

Q1

C^{-1}

$$C = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 4 & 5 \\ 0 & -1 & 2 \end{bmatrix}$$

$$|C| = 3(8+5) - 0(4-0) - 1(-2-0) = 41$$

$$C_{1,1} = 1 \times 4 \times 2 - 5 \times 1 = 13$$

$$C_{1,2} = -1 \times 2 \times 2 - 5 \times 0 = -4$$

$$C_{1,3} = 1 \times 2 \times -1 - 4 \times 0 = -2$$

$$C_{2,1} = 1 \times 0 \times 2 - 1 \times -1 = 1$$

$$C_{2,2} = 1 \times 3 \times 2 - 1 \times 0 = 6$$

$$C_{2,3} = -1 \times 3 \times -1 - 0 \times 0 = 3$$

$$C_{3,1} = 1 \times 0 \times 5 - 1 \times 4 = -4$$

$$C_{3,2} = -1 \times 3 \times 5 - 1 \times 2 = -17$$

$$C_{3,3} = 1 \times 3 \times 4 - 0 \times 2 = 12$$

$$C^{-1} = \frac{1}{|C|} \times C^T = \frac{1}{41} \begin{bmatrix} 13 & -4 & -2 \\ 1 & 6 & 3 \\ -4 & -17 & 12 \end{bmatrix}$$

Q1

$$C^{-1} = \begin{bmatrix} 13/41 & 1/41 & 4/41 \\ -4/41 & 6/41 & -17/41 \\ -2/41 & 3/41 & 12/41 \end{bmatrix}$$

$$b) A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 4 & 5 \\ 0 & -1 & 2 \end{bmatrix}$$

$$AC^{-1} + C^T$$

$$AC^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 5 \end{bmatrix} \begin{bmatrix} 13/41 & 1/41 & 4/41 \\ -4/41 & 6/41 & -17/41 \\ -2/41 & 3/41 & 12/41 \end{bmatrix}$$

$$AC^{-1} = \begin{bmatrix} 13/41 & 1/41 & 45/41 \\ -22/41 & 33/41 & 9/41 \end{bmatrix}$$

$$AC^{-1} + C^T = \begin{bmatrix} 13/41 & 1/41 & 45/41 \\ -22/41 & 33/41 & 9/41 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 0 \\ 0 & 4 & -1 \\ -1 & 5 & 2 \end{bmatrix}$$

Not possible to add. The matrices must be of the same size.

$$c) 3A + B^T$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$$

$$3A = 3 \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & -3 & 6 \\ 0 & 9 & 15 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$3A + B^T = \begin{bmatrix} 3 & -3 & 6 \\ 0 & 9 & 15 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}$$

Not possible to add.

The two matrices are not of the same size

$$d) AA^T - 2B^2$$

$$AA^T = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 7 & 34 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 19 & 24 \\ 40 & 51 \end{bmatrix}$$

$$-2B^2 = \begin{bmatrix} 0 & 70 \\ 7 & 34 \end{bmatrix} - \begin{bmatrix} 38 & 48 \\ 40 & 102 \end{bmatrix}$$

$$= \begin{bmatrix} -38 & -41 \\ -73 & -68 \end{bmatrix}$$

$$e) ((B^T)^{-1})^T - ((B^{-1})^T)^{-1}$$

$$B^T = \begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$(B^T)^{-1} = \frac{1}{|B^T|} \begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix}$$

$$(B^T)^{-1} = \frac{1}{2 \times 6 - 5 \times 3} \begin{bmatrix} 6 & -5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 5/3 \\ 1 & -2/3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{2 \times 6 - 3 \times 5} \begin{bmatrix} 6 & -3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 5/3 & -2/3 \end{bmatrix}$$

$$(B^{-1})^T = \begin{bmatrix} -2 & 5/3 \\ 1 & -2/3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = A$$

$$(B^T)^{-1} - (B^{-1})^T = \begin{bmatrix} -2 & 5/3 \\ 1 & -2/3 \end{bmatrix} - \begin{bmatrix} -2 & 5/3 \\ 1 & -2/3 \end{bmatrix}$$

$$((B^T)^{-1} - (B^{-1})^T)^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (2 \times 2 - 2 \times 0) \times 1 = 0$$

$$0 = (2 \times 0 - 0 \times 0) \times 1 = 0$$

$$\text{Determinant} = 0 \quad (2 \times 1 - 0 \times 0) \times 1 = 0$$

The matrix is not invertible.

$$0 = (2 \times 0 - 0 \times 1) \times 1 = 0$$

$$0 = (0 \times 1 - 0 \times 0) \times 1 = 0$$

$$0 = (0 \times 0 - 0 \times 1) \times 1 = 0$$

$$0 = (0 \times 0 - 0 \times 1) \times 1 = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = A$$

$$\begin{bmatrix} 2/3 & 0 \\ 2/3 & 2/3 \end{bmatrix} \begin{bmatrix} 2/3 & 0 \\ 2/3 & 2/3 \end{bmatrix} = \begin{bmatrix} 2/3 & 0 \\ 2/3 & 2/3 \end{bmatrix} = A$$

Q 2.

$$a) A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \\ 5 & 6 & 9 \end{bmatrix}$$

$$|A| = -36$$

$$C_{1,1} = 1 \times (0 \times 9 - 4 \times 6) = -24$$

$$C_{1,2} = -1 (2 \times 9 - 4 \times 5) = 2$$

$$C_{1,3} = 1 (2 \times 6 - 0 \times 5) = 12$$

$$C_{2,1} = -1 (3 \times 9 - (-1) \times 6) = -33$$

$$C_{2,2} = 1 (1 \times 9 - (-1) \times 5) = 14$$

$$C_{2,3} = -1 (1 \times 6 - 2 \times 5) = 9$$

$$C_{3,1} = 1 (3 \times 4 - (-1) \times 0) = 12$$

$$C_{3,2} = -1 (1 \times 4 - (-1) \times 2) = -6$$

$$C_{3,3} = 1 (1 \times 0 - 3 \times 2) = -6$$

$$A^{-1} = \frac{1}{|A|} \times C^T = \frac{1}{-36} \begin{bmatrix} -24 & -33 & 12 \\ 2 & 14 & -6 \\ 12 & 9 & -6 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2/3 & 11/12 & -1/3 \\ -1/18 & -7/18 & 1/6 \\ -2/3 & -3/12 & 1/6 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I = A A^{-1}$$

b) $A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $\begin{cases} 2x - y = 5 \\ y + 3z = 8 \\ 4x + 5y + 6z = 2 \end{cases}$

$$|A| = -30$$

$$C_{1,1} = 1(6 - 15) = -9$$

$$C_{1,2} = -1(0 - 12) = 12$$

$$C_{1,3} = 1(0 - 4) = -4$$

$$C_{2,1} = -1(-6 - 0) = 6$$

$$C_{2,2} = 1(12 - 0) = 12$$

$$C_{2,3} = -1(10 + 4) = -14$$

$$C_{3,1} = 1(-3 - 0) = -3$$

$$C_{3,2} = -1(6 - 0) = -6$$

$$C_{3,3} = 1(2 - 0) = 2$$

$$A^{-1} = -\frac{1}{30} \begin{bmatrix} -9 & 6 & -3 \\ 12 & 12 & -6 \\ -4 & -14 & 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{10} & -\frac{1}{5} & \frac{1}{10} \\ -\frac{2}{5} & -\frac{2}{5} & \frac{1}{5} \\ \frac{2}{15} & \frac{7}{15} & -\frac{1}{15} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Q3

9. $x + 3y - z = 2$

$2x + 4z = 7$

$5x + 6y + 9z = 0$

In matrix form. $P = (21-0)1 = 1, 0$

$$\begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \\ 5 & 6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \\ 5 & 6 & 9 \end{bmatrix}^{-1}$$

Determinant = -36

$C_{1,1} = -24$

$C_{1,2} = 2$

$C_{1,3} = 12$

$C_{2,1} = -33$

$C_{2,2} = 14$

$C_{2,3} = 9$

$C_{3,1} = 12$

$C_{3,2} = -6$

$C_{3,3} = -6$

Inverse of the matrix

$$= \frac{1}{-30} \begin{bmatrix} -24 & -33 & 12 \\ 2 & 14 & -6 \\ 12 & 9 & 6 \end{bmatrix} = \begin{bmatrix} 4/5 & 11/10 & -2/5 \\ -1/5 & -7/15 & 1/5 \\ -2/5 & -3/10 & 1/5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4/5 & 11/10 & -2/5 \\ -1/5 & -7/15 & 1/5 \\ -2/5 & -3/10 & 1/5 \end{bmatrix} \begin{bmatrix} 2 \\ 7 \\ 0 \end{bmatrix} = \begin{bmatrix} 93/10 \\ -17/5 \\ -29/10 \end{bmatrix}$$

$$x = 93/10 \quad y = -17/5 \quad z = -29/10$$

b). $2x - y = 9$

$$y + 3z = 6$$

$$4x + 5y + 6z = 0$$

In matrix form.

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 0 \end{bmatrix}$$

Inverse of the 3×3 matrix.

Determinant $= -30$

$$C_{1,1} = -9$$

$$C_{1,2} = 12$$

$$C_{1,3} = -4$$

$$C_{2,1} = 6$$

$$C_{2,2} = 12$$

$$C_{2,3} = -14$$

$$C_{3,1} = -3$$

$$C_{3,2} = -6$$

$$C_{3,3} = 2$$

Inverse of the matrix

$$= \frac{-1}{30} \begin{bmatrix} -9 & 6 & -3 \\ 12 & 12 & -6 \\ -4 & -14 & 2 \end{bmatrix} = \begin{bmatrix} 3/10 & -1/5 & 1/10 \\ -2/5 & -2/5 & 1/5 \\ 2/15 & 7/15 & -1/15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3/10 & -1/5 & 1/10 \\ -2/5 & -2/5 & 1/5 \\ 2/15 & 7/15 & -1/15 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{3a-2b+c}{10} \\ \frac{-2a-2b+c}{5} \\ \frac{2a+7b-c}{15} \end{bmatrix}$$

$$x = \frac{3a-2b+c}{10}$$

$$y = \frac{-2a-2b+c}{5}$$

$$z = \frac{2a+7b-c}{15}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 10 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

Q4

$$a) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 7 & 0 & 2 \end{bmatrix}$$

Eliminating elements in the first column.

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & -2 & -4 \\ 0 & -21 & -33 \end{bmatrix}$$

Eliminate elements in the second column.

$$\begin{bmatrix} 1 & 3 & 5 \\ 0 & -2 & -4 \\ 0 & 0 & 9 \end{bmatrix}$$

Determinant = product of the elements in the main diagonal.

$$= -1 \times (-2) \times 9$$

$$= -18$$

$$2 \times 8 = 16 \times 2 \times (8-2) \times 1 = \text{Determinant}$$

$$b) \begin{vmatrix} 1 & 3 & 4 & 5 \\ 3 & 1 & 2 & 3 \\ 0 & 0 & 3 & 0 \\ -1 & 2 & 1 & 4 \end{vmatrix}$$

Eliminate elements in the first column.

$$\begin{vmatrix} 1 & 3 & 4 & 5 \\ 0 & -8 & -10 & -12 \\ 0 & 0 & 3 & 0 \\ 0 & 5 & 5 & 9 \end{vmatrix}$$

Eliminate elements in the second column.

$$\begin{vmatrix} 1 & 3 & 4 & 5 \\ 0 & -8 & -10 & -12 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & -5/4 & 3/2 \end{vmatrix}$$

Eliminate elements in the 3rd column.

$$\begin{vmatrix} 1 & 3 & 4 & 5 \\ 0 & -8 & -10 & -12 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5/2 \end{vmatrix}$$

$$\text{Determinant} = 1 \times (-8) \times 3 \times 5/2 = -36$$

~~Q4~~ Q5

$$a) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 7 & 0 & 2 \end{bmatrix}$$

$$\begin{aligned} \text{Determinant} &= 1 \begin{vmatrix} 4 & 6 \\ 0 & 2 \end{vmatrix} - 3 \begin{vmatrix} 2 & 6 \\ 7 & 2 \end{vmatrix} + 5 \begin{vmatrix} 2 & 4 \\ 7 & 0 \end{vmatrix} \\ &= 1(8 - 0) - 3(4 - 42) + 5(0 - 28) \\ &= -18 \end{aligned}$$

$$b) \begin{vmatrix} 1 & 3 & 4 & 5 \\ 3 & 1 & 2 & 3 \\ 0 & 0 & 3 & 0 \\ -1 & 2 & 1 & 4 \end{vmatrix}$$

$$\begin{aligned} \text{Determinant} &= 1 \begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} - 3 \begin{vmatrix} 3 & 3 \\ 0 & 3 \end{vmatrix} + 4 \begin{vmatrix} 3 & 1 \\ 0 & 0 \end{vmatrix} \\ &\quad - 5 \begin{vmatrix} 3 & 1 \\ 0 & 0 \end{vmatrix} \\ &= 1(2 \cdot 3 - 3 \cdot 3) - 3(3 \cdot 3 - 0 \cdot 3) + 4(3 \cdot 0 - 1 \cdot 0) \\ &\quad - 5(3 \cdot 1 - 0 \cdot 0) \\ &= 1(6 - 9) - 3(9 - 0) + 4(0 - 0) - 5(3 - 0) \\ &= 1(-3) - 3(9) + 4(0) - 5(3) \\ &= -3 - 27 + 0 - 15 \\ &= -45 \end{aligned}$$

$$1 \mid 3 \ 0 \mid -2 \mid 0 \ 0 \mid +3 \mid 0 \ 3 \mid -9 \mid 3 \ 0 \mid +6 \mid 0 \ 0 \mid$$

$$\mid 1 \ 4 \mid \mid 2 \ 4 \mid \mid 2 \ 1 \mid \mid 0 \ 1 \ 4 \mid \mid -1 \ 4 \mid$$

$$-9 \mid 0 \ 3 \mid +12 \mid 0 \ 0 \mid -4 \mid 0 \ 0 \mid +12 \mid 0 \ 0 \mid -15 \mid 0 \ 3 \mid$$

$$\mid -1 \ 1 \mid \mid 2 \ 4 \mid \mid -1 \ 4 \mid \mid 2 \ -1 \ 2 \mid \mid 2 \ 1 \mid$$

$$+5 \mid 0 \ -3 \mid -10 \mid 0 \ 0 \mid$$

$$\mid -1 \ 1 \mid \mid -1 \ 2 \mid$$

$$= -30$$

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$$\begin{vmatrix} 2 & 4 & 3 & 1 \\ 3 & 5 & 1 & 5 \\ 0 & 3 & 0 & 0 \\ 4 & 1 & 3 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 3 & 1 & 3 \\ 0 & 0 & 0 \\ 4 & 5 & 1 \end{vmatrix} + \begin{vmatrix} 3 & 3 & 3 \\ 0 & 3 & 0 \\ 7 & 1 & 1 \end{vmatrix} - \begin{vmatrix} 3 & 3 & 1 \\ 0 & 3 & 0 \\ 4 & 1 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 3 & 1 \\ 0 & 3 & 0 \\ 4 & 1 & 5 \end{vmatrix} = 4$$

$$\begin{vmatrix} 3 & 1 & 3 \\ 3 & 0 & 0 \\ 1 & 3 & 1 \end{vmatrix} = 2$$

$$b) \begin{vmatrix} 1 & 3 & 4 & 5 \\ 3 & 1 & 2 & 3 \\ 0 & 0 & 3 & 0 \\ -1 & 2 & 1 & 4 \end{vmatrix}$$

Eliminate elements in the first column.

$$\begin{vmatrix} 1 & 3 & 4 & 5 \\ 0 & -8 & -10 & -12 \\ 0 & 0 & 3 & 0 \\ 0 & 5 & 5 & 9 \end{vmatrix}$$

Eliminate elements in the second column.

$$\begin{vmatrix} 1 & 3 & 4 & 5 \\ 0 & -8 & -10 & -12 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & -5/4 & 3/2 \end{vmatrix}$$

Eliminate elements in the 3rd column.

$$\begin{vmatrix} 1 & 3 & 4 & 5 \\ 0 & -8 & -10 & -12 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5/2 \end{vmatrix}$$

$$\text{Determinant} = 1 \times (-8) \times 3 \times 5/2 = -36$$

Q6

$$x + 3y - z = 2$$

$$2x + 4z = 7$$

$$5x + 6y + 9z = 0$$

Matrix

$$\text{Determinant} = \begin{vmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \\ 5 & 6 & 9 \end{vmatrix} = -36$$

$$\begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \\ 5 & 6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ 0 \end{bmatrix}$$

$$\text{Determinant } x = \begin{vmatrix} 2 & 3 & -1 \\ 7 & 0 & 4 \\ 0 & 6 & 9 \end{vmatrix} = -279$$

$$\text{Determinant } y = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 7 & 4 \\ 5 & 0 & 9 \end{vmatrix} = 102$$

$$\text{Determinant } z = \begin{vmatrix} 1 & 3 & 2 \\ 2 & 0 & 7 \\ 5 & 6 & 0 \end{vmatrix} = 87$$

$$X = \frac{\text{Determinant } X}{\text{Determinant}} = \frac{-279}{-30} = \frac{93}{10}$$

$$Y = \frac{102}{-30} = \frac{-17}{5}$$

$$Z = \frac{87}{-30} = \frac{-29}{10}$$

$$X = \frac{93}{10}$$

$$Y = \frac{-17}{5}$$

$$Z = \frac{-29}{10}$$

$$b) \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & -3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ c \end{bmatrix}$$

$$\text{Determinant } \Delta = -30$$

$$\Delta_1 = \begin{vmatrix} 9 & -1 & 0 \\ 6 & 1 & 3 \\ c & 5 & 6 \end{vmatrix} = -99 + 66 - 3c$$

$$\frac{1}{2} (-25 f(x_0) + 48 f(x_0 + h) - 36 f(x_0 + 2h) + 16 f(x_0 + 3h))$$

Q7

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$$

a) $\begin{vmatrix} d & e & f \\ a & b & c \\ 3g & 3h & 3i \end{vmatrix}$

Determinant

$$= d \begin{vmatrix} b & c \\ 3h & 3i \end{vmatrix} - e \begin{vmatrix} a & c \\ 3g & 3i \end{vmatrix} + f \begin{vmatrix} a & b \\ 3g & 3h \end{vmatrix}$$

$$= d(3bi - 3hc) - e(3ai - 3gc) + f(3bh - 3gb)$$

$$= 3dbi - 3hcd - 3aie - 3gce + 3fah - 3gbf$$

$$= -3a(ei - fh) + 3b(di - gf) - 3c(hd + ge)$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= 3a(ei - fh) - 3b(di - gf) + 3c(hd + ge)$$

$$3a(ei - fh) - 3b(di - gf) + 3c(hd + ge) = 3$$

$$\begin{vmatrix} a & e & f \\ b & d & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix} = 3$$

$$\Delta_2 = \begin{vmatrix} 2 & a & 0 \\ 0 & b & 3 \\ 4 & c & 6 \end{vmatrix} = 12a + 12b - 6c$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & a \\ 0 & 1 & b \\ 4 & 5 & c \end{vmatrix} = -4a - 14b + 20$$

$$X = \frac{\Delta_1}{\Delta} = \frac{-9a + 6b - 3c}{-30} = \frac{3a - 2b + c}{10}$$

$$Y = \frac{\Delta_2}{\Delta} = \frac{12a + 12b - 6c}{-30} = \frac{-2a - 2b + c}{5}$$

$$Z = \frac{\Delta_3}{\Delta} = \frac{-4a - 14b + 20}{-30} = \frac{2a + 7b - c}{15}$$

$$X = \frac{3a - 2b + c}{10}$$

$$Y = \frac{-2a - 2b + c}{5}$$

$$Z = \frac{2a + 7b - c}{15}$$

$$a(ei - hf) + b(di - gf) + c(dh - eg) = 3$$

$$-a(ei - hf) + b(di - gf) - c(dh - eg) = -3$$

$$-3a(ei - hf) + 3b(di - gf) - 3c(dh - eg)$$

$$\text{Determinant} = -\frac{1}{3}$$

$$b) \begin{vmatrix} a & b & c \\ a+2d & b+2e & c+2f \\ e & f & g \end{vmatrix}$$

$$\text{Determinant} = a \begin{vmatrix} b+2e & c+2f \\ h & i \end{vmatrix} - b \begin{vmatrix} a+2d & c+2f \\ g & i \end{vmatrix} + c \begin{vmatrix} a+2d & b+2e \\ g & h \end{vmatrix}$$

$$a[i(b+2e) - h(c+2f)] - b[i(a+2d) - g(c+2f)] +$$

$$c[h(a+2d) - g(b+2e)]$$

$$a[ib + 2ei - hc - 2fh] - b[ai + 2di - gc - 2fg] +$$

$$c[hg + 2dh - gb - 2eg]$$

$$[ib - hc - 2fh]a - [ai - gc - 2fg]b + [hg - gb - 2eg]c$$

$$c) \begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} = (ae - db)h - (ad - bc)f + (bh - ec)a$$

$$\text{Determinant} = a \begin{vmatrix} e & h \\ f & i \end{vmatrix} - d \begin{vmatrix} b & h \\ c & i \end{vmatrix} + g \begin{vmatrix} b & e \\ c & f \end{vmatrix}$$

$$\begin{aligned} &= a(ei - fh) - d(bi - ch) + g(bf - ec) \\ &= aei - afh - dbi + dch + gbf - gec \\ &= a(ei - fh) - b(di - gf) + c(dh - ge) \end{aligned}$$

$$\text{Determinant} = 3.$$

$$d) \begin{vmatrix} a & b & c \\ d & f & g \\ a-d & b-f & c-g \end{vmatrix}$$

$$= a \begin{vmatrix} f & g \\ b-f & c-g \end{vmatrix} - b \begin{vmatrix} d & g \\ a-d & c-g \end{vmatrix} + c \begin{vmatrix} d & f \\ a-d & b-f \end{vmatrix}$$

$$\begin{aligned} &= a[f(c-g) - g(b-f)] - b[d(c-g) - g(a-d)] + \\ &\quad c[d(b-f) - f(a-d)] \end{aligned}$$

$$= a(fc - gf - gb + gf) - b(dc - dg - ga + dg) + c(db - df - fa + fd)$$

$$= a(fc - gb) - b(dc - ga) + c(db - fa)$$

$$= 0$$

$$= a(fc - gb)$$