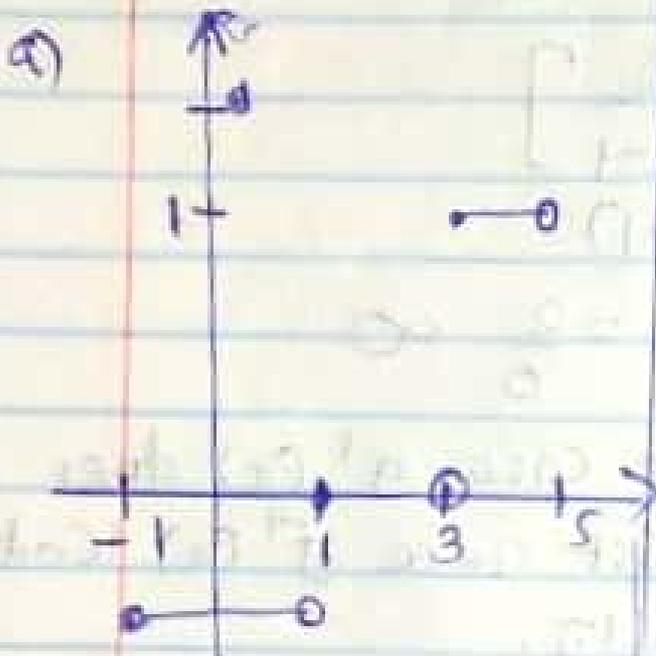


Problem 5

$g: \mathbb{R} \rightarrow \mathbb{R}$

$$g(x) = \left[ \frac{x-1}{2} \right]$$



If  $g(x)$  is positive  
if  $x = 3$

$$g(x) = \left[ \frac{3-1}{2} \right]$$

$$\begin{aligned} &= \left[ \frac{2}{2} \right] \\ &= \left[ 1 \right] \end{aligned}$$

Let  $x = -1$

$$g(x) = \left[ \frac{-1-1}{2} \right]$$

$$= \left[ \frac{-2}{2} \right]$$

$$g(x) = -1$$

Therefore  $g(x)$   
is  $(1, -1)$ .

b) Yes  $g$  is on to  $\mathbb{R}$  because for any real number  $y$  there exist rational and irrational number. On real number line both  $-1$  and  $1$  can be accommodated.

Does the solution  $g^{-1}(y)$  exist?

$$g(x) = \left[ \frac{x-1}{2} \right]$$

$$g^{-1}(y) = \left[ \frac{2}{x-1} \right]$$

At  $g(1, -1)$

$$\frac{2}{1-1} = \frac{2}{0} = \infty$$

for this case,  $g^{-1}(y)$  does not exist since  $g^{-1}(x)$  tends to infinity.

### Problem 6

$$g: A \rightarrow B$$

$$f: B \rightarrow C$$

$$f \circ g \text{ is } 1-1$$

$$g \text{ is } 1-1$$

$$f(a, b) \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$a = 1$$

$$-b = -1$$

$$\frac{-1b}{-1} = \frac{-1}{-1}$$

$$b = 1$$

$$f(a, b) = (1, 1)$$

It is not must for  $f$  to be  $(1, -1)$ .  
Since  $f$  is  $(1, 1)$ .

### Problem 7

For any positive real numbers  $x, y$

$$|x \cdot y| = \begin{cases} xy & \text{if } xy \geq 0 \\ -xy & \text{if } xy < 0 \end{cases}$$

$$|x \cdot y| \leq |x| |y|$$

Absolute value of  $x$  is  $-x$   
and absolute value of

$y$  is  $-y$

Multiplying them the negative

sign cancels out hence

$x \cdot y$  is positive.

Both  $x$  and  $y$  are non-negative

therefore if

$$xy \geq 0$$

I will disapprove the result.

Since both  $x$  and  $y$

are non-negative

therefore  $xy \geq 0$

therefore

$$|x \cdot y| \leq |x| |y| \text{ is}$$

wrong.

$$|x \cdot y| \geq |x| |y|$$

is wrong.

$$x^2 + y^2 \geq 2xy$$

$$x^2 + y^2 - 2xy \geq 0$$

$$(x - y)^2 \geq 0$$

$$x^2 - 2xy + y^2 \geq 0$$

$$x^2 + y^2 \geq 2xy$$

$$x^2 + y^2 - 2xy \geq 0$$

$$(x - y)^2 \geq 0$$

### Problem 8

$$g: A \rightarrow B$$

$$f: B \rightarrow C$$

$$A = B = C = \{1, 2, 3, 4\}$$

$$g = \{(1, 2), (2, 3), (3, 2), (4, 4)\}$$

$$f = \{(1, 2), (2, 4), (3, 2), (4, 3)\}$$

in Matrix form

$$a) \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 2 & 4 \end{bmatrix}$$

$$fg = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 22 & 33 & 22 & 44 \end{bmatrix}$$

$$b) g \cdot f = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 2 & 3 \end{bmatrix}$$

$$g \cdot f = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 22 & 44 & 22 & 33 \end{bmatrix}$$

$$c) f^g$$

$$c) g \cdot g$$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 20 & 30 & 40 \\ 22 & 33 & 22 & 44 \end{bmatrix}$$

$$d) g[g \cdot g]$$

$$\text{But } [g \cdot g] = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 22 & 33 & 22 & 44 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 10 & 20 & 30 & 40 \\ 22 & 33 & 22 & 44 \end{bmatrix}$$

$$= \begin{bmatrix} 100 & 200 & 300 & 400 \\ 242 & 363 & 242 & 484 \end{bmatrix}$$

### Problem 9.

a)  $a_n = 7$

$n = 7$

$$a_7 = a + (7-1)d$$

$$a_7 = a + 6d$$

b)

$$a_n = 2 - (-1)^n$$

$$a_n = 2 + 1^n$$

$$a_n = \log 2 + n \log 1$$

c) 5, 7, 9, 11, 13

$$a = 5$$

$$d = 2$$

$$a_n = 5 + (n-1)2$$

$$a_n = 5 + 2n - 2$$

$$a_n = 3 + 2n$$

d) 5, 5, 5, 5, 5

$$a_n = ar^{n-1}$$

$$r = \frac{5}{5} = 1$$

$$c = a$$

$$a_n = 5 \times 1^{n-1}$$

$$a_n = 5^n$$

$$a_n = n \log 5$$

e)

7, 3, 7, 3, 7, 3

$$a = 7$$

$$d = 4$$

$$a_n = 7 + (n-1)4$$

$$a_n = 7 + 4n - 4$$

$$a_n = 3 + 4n$$

$$a_n = 3 + 4n$$

a) Find  $\sum_{i=1}^{1000} i$

$$i^2 = 1000 \quad \text{But } i =$$

$$n \log 1 = \log 1000$$

$$i \log 1 = \log 1000$$

$$i = \frac{\log 1000}{\log 1}$$

$$i = \infty$$

$$\sum_{i=1}^6 ((-2)^i - 2^i)$$

But  $i=1$

$$\sum_{i=1}^6 ((-2)^1 - 2^1)$$

$$\sum_{i=1}^6 4$$

$6P_4$ .

$P$ -permutation.

$$\frac{6!}{4!}$$

$$= \underline{\underline{30}}$$