Q.1

$-4x + 8y = -9$

$6x - 12y = 13.5$

$x = \frac{Dx}{D}$

$y = \frac{Dy}{D}$

$$D = \begin{vmatrix} -4 & 8 \\ 6 & -12 \end{vmatrix} = 48 - 48 = 0$$

$$Dx = \begin{vmatrix} -9 & 8 \\ 13.5 & -12 \end{vmatrix} = 108 - 108 = 0$$

$$Dy = \begin{vmatrix} -4 & -9 \\ 6 & 13.5 \end{vmatrix} = -54 + 54 = 0$$

$0 = 0$

There are infinitely many solutions.

$-4x = -8y - 9$

$x = 2y + 2.25$ $\quad 8y = -9 + 4x$ $\quad y = 0.5x - 1.125$

solution

$(2y + 2.25, y)$ or $(x, 0.5x - 1.125)$

Q. 2.

$-4x + 8y = -9$

$5x - 10y = 20$

$x = \frac{Dx}{D}$ $\quad D = \begin{vmatrix} -4 & 8 \\ 5 & -10 \end{vmatrix} = 40 - 40 = 0.$

$y = \frac{Dy}{D}$

$$Dx = \begin{vmatrix} -9 & 8 \\ 20 & -10 \end{vmatrix} = +90 - 160 = -70$$

$$Dy = \begin{vmatrix} -4 & -9 \\ 5 & 20 \end{vmatrix} = -80 + 45 = -35$$

$-70 \neq 0$ and $-35 \neq 0$

There is no solution.

Q. 3

$-4x + 8y = -28$

$6x - 10y = 38$

$x = \frac{Dx}{D}$ $y = \frac{Dy}{D}$

$$D = \begin{vmatrix} -4 & 8 \\ 6 & -10 \end{vmatrix} = 40 - 48 = -8$$

$$Dx = \begin{vmatrix} -28 & 8 \\ 38 & -10 \end{vmatrix} = 280 - 304 = -24.$$

$$Dy = \begin{vmatrix} -4 & -28 \\ 6 & 38 \end{vmatrix} = -152 + 168 = 16$$

$x = \frac{+24}{+8}$ $y = \frac{16}{-8}$

$= 3$ $= -2$

There is one solution $(3, -2)$

Q. 4

$$A=\begin{bmatrix}1 & -1 & 2\\ 3 & -2 & 4\end{bmatrix},\quad B=\begin{bmatrix}0 & -3\\ 2 & 4\\ 3 & 1\end{bmatrix},\quad C=\begin{bmatrix}7 & 4\\ 5 & 0\\ -5 & 6\end{bmatrix},\quad D=\begin{bmatrix}1 & 3\\ 5 & 7\end{bmatrix}$$

a) A+B cannot be done because A ~~has~~ is a 2×3 matrix and B is a 3×2 matrix and to add two matrices they ~~have~~ ought to have ~~two~~ equal number of rows and columns.

b) $$B-C=\begin{bmatrix}0 & -3\\ 2 & 4\\ 3 & 1\end{bmatrix}-\begin{bmatrix}7 & 4\\ 5 & 0\\ -5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -7\\ -3 & 4\\ 8 & -5\end{bmatrix}$$

c) $$AB=\begin{bmatrix}1 & -1 & 2\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}0 & -3\\ 2 & 4\\ 3 & 1\end{bmatrix}=\begin{bmatrix}0-2+6 & -3+-4+2\\ 6-4+12 & -9-8+4\end{bmatrix}$$

$$=\begin{bmatrix}4 & -5\\ 8 & -13\end{bmatrix}$$

d) $$BA=\begin{bmatrix}0 & -3\\ 2 & 4\\ 3 & 1\end{bmatrix}\begin{bmatrix}1 & -1 & 2\\ 3 & -2 & 4\end{bmatrix}\neq\begin{matrix}0-9 & 0+6\\ 2+12 & -2-8\\ 3+3 & 6+4\end{matrix}$$

$$=\begin{bmatrix}0-9 & 0+6 & 0-12\\ 2+12 & -2-8 & 4+16\\ 3+3 & -3-2 & 6+4\end{bmatrix}=\begin{bmatrix}-9 & 6 & -12\\ 14 & -10 & 20\\ 9 & -5 & 10\end{bmatrix}$$

e) $$D^2=\begin{bmatrix}1 & 3\\ 5 & 7\end{bmatrix}\begin{bmatrix}1 & 3\\ 5 & 7\end{bmatrix}=\begin{bmatrix}1+15 & 3+21\\ 5+35 & 15+49\end{bmatrix}=\begin{bmatrix}16 & 24\\ 40 & 64\end{bmatrix}$$

Q. 5.

$(x - 5y + 8z = -9 \ldots) \times 2 \ldots ①$

$2x - 9y + 14z = -19 \ldots ②$

$-3x + 15y - 23z = 25 \ldots ③$

By elimination:

$2x - 10y + 16z = -18$

$2x - 9y + 14z = -19$ $-$

$-y + 2z = 1$

Multiply ① by 3

$3x - 15y + 24z = -27$

$-3x + 15y - 23z = 25$ $+$

$z = -2$

$-y - 4 = 1$

$-y = 5 \quad y = -5$

Using ①,

$x + 25 - 16 = -9$

$x + 9 = -9$

$x = -18$

solution: $(-18, -5, -2)$

By inverse method:

$$\begin{bmatrix} 1 & -5 & 8 \\ 2 & -9 & 14 \\ -3 & 15 & -23 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -9 \\ -19 \\ 25 \end{bmatrix}$$

A

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & 5 & 2 \\ 4 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}}_{A^{-1}} \begin{bmatrix} -9 \\ -19 \\ 25 \end{bmatrix} = \begin{bmatrix} 27 - 95 + 50 \\ -36 - 19 + 50 \\ -27 + 0 + 25 \end{bmatrix} = \begin{bmatrix} -18 \\ -5 \\ -2 \end{bmatrix}$$

Q.6.

$2x + y - 5z = -12$

$x + 3y + z = 11$

$x + 6y + z = 23$

Cramer's rule:

$x = \frac{Dx}{D} \qquad y = \frac{Dy}{D} \qquad z = \frac{Dz}{D}$

$$D = \begin{vmatrix} 2 & 1 & -5 \\ 1 & 3 & 1 \\ 1 & 6 & 1 \end{vmatrix} = 2\begin{vmatrix} 3 & 1 \\ 6 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 5\begin{vmatrix} 1 & 3 \\ 1 & 6 \end{vmatrix}$$

$= 2(3-6) - 1(1-1) - 5(6-3)$

$= -6 - 15$

$= -21$

$$Dx = \begin{vmatrix} -12 & 1 & -5 \\ 11 & 3 & 1 \\ 23 & 6 & 1 \end{vmatrix} = -12\begin{vmatrix} 3 & 1 \\ 6 & 1 \end{vmatrix} - 1\begin{vmatrix} 11 & 1 \\ 23 & 1 \end{vmatrix} - 5\begin{vmatrix} 11 & 3 \\ 23 & 6 \end{vmatrix}$$

$= -12(3-6) - 1(11-23) - 5(66-69)$

$= 36 + 12 + 15$

$= 63$

$$Dy = \begin{vmatrix} 2 & -12 & -5 \\ 1 & 11 & 1 \\ 1 & 23 & 1 \end{vmatrix} = 2\begin{vmatrix} 11 & 1 \\ 23 & 1 \end{vmatrix} + 12\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 5\begin{vmatrix} 1 & 11 \\ 1 & 23 \end{vmatrix}$$

$= 2(11-23) + 12(0) - 5(23-11)$

$= -24 - 60$

$= -84$

$$z = \begin{vmatrix} 2 & 1 & -12 \\ 1 & 3 & 11 \\ 1 & 6 & 23 \end{vmatrix} = 2\begin{vmatrix} 3 & 11 \\ 6 & 23 \end{vmatrix} - 1\begin{vmatrix} 1 & 11 \\ 1 & 23 \end{vmatrix} - 12\begin{vmatrix} 1 & 3 \\ 1 & 6 \end{vmatrix}$$

$2(69-66) - 1(23-11) - 12(6-3)$

$= 6 - 12 - 36$

$= -42$

$x = \frac{63}{-21}$ $\quad y = \frac{-54}{-21}$ $\quad z = \frac{-42}{-21}$

$x = -3$ $\quad y = 4$ $\quad z = 2$

solution: $(-3, 4, 2)$