We want to solve the following system of equation numerically:

$$\begin{cases} \frac{\partial u}{\partial t} + g\frac{\partial h}{\partial x} = 0 \\ \frac{\partial h}{\partial t} + H\frac{\partial u}{\partial x} = 0 \end{cases}$$

where, g=9.81m/s2 and H=10km are the gravity and the mean height of the atmosphere, respectively.

1- By considering analytical solutions of the form $u(x,t) = u_0 e^{i(kx+\omega t)}$ and. $h(x,t) = h_0 e^{i(kx+\omega t)}$,

establish the relation which connects the amplitudes u0 and h0. Deduce the expression for the dispersion relation.

2- Establish the stability criterion for a diagram in centered differences. What is the maximum value of the step time for the pattern to be stable.

3- Application:

The atmosphere is represented on a scale of length Lx = 1200 km and at the initial time, it is assumed that the height of the atmosphere to a Gaussian form of the form:

$$h(X,t) = 100 \times \exp\left[-\frac{1}{2}\left(\frac{X - X_0}{s}\right)^2\right]$$

$$X_0 = 400km \text{ et } s = 80km.$$

a) Make a Matlab script that solves the system of equations (2) for a difference diagram centered. The spatial resolution is Δx = 1km and the time step is the value obtained at point 2.Represent on different graphs the amplitudes of the speed and height of the atmosphere after a period of 10 min; also display analytical solutions for comparison. How far away are the maximum amplitudes?

b) For Δt = 4s, represent on different graphs the amplitude of the speed or the height of the atmosphere after a period of 4 min. Explain the results obtained in comparison with the point (a).