1. A normal logistic growth model is described by this ODE

$$\frac{dp}{dt} = kp\left(1 - \frac{p}{K}\right)$$

Solving this by seperation of variables

$$\int \frac{dp}{p(1-p/K)} = \int k\,dt$$

To evaluate the left hand side

$$\frac{1}{p(1-p/K)} = \frac{K}{p(K-p)} = \frac{1}{p} + \frac{1}{K-p}.$$

$$\text{Hence } \int \frac{dp}{p} + \int \frac{dp}{K-p} = \int k\,dt$$

$$\ln|p| - \ln|K-p| = kt + c$$

$$\ln\left|\frac{p}{K-p}\right| = +kt + c$$

$$\ln\left|\frac{K-p}{p}\right| = -kt - c$$

$$\frac{K-p}{p} = e^{-kt-c} \qquad \text{Where } \pm e^{-c} = A$$

$$\frac{K-p}{p} = Ae^{-kt}$$

$$\text{Therefore } p(t) = \frac{K}{1+Ae^{-kt}} \qquad \text{Where } A = \frac{K-P_0}{P_0}$$

From our eqn

$$\frac{dp}{dt} = 0.04P\left(1 - \frac{P}{20000}\right) \Rightarrow$$

$K = 20000$
$k = 0.04$

$P(t) = 0$ because the function describing the derivative is free of t

b) $p(t) = \dfrac{K}{1+Ae^{-kt}} = \dfrac{20000}{1+Ae^{0.04(0.5)}}$.

$$A = \frac{K-P_0}{P_0} = \frac{20000-1000}{1000} = 19.$$

$$P(t) = \frac{20000}{1+19e^{-0.04(0.5)}} = 1020$$

c) There is continuous time growth

$P'(t) = 0.04P\left(1-\dfrac{P}{20000}\right)$. population in the middle of 2011 = P

$P = 1020$

$$\frac{dp}{dt} = 0.04(1020)\left(1-\frac{1020}{20000}\right) = 37$$

This show continuous growth of population with time.

d) $P_0 = 1000$

$P(t) = \dfrac{20000}{1+19e^{-0.04 \times 1/12}} = 1004$ or $1003$.

~~The~~ Using this value of P at 1/12 years

$$P(t) = 0.04(1003)\left(1-\frac{1003}{20000}\right) = 38$$

Rate of change is constant as per the figure in part (c).

e) $P(t)$

step size = ~~0.08333 years~~ $= \frac{1}{2}$

$P(t) = \dfrac{20000}{1+19e^{-kt}} = f(t,y)$

$f(0,1000) = 1000$

Taking a step size 0.1

$y(1) = y_0 + hf_0 = 1$

| Time $t_n$ | approximation | time | approximation |
|---|---|---|---|
| $t_0 = 0$ | $P(0) = 1000$ | $t_{13}$ | $P = 1053.2$ |
| $t_1 = 0.0833$ | $y_1 = 1003.8$ | $t_{14}$ | $P = 1057$ |
| $t_2 = \frac{1}{6}$ | $y_2 = 1007.6$ | $t_{15}$ | $P = 1060.8$ |
| $t_3 = \frac{1}{4}$ | $y_3 = 1011.4$ | $t_{16}$ | $P = 1064.6$ |
| $t_4 = \frac{1}{3}$ | $y_4 = 1015.2$ | $t_{17}$ | $P = 1068.4$ |
| $t_5 = \frac{5}{12}$ | $y_5 = 1019$ | $t_{18}$ | $P = 1072.2$ |
| $t_6 = \frac{1}{2}$ | $y_6 = 1022.8$ | $t_{19}$ | $P = 1076$ |
| $t_7 = \frac{7}{12}$ | $y_7 = 1026.6$ | $t_{20}$ | $P = 1079.8$ |
| $t_8 = \frac{8}{12}$ | $y_8 = 1030.4$ | $t_{21}$ | $P = 1083.6$ |
| $t_9 = \frac{3}{4}$ | $y_9 = 1038$ | $t_{22}$ | $P = 1087.4$ |
| $t_{10} = \frac{5}{6}$ | $y_{10} = 1041.8$ | $t_{23}$ | $P = 1091.4$ |
| $t_{11} = \frac{11}{12}$ | $y_{11} = 1045.6$ | $t_{24}$ | $P = 1095$ |
| $t_{12} = 1$ | $y_{12} = 1049.4$ | $t_{25}$ | $P = 1098.8$ |
| | | $t_{26}$ | $P = 1102.6$ |

Continued next page

Given $y' = 0.04 \times 100 \left(1 - \frac{1000}{20000}\right)$

3×22

$y_1 = y_0 + hf(x_0, y_0) = 1000 + 0.1\, f(0.0833, 1000) = 1003.8$

$y_2 = y_1 + hf(x_1, y_1) = 1003.8 + 0.1\, f(0.1833, 1003.8) = 1007.6$

$y_3 = y_2 + hf(x_2, y_2) = 1007.6 + 0.1\, f(0.2833, 1007.6) = 1011.4$

$y_4 = y_3 + hf(x_3, y_3) = 1011.4 + 0.1\, f(0.3833, 1011.4) = 1015.2$

$y_5 = y_4 + hf(x_4, y_4) = 1015.2 + 0.1\, (f(0.4833, 1015) = 1019$

$y_6 = y_5 + hf(x_5, y_5) = 1019 + 0.1\, f(0.5833, 1019) = 1022.8$

~~P(x) = y_{47} + 0.1 f(x_{47}, y_{47}) = y(1.9833) = 1072.2~~

Population

| | |
|---|---|
| $t_{27}$ | $P = 1106.4$ |
| $t_{28}$ | $P = 1110.2$ |
| $t_{29}$ | $P = 1114$ |
| $t_{30}$ | $P = 1117.8$ |
| $t_{31}$ | $P = 1121.6$ |
| $t_{32}$ | $P = 1125.4$ |
| $t_{33}$ | $P = 1129.2$ |
| $t_{34}$ | $P = 1133$ |
| $t_{35}$ | $P = 1136.8$ |
| $t_{36}$ | $P = 1140.6$ |
| $t_{37}$ | $P = 1144.4$ |
| $t_{38}$ | $P = 1148.2$ |
| $t_{39}$ | |

$$y_{39} = y(3.9833) = 1148.2$$

(f) ✓

$y_0 = 1000$
$y_1 = 1038$
$y_2 = 1076$
$y_3 = 1114$
$y_4 = 1152$
$y_5 = 1190$
$y_6 = 1228$
$y_7 = 1266$
$y_8 = 1304$
$y_9 = 1342$
$y_{10}$ 1380
$y_{11} = 1418$
✓ $y_{12} = 1456$
$y_{13} = 1494$
$y_{14} = 1532$
$y_{15} = 1570$
$y_{16} = 1608$

$y_{17} = 1646$
$y_{18} = 1684$
$y_{19} = 1722$
$y_{20} = 1760$
$y_{21} = 1798$
$y_{22} = 1836$
$y_{23} = 1874$
✓ $y_{24} = 1912$
$y_{25} = 1950$
$y_{26} = 1988$
$y_{27} = 2026$
$y_{28} = 2064$
$y_{29} = 2102$
$y_{30} = 2140$
$y_{31} = 2178$
$y_{32} = 2216$
$y_{33} = 2254$

$y_{34} = 2292$
$y_{35} = 2330$
✓ $y_{36} = 2368$
$y_{37} = 2406$
$y_{38} = 2444$
$y_{39} = 2482$
$y_{40} = 2520$
$y_{41} = 2558$
$y_{42} = 2596$
$y_{43} = 2634$
$y_{44} = 2672$
$y_{45} = 2710$
$y_{46} = 2748$
$y_{47} = 2786$
✓ $y_{48} = 2824$
$y_{49} = 2862$
$y_{50} = 2900$
$y_{51} = 2938$
$y_{52} = 2976$
$y_{53} = 3014$
$y_{54} = 3052$
$y_{55} = 3090$
$y_{56} = 3128$
$y_{57} = 3166$

$y_{58} = 3204$
$y_{59} = 3242$
✓ $y_{60} = 3280$
$y_{61} = 3318$
$y_{62} = 3356$
$y_{63} = 3394$
$y_{64} = 3432$
$y_{65} = 3470$
$y_{66} = 3508$
$y_{67} = 3546$
$y_{68} = 3584$
$y_{69} = 3622$
$y_{70} = 3660$
$y_{71} = 3698$
✓ $y_{72} = 3736$
$y_{73} = 3774$
$y_{74} = 3812$
$y_{75} = 3850$
$y_{76} = 3888$
$y_{79} = 3926$
$y_{80} = 3964$
$y_{81} = 4002$
$y_{82} = 4040$
$y_{83} = 4078$
✓ $y_{84} = 4116$
$y_{85} = 4154$
$y_{86} = 4192$
$y_{87} = 4230$
$y_{88} = 4268$
$y_{89} = 4306$
$y_{90} = 4344$
$y_{91} = 4382$
$y_{92} = 4420$

$y_{93} = 4458$
$y_{94} = 4496$
$y_{95} = 4534$
✓ $y_{96} = 4572$
$y_{97} = 4610$
$y_{98} = 4648$
$y_{99} = 4686$
$y_{100} = 4724$
$y_{101} = 4762$
$y_{102} = 4800$
$y_{103} = 4838$
$y_{104} = 4876$
$y_{105} = 4914$
$y_{106} = 4952$
$y_{107} = 4990$
✓ $y_{108} = 5028$
$y_{109} = 5066$
$y_{110} = 5104$
$y_{111} = 5142$
$y_{112} = 5180$
$y_{113} = 5218$
$y_{114} = 5256$
$y_{115} = 5294$
$y_{116} = 5332$

$y_{117} = 5370$
$y_{118} = 5408$
$y_{119} = 5446$
✓ $y_{120} = 5484$

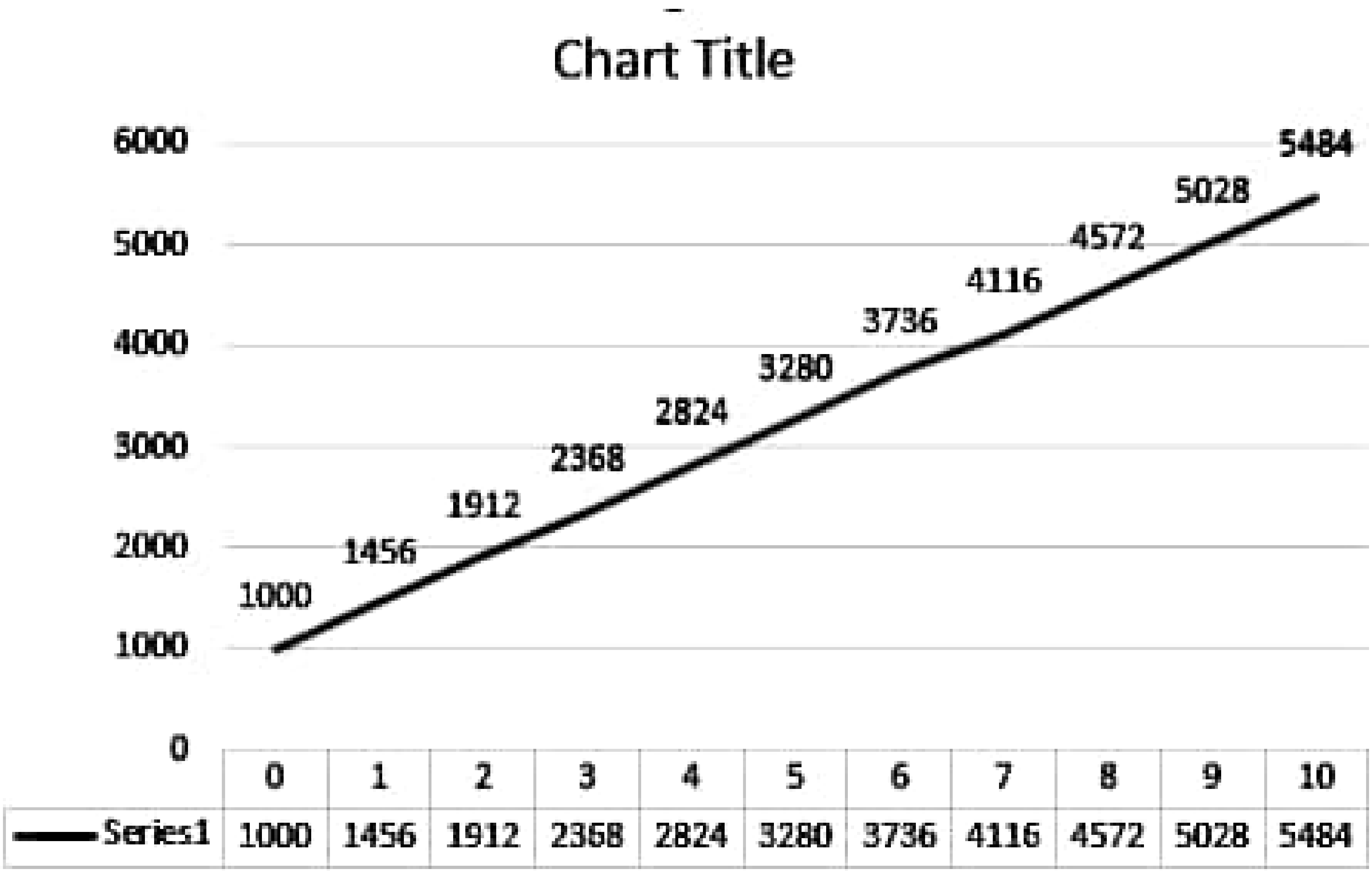


Chart Title
6000
5000
4000
3000
2000
1000
0
1000
1456
1912
2368
2824
3280
3736
4116
4572
5028
5484
0 1 2 3 4 5 6 7 8 9 10
Series1 1000 1456 1912 2368 2824 3280 3736 4116 4572 5028 5484

(f) $Q = 238(0.8^{t/4})$

$$Q'(t) = \frac{-119\ln(5) - \ln(4) \cdot 4^{t/4}}{2 \cdot 5^{t/4}}$$

And with $t = 2$

$$Q'(t) = \frac{-119\ln 5 - \ln 4 \cdot 4^{1/2}}{2 \cdot 5^{2/4}} = 119.0941$$

(g) 10% of 238 = 23.8

$$23.8 = 238\,(0.8^{t/4})$$

$$\frac{23.8}{238} = 0.8^{t/4}$$

$$0.1 = 0.8^{t/4}$$

$$\log 0.1 = (\log .8)\, t/4$$

$$\frac{\log 0.1}{\log 0.8} = t/4$$

$$10.32 = t/4 \Rightarrow t = 41.28 \text{ hours later}$$

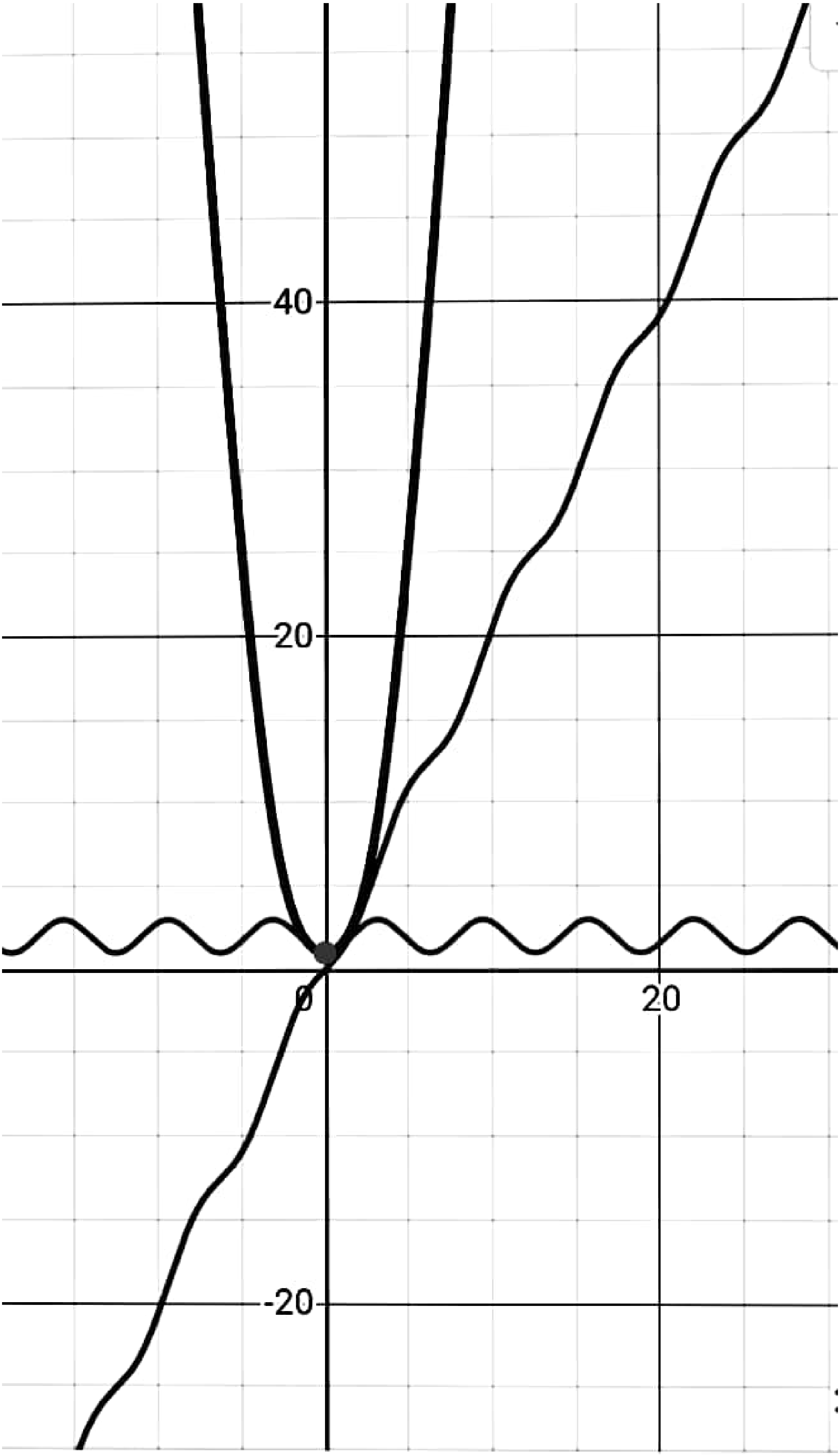
3. $y = x^2 + \cos x$

$y' = 2x + \sin x$ $\quad$ $y'$ is a graph of $y = 2x$ ie the graph has

$y'' = 2 - \cos x$ $\quad$ a gradient of 2

$y''$ is a continuous and intercepts the other curves at (0, 1). It is a continuous sine wave oscillating at $x = 2$

Scanned with CamScanner


40
20
0
20
-20

4. $N = a(1 - e^{-kt})$

$N = 5 \times 10^6$ when $t = 1$, $5 \times 10^5$ people heard the message

$500\,000 = a(1 - e^{-k(1)})$

$500\,000 = a - ae^{-k}$ --(1)

when $t = \infty$ $N = 5000\,000$

$5000000 = a(1 - e^{-k\infty}) = a$

Scanned with CamScanner

$\therefore a = 5\times10^6$

in eqn (i) $500000 = 5000000(1-e^{-k}) \Rightarrow 0.1 = 1-e^{-k}$.

$0.1 = 1-e^{-k}$.

$e^{-k} = 1-0.1 = 0.9$

$-k = \ln 0.9$

$-k = -0.1053$

$K = 0.1053$

N for $t=2$

$= 5000000\left(1-e^{-0.1053(2)}\right)$

$= 5000000\ (1-0.8101)$

$= 5000000\ (0.1899)$

$= 949500$

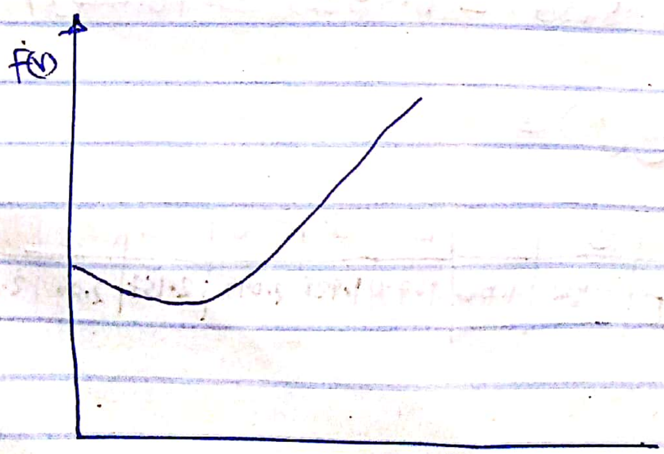
percentage of people that heard $= 18.99\%$

5. $f(v)$ = Amt of energy

a) The energy in the bird reduces as the bird is gaining momentum along the wind. The bird's speed increases as it moves along the wind. Energy increases when birds increase the frequency of their wing strokes and thereby fly faster.

b) $F(v) = Ae^{(v)}$ because the graph is that of an exponent.

c)




a(v) is minimum at zero

(d) It should minimize $f(v)$. This automatically controls $a(v)$ because the bird without flapping its wings speed won't increase

6. a)

$$A = \int_2^{10} (x + \log_e^x)^{1/3} \, dx$$

let $x + \log_e^x = u$. $\Rightarrow \frac{du}{dx} = 1 + 1 = 2$.

$du = 2dx$

$dx = \frac{du}{2}$

$$A = \int_2^{10} u^{1/3} \frac{du}{2}.$$

$$A = \frac{1}{2}\int_2^{10} u^{1/3} du.$$

$$A = \frac{1}{2}\left[\frac{u^{4/3}}{4/3}\right]_2^{10}$$

$$A = \frac{1 \times 3}{2 \cdot 4}\left[u^{4/3}\right]_2^{10}.$$

$$A = 3/8\left[\left[x + \log e^x\right]^{4/3}\right]_3^{10}$$

$$A = 3/8\left[\left((10 + 10\log e)^{4/3}\right) - (3 + 3\log e)^{4/3}\right]$$

$$3/8(34.8485 - 6.9986) = 10.4437$$

b) $y = (x + \log_e^x)^{1/3}$

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| y | 0 | 1.427 | 1.42 | 1.606 | 1.79 | 1.928 | 2.049 | 2.157 | 2.26 | 2.345 | 2.43 |

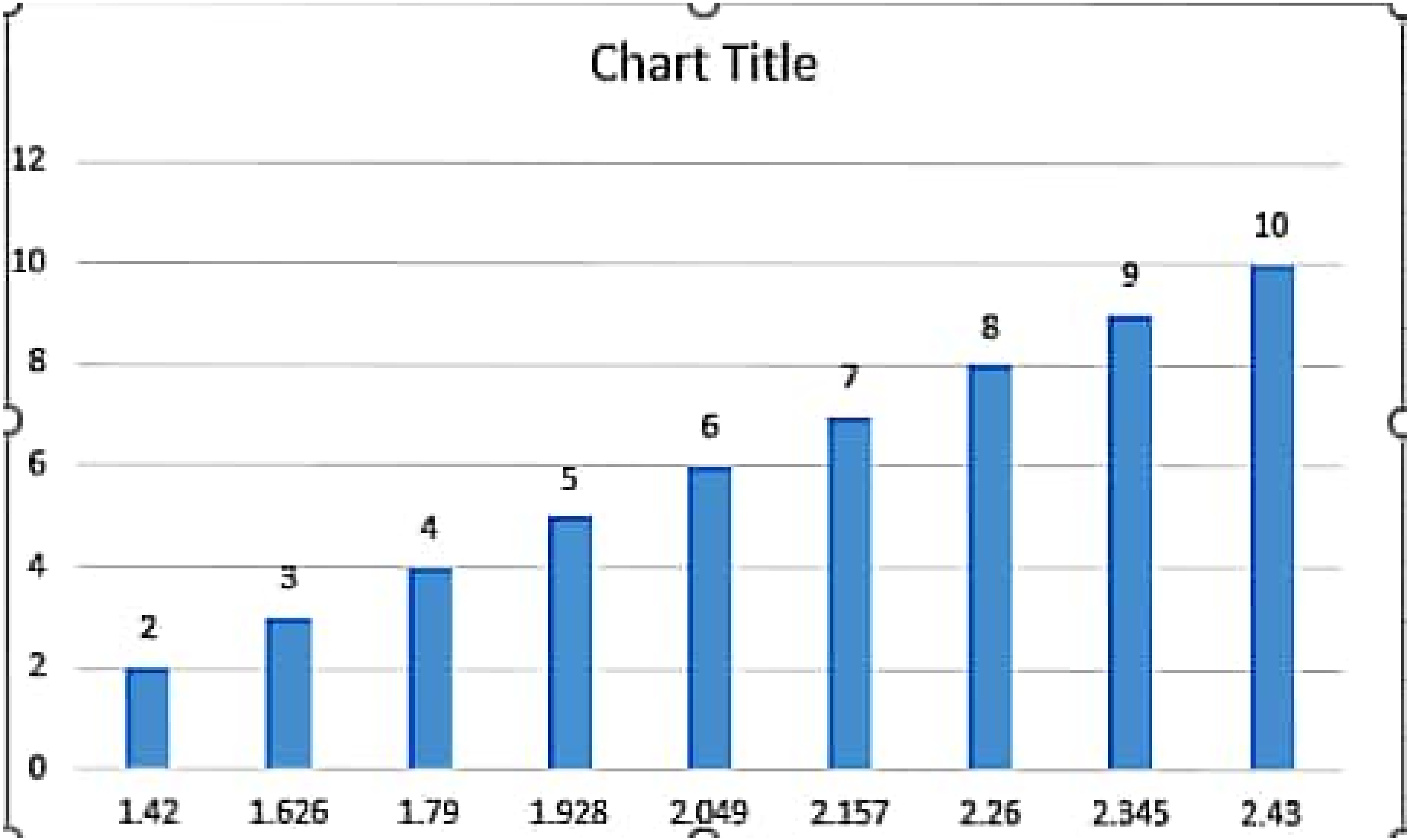


Chart Title
12
10
8
6
4
2
0
2
3
4
5
6
7
8
9
10
1.42
1.626
1.79
1.928
2.049
2.157
2.26
2.345
2.43

Interval $(2, 10)$ 8 strips

$$\Delta x = \frac{10-2}{8} = 1$$

$$LRS = \sum_{r=2}^{10} f(x)\,\Delta x$$

$$= \Delta x \{ f(2) + f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10) \}$$

$$= 1 \cancel{[1.42 + 1.626 + 1.79 + 1.928 + 2.049 + 2.157 + 2.26 + 2.345 + 2.43]} \cancel{= 18.005}$$

$$= 1 (1.42 + 1.6 + 1.75 + 1.90 + 1.98 + 2.10 + 2.20 + 2.30 + 2.40) = 17.63$$

Right Riemann Sum

$$RRS = \sum_{r=2}^{10} f(x)\,\Delta x$$

$$= \Delta x \{ f(2) + f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10) \}$$

$$= 1 \{ 1.45 + 1.63 + 1.84 + 1.94 + 2.061 + 2.176 + 2.31 + 2.40 + 2.48 ) = 19.417$$

$$\text{Actual} = 1 [ 1.427 + 1.42 + 1.626 + 1.79 + 1.928 + 2.049 + 2.157 + 2.26 + 2.345 + 2.43 ) = 19.132.$$

It's evident that LRC is an understatement and RRV is an overstatement.

b) $\Delta x = \frac{10-2}{800} = 0.01$

$$LRS = \sum_{r=2}^{10} (x + \log_e x)^{1/3}\,\Delta x$$

$$= 0.01 \{ f(2) + f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10)$$

$LRS = 0.01\{1.40 + 1.61 + 1.75 + 1.918 + 2.01 + 2.157 + 2.28 + 2.38 + 2.4$

$LRS = 0.1775$

$RRS = \Delta x\{f(2) + f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10)\}$

$= 0.01\{1.45 + 1.630 + 1.82 + 1.928 + 2.053 + 2.160 + 2.2$
$2.349 + 2.436\}$

$RRS = 0.1809$

Continuation of q to 1 (f)

1(f) $h = 1$

$y(150) = y(150)$

$X_{1800} = X_{1799} + 1 = 1800$

$y(1799) = y(1799) + h \cdot f(1799)$

$= 69362 + 1 \cdot (38) = 69400$