-4)}} \right)$$

Step 6: Write the remaining terms. $2(x-4) + 1(x+4) = 17$

Step 7: Solve the equation remaining. (This one is not quadratic, but if it is, you would have to factor or use other methods to solve.)

$$2x - 8 + x + 4 = 17 \quad \text{Linear Equation: Solve for } x.$$

$$3x - 4 = 17$$

$$\begin{array}{r} +4 \quad +4 \\ \hline 3x = 21 \\ \hline 3 \quad 3 \end{array}$$

$$x = 7$$

Step 8: Check your answer! Will it make the denominator equal zero?

The LCD is $(x+4)(x-4)$.

Plug the possible solution(s) into the LCD & see if it makes one of the factors = 0.

Let's test $x = 7$ in the LCD: $(7+4)(7-4) = (11)(3) = 33$ which is NOT zero, so 7 works and is a solution!

Therefore, we can use 7 as our answer.

Final Solution: $x = 7$

*Note, some equations are quadratics after eliminating the fractions. Factor and solve. Check EACH possible answer in the LCD to see if it makes it = 0. If one does and one does not, eliminate the one that makes the LCD = 0. You would only have one solution. If BOTH make the LCD = 0, your answer is NO SOLUTION (DNE).

8. [10 pts] Solve the following rational equation using the previous reference page as a guide.

$$\frac{2}{x+3} + \frac{5}{x-3} = \frac{37}{x^2-9}$$

9. [10 points] Solve.

$$\frac{1}{5} + \frac{3}{x+3} = \frac{9}{2x+6}$$