

Math104 Part II Review Solutions

1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ be the universal set. $A = \{1, 4, 8, 10\}$, $B = \{1, 3, 5, 9\}$, $C = \{2, 4, 6, 8, 10\}$. Find $A \cap B$, $A \cup B$, $(A \cap B) \cup C$, A^c , $(A \cup B)^c$.

$A \cap B = \{1\}$ (common elements of A and B), $A \cup B = \{1, 3, 4, 5, 8, 9, 10\}$ (including all elements in A or B).

$(A \cap B) \cup C = \{1, 2, 4, 6, 8, 10\}$

$A^c = \{2, 3, 5, 6, 7, 9\}$ (cross the elements in A from U).

$(A \cup B)^c = \{2, 6, 7\}$.

2. A leading cosmetics manufacturer advertises its products in three magazines: *Allure*, *Cosmopolitan*, and the *Ladies Home Journal*.

A survey of 500 customers by the manufacturer reveals the following information:

180 learned of its products from *Allure*.

200 learned of its products from *Cosmopolitan*.

192 learned of its products from the *Ladies Home Journal*.

84 learned of its products from *Allure* and *Cosmopolitan*.

52 learned of its products from *Allure* and the *Ladies Home Journal*.

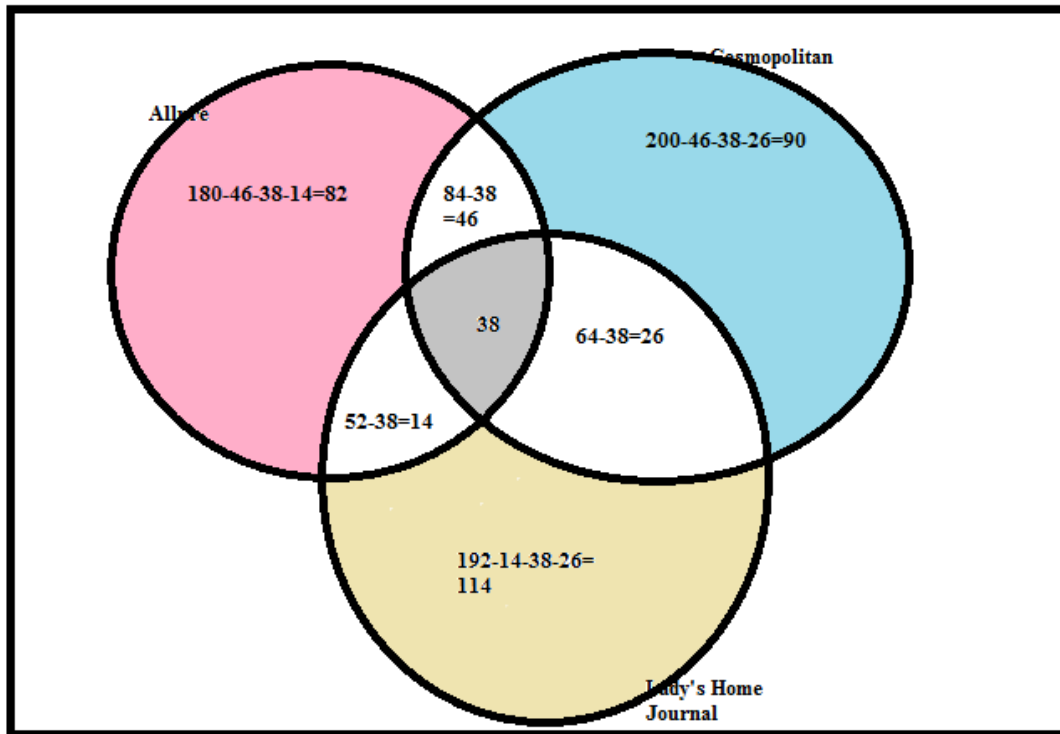
64 learned of its products from *Cosmopolitan* and the *Ladies Home Journal*.

38 learned of its products from all three magazines.

How many of the customers saw the manufacturer's advertisement in

a. At least one magazine?

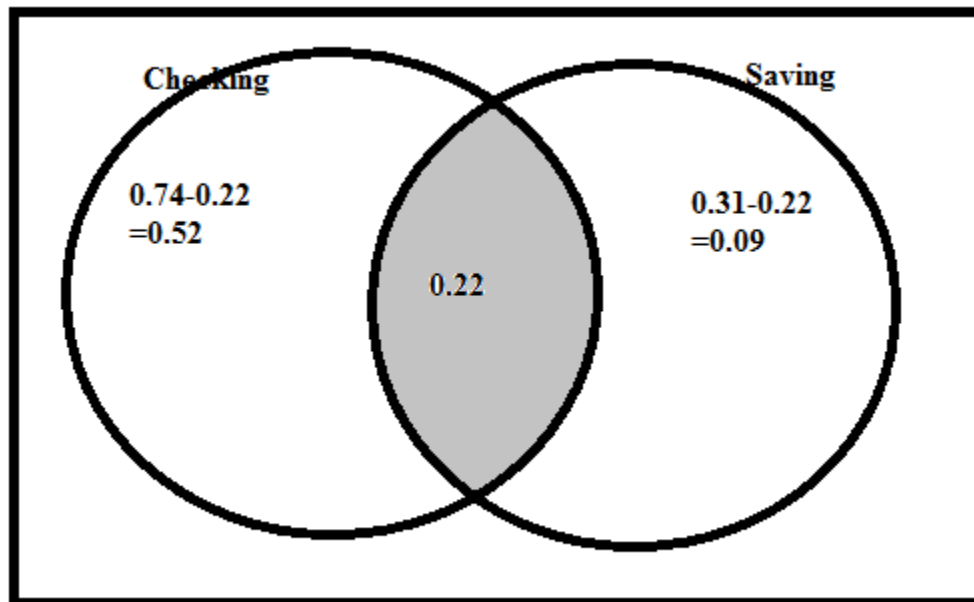
b. Exactly one magazine?



$N(\text{at least one}) = \text{sum of all the pieces} = 38 + 46 + 14 + 26 + 82 + 90 + 114 = 410$

$N(\text{exactly one}) = N(\text{A only}) + N(\text{B only}) + N(\text{C only}) = 82 + 90 + 114 = 286$

- The probability that a person has a checking account is 0.74, a saving account is 0.31, and both accounts is 0.22. Find the probability that a randomly selected person has a checking account or a saving account.



$P(\text{Checking})=0.74$, $P(\text{Saving})=0.31$ $P(\text{both})=0.22$
 $P(\text{Checking or saving}) = P(\text{checking})+P(\text{saving}) - P(\text{both}) = 0.74+0.31-0.22=0.83$

Or add the three pieces together in the picture= $0.52+0.22+0.09=0.83$

4. Among the 2500 college graduates in a survey, 1300 are male and 1200 are female. Among the male graduates, 1000 got job offers, and 300 did not get any. Among the female graduates, 1000 got job offers and 200 did not get any.

- Find the probability that a randomly selected college graduate will get job offers.
- Find the probability that a randomly selected graduate get job offer if the selected student is a male student.
- Find the probability that a randomly selected graduate get is male if the selected student will get job offer. .

	got Job	No Job	total
Male	1000	300	1300
female	1000	200	1200
total	2000	500	2500

- Total number of graduates = 2500, total number that got jobs = $1000+1000=2000$.
 $P(\text{get job})= 2000/2500=0.8$
- There are 1300 male graduates, 1000 got jobs. $P(\text{got job} | \text{male})= 1000/1300 = 0.77$
- $P(\text{male} | \text{got Job})= 1000/2000=0.5$

5. An experiment consists of selecting a card at random from a standard 52-card deck. A). Find the probability that the card is an Ace or a spade. B). Find the probability that the card is an ace knowing that it is a heart?

A). $P(\text{Ace})= 4/52$. $P(\text{Spade})= 13/52$, $P(\text{Spade and Ace})=1/52$.

$P(\text{Spade or Ace})= 4/52+13/52-1/52=16/52$.

B). There are 13 hearts, $P(\text{Ace} | \text{knowing heart})= 1/13$.

6. A). A committee has 15 members. How many ways can a chairman and a treasure be selected? If we know that there are 9 men and 6 women in the committee. B). How many ways can a chairman and a treasure be selected so that the chairman is a man and the treasure is a women?

A). $15 \times 14 = 210$

B). $9 \times 6 = 54$.

7. A nationwide survey conducted by the National Cancer Society revealed the following information: Of 10,000 people surveyed, 3,300 were "heavy coffee drinkers" and 160 had cancer of the pancreas. Of those who had cancer of the pancreas, 137 were heavy coffee drinkers. A). If a person is randomly selected, what is the probability that the person is has cancer of the pancreas? B). What is

the probability that the person has cancer of the pancreas knowing that the person is a heavy coffee drinker? C). Are the two events "being a heavy coffee drinker" and "having cancer of the pancreas" independent?

	Cancer	No Cancer	total
Heavy Drinker	137	3163	3300
No Heavy Drinker	160-137=23	6677	6700
total	160	9840	10000

- A). $P(\text{Cancer}) = 160/10,000$
 B). $P(\text{Cancer} | \text{Heavy drinker}) = 137/3300$
 C). $P(\text{Cancer})$ is different from $P(\text{Cancer} | \text{Heavy drinker})$, so the two events are not independent.

8. Customers are asked to make a determination about their satisfaction following a visit to a convenience store. The following table reports how many new and returning customers are either satisfied or unsatisfied.

	Satisfied	Unsatisfied
New	400	200
Returning	500	100

A customer is selected at random. Let N represent the event that the customer is a new customer, R represent the event that the customer is a returning customer, S represent the event that the customer is a satisfied customer, and U represent the event that the customer is an unsatisfied customer.

- A. Find $P(N)$. B. Find $P(S|N)$. C. Find $P(N|S)$.

Total customers = $400 + 500 + 200 + 100 = 1200$, new customer = $400 + 200 = 600$
 Total satisfied customer = $400 + 500 = 900$.

$$P(N) = 600/1200 = 0.5$$

$$P(S|N) = 400/600$$

$$P(N|S) = 400/900$$

9. Let A and B be two independent events in a sample space S such that $P(A) = 0.5$, $P(B) = 0.2$, Find $P(A \cap B)$, $P(A \cup B)$ and $P(A|B)$.
 A , B are independent, so $P(A \cap B) = P(A)P(B) = 0.5 \times 0.2 = 0.1$
 $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.2 - 0.1 = 0.6$
 $P(A|B) = P(A) = 0.5$ since B won't affect A .

10. Given the following data: 2, 5, 3, 7, 9, 9, 14, 15, 3, 9, 13.

a). Find the mean, median, variance and standard deviation of the data set if the data is collected from a sample.

b). Find the mean, median, variance and standard deviation of the data set if the data is collected from a population.

Put the numbers in order: 1, 3, 3, 5, 7, 9, 9, 9, 13, 14, 15.

In both cases mean = sum of all the numbers / 11 = 8, median = 9, mode = 9.

If it is sample data, variance = $(\sum(x - \text{mean})^2) / (n - 1) = 22.2$, standard deviation = square root of 22.2 = 4.71

If it is population data, variance = $(\sum(x - \text{mean})^2) / n = 20.18$, standard deviation = 4.49

11. A class of 40 students took a 5-point math quiz. The following frequency distribution describes the scores received on the quiz.

Score	0	1	2	3	4	5
Frequency	1	3	6	15	10	5

A. Find the probability distribution of X , where X denotes the score received on the quiz.

Score	0	1	2	3	4	5
Probability	1/40	3/40	6/40	15/40	10/40	5/40

B. Find the probability that a randomly selected student got a score more than 3.

$$P(x > 3) = 10/40 + 5/40 = 0.375$$

C. Find the mean and variance of the probability distribution of X .

$$\text{Mean} = \sum xp(x) = 0\left(\frac{1}{40}\right) + 1\left(\frac{3}{40}\right) + 2\left(\frac{6}{40}\right) + 3\left(\frac{15}{40}\right) + 4\left(\frac{10}{40}\right) + 5\left(\frac{5}{40}\right) = 3.125$$

$$\begin{aligned} \text{Variance} &= \sum (x - \text{mean})^2 p(x) = (0 - 3.125)^2 \left(\frac{1}{40}\right) + (1 - 3.125)^2 \left(\frac{3}{40}\right) + \\ &+ (2 - 3.125)^2 \left(\frac{6}{40}\right) + (3 - 3.125)^2 \left(\frac{15}{40}\right) + (4 - 3.125)^2 \left(\frac{10}{40}\right) + \\ &+ (5 - 3.125)^2 \left(\frac{5}{40}\right) = 1.40 \end{aligned}$$

12. A local marathon runner estimates that the probability he will win his next race is 0.4.

A. What are the odds that he will win his next race? $0.4:0.6 = 4:6$

B. What are the odds that he will not win his next race? $0.6:0.4 = 6:4$

13. What should you pay to play a game in which you receive \$5 for drawing an ace from a deck of cards, \$1 for a face-card, and 50¢ for any other card so that the game is fair?

Let X be the amount of money you will get. The probability distribution of X is

X	\$5	\$1	\$0.5
$P(X)$	$4/52 = 1/13$	$12/52 = 3/13$	$36/52 = 9/13$

$$\text{Mean of } X = 5(1/13) + 1(3/13) + 0.5(9/13) = 12.5/13 = 0.961$$

It will be a fair game if you pay 0.96 to play a game.

14. Ms. Wiggins purchases a \$10,000 one-year term life insurance policy for \$105. Find the company's expected gain (to the nearest cent) if the probability that she will live for another year is 0.9915.

If Ms. Wiggins dies (probability is $1 - 0.9915 = 0.0085$), the net amount the insurance company will loss is $10,000 - 105 = 9895$

If Ms. Wiggins lives for another year (probability is 0.9915), the net amount the insurance company will make is 105.

	Live for another year	Die within a year
Probability	0.9915	0.0085
Company gain	105	$-10,000 + 105 = -9895$

$$\text{Expected gain} = 0.9915 \cdot 105 + 0.0085(-9895) = 20.$$

15. Market research shows that 27% of college students have at least one major credit card. If 10 students are selected at random, calculate the probability that fewer than 2 of them have at least one major credit card. What about the probability that exactly 5 out of the 10 students have at least one major credit card? Find the mean and standard deviation of this probability distribution.

Use binomial distribution, $p = 0.27$, $q = 0.73$, $n = 10$

$$P(X < 2) = P(x=0) + P(x=1) = {}_{10}C_0 \cdot (0.27)^0 \cdot (0.73)^{10} + {}_{10}C_1 \cdot (0.27)^1 \cdot (0.73)^9 = 0.202$$

$$P(x=5) = {}_{10}C_5 \cdot (0.27)^5 \cdot (0.73)^5 = 0.075$$

$$\text{Mean} = 10 * 0.27 = 2.7$$

$$\text{Standard deviation} = \sqrt{10 * 0.27 * 0.73} = 1.404$$

16. A new drug has been found to be effective in treating 75% of the people afflicted by a certain disease. If the drug is administered to 700 people who have this disease, what are the mean and the standard deviation of the number of people for whom the drug can be expected to be effective? (Round your standard deviation to two decimal places.)

$$\text{Mean} = 700(75\%) = 525. \text{ Standard deviation} = \text{square root (npq)} = \text{square root of } (700 * 75\% * 25\%) = 11.46$$

17. IQ's are normally distributed with a mean of 100 and a standard deviation of 10. If a person is selected at random, what is the probability that his/her IQ is between 100 and 115? What is the probability that his/her IQ is higher than 120? What about lower than 100?

Use normal distribution. Mean = 100, standard deviation = 10

$$P(100 < X < 115) = P\left(\frac{(100-100)}{10} < Z < \frac{(115-100)}{10}\right) = P(0 < Z < 1.5) = 0.4332$$

$$P(X > 120) = P\left(Z > \frac{(120-100)}{10}\right) = P(Z > 2) = 1 - 0.9772 = 0.0228$$

$$P(X < 100) = P(Z < 0) = 0.5$$

18. According to data released by the Chamber of Commerce of a certain city, the weekly wages (in dollars) of female factory workers are normally distributed with a mean of 575 and a standard deviation of 50. Find the probability that a female factory worker selected at random from the city makes a weekly wage of \$475 to \$625.

Use normal distribution. Mean = 575, standard deviation = 50

$$P(475 < X < 625) = P\left(\frac{(475-575)}{50} < Z < \frac{(625-575)}{50}\right) = P(-2 < Z < 1) = 0.8185.$$

19. The average grade for an exam is 74, and the standard deviation is 7. If 12% of the class is given As, and the grades are curved to follow a normal distribution, what is the lowest possible A? If D is assigned to a grade of 65 or below. What percent of the class received a D?

We have $\mu = 74, \sigma = 7$. We are looking for A such that $P(x > A) = 0.12$, or $P(x < A) = 0.88$. The corresponding z value is 1.18 because $P(z < 1.18) = 0.8810$, $P(z < 1.17) = 0.8790$ (you can use $z = 1.18$ or $z = 1.17$)

$$\text{Thus } A = 74 + 7 * 1.18 = 82.26 = 82 \text{ and } B = 81.$$

$$\text{Meanwhile, } P(D) = P(x < 65) = P\left(z < \frac{(65-74)}{7}\right) = P(z < -1.29) = 0.0985.$$

9.85% of the class will receive a D.

20. A probability distribution has a mean of 20 and a standard deviation of 2. Use Chebychev's inequality to estimate the probability that an outcome of the experiment lies between 16 and 24.

$16 = 20 - 2 \cdot 2$, $24 = 20 + 2 \cdot 2$, therefore, 16 to 24 is two standard deviation about the mean. Based on Chebychev's inequality, there are $1 - (1/k^2) = 1 - (1/2^2) = \frac{3}{4}$ or 75% of data are between 16 and 24.