

MU123

TMA 04

2019J

Covers Units 10, 11 and 12

Cut-off date 14 April 2020

Submission instructions

You will find instructions for completing TMAs in the ‘Assessment’ area of the MU123 website. Please read these instructions before beginning work on this TMA.

Reviewing your tutor’s comments on your previous TMA will help you as you work on this one.

Special instructions

Remember that you need to explain your reasoning and communicate your ideas clearly, as described in Subsection 5.3 of Unit 1. This includes:

- explaining your mathematics in the context of the question
- the correct use of notation and units
- appropriate rounding.

Your score out of 5 for good mathematical communication (GMC) will be recorded against Question 8. You do not have to submit any work for Question 8.

Question 1 – 18 marks

This question is based on your work on MU123 up to and including Unit 10.

(a) This part of the question concerns the quadratic equation

$$4x^2 - 4x + 1 = 0.$$

- (i) Find the discriminant of the quadratic expression $4x^2 - 4x + 1$. [2]
- (ii) What does this tell you about the number of solutions of the equation? Explain your answer briefly. [2]
- (iii) What does this tell you about the graph of $y = 4x^2 - 4x + 1$? [1]

(b) Figure 1 shows part of the graph of the quadratic function $y = 3x^2 - 9x - 30$.

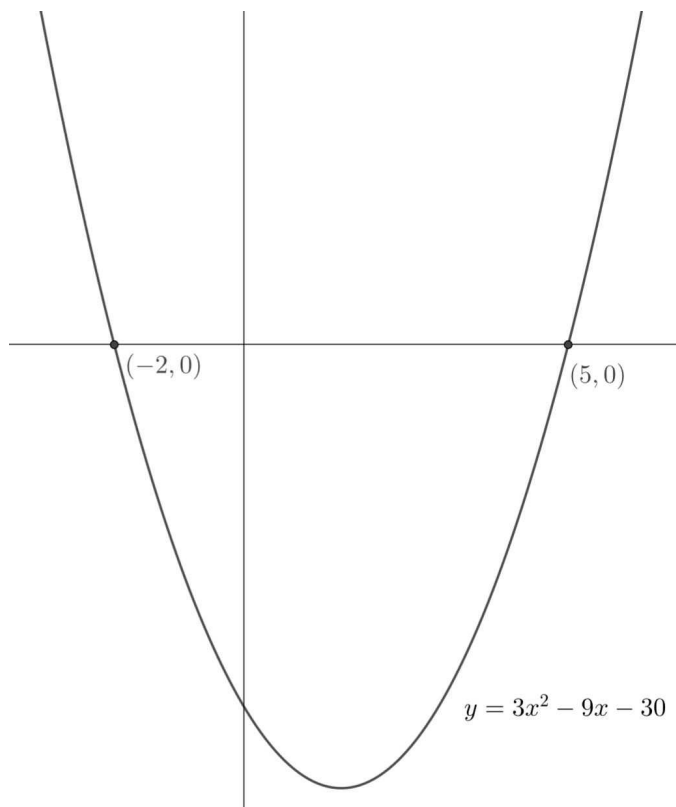


Figure 1

- (i) Find the equation of the axis of symmetry of the parabola, explaining your method. [2]
 - (ii) Use your answer to part (b)(i) to find the coordinates of the vertex of the parabola. [2]
- (c) This part of the question concerns the quadratic function

$$y = x^2 - 12x - 22.$$

- (i) Write the quadratic expression $x^2 - 12x - 22$ in completed-square form. [2]
- (ii) Use the completed-square form from part (c)(i) to solve the equation $x^2 - 12x - 22 = 0$, leaving your answer in exact (surd) form, simplified as far as possible. [3]

- (iii) Use the completed-square form from part (c)(i) to write down the vertex of the parabola $y = x^2 - 12x - 22$. [1]
- (iv) Provide a sketch of the graph of the parabola $y = x^2 - 12x - 22$, either by hand or by using Graphplotter. [3]
- If you intend to go on to study more mathematics, you are advised to sketch by hand. Whichever method you choose, you should refer to the graph-sketching strategy box in Subsection 2.4 of Unit 10 for information on how to sketch and label a graph correctly.*

Question 2 – 12 marks

This question is based on your work on MU123 up to and including Unit 10.

Hannah is a computer games designer. She is developing a simple golf simulator which will allow a user to select a golf club and then, using a computer-generated golfer, hit a typical golf shot for the club selected. The trajectory of the golf ball after it is struck by the virtual golfer is modelled by a quadratic equation of the form

$$y = ax^2 + bx + c,$$

where y represents the height (in metres) of the ball above the ground, and x represents the horizontal distance (in metres) of the ball from the position where it is struck. The values of a , b and c are determined by the computer code, depending on which club is selected, and the angle and force at which the ball is struck. (Assume that the golf course is completely flat and horizontal during the development.)

- (a) Hannah wants to check that her code is working correctly so she simulates a test golf swing for a driver. The quadratic equation generated by her code is

$$y = 0.0025x^2 + 8.0227x + 0.0491.$$

By considering the coefficient of x^2 , explain why Hannah's code must contain an error. [1]

- (b) Hannah corrects the error, and runs the code again to simulate another test swing from a driver. This time the quadratic equation generated by the code is

$$y = -0.0016x^2 + 0.4801x + 0.0489.$$

In this part of the question, you are asked to consider the trajectory of the golf ball modelled by this equation. (The graph of $y = -0.0016x^2 + 0.4801x + 0.0489$ is a parabola.)

- (i) Explain why the y -intercept is 0.0489. [1]
- (ii) What does the y -intercept represent in the context of this model? [1]
- (iii) Use the quadratic formula to find the x -intercepts. Give your answers correct to four decimal places. [4]
- (iv) In the simulation, the hole is 325 metres away from where the virtual golfer teed off. Explain why the golfer can't possibly have scored a hole-in-one. [2]
- (v) Hannah decides to add a tree which stands 10 metres tall to the virtual golf course. She positions it directly in the path of the test shot, at a distance of 270 metres horizontally from where the ball is struck. Will the golf ball clear the tree? Explain your answer. [3]

Question 3 – 20 marks

This question is based on your work on MU123 up to and including Unit 11.

Jakub is a teacher in a secondary school. He teaches two different classes at the same level and stage. He is interested in comparing the performance of his two classes on the same exam. The raw data is shown in Table 1.

Table 1 Marks (out of 100) for a class test

Class A	Class B
35	0
37	12
41	20
42	30
49	31
50	33
51	40
55	45
56	48
59	51
60	60
62	65
62	65
64	66
66	70
79	72
80	76
83	80
85	81
90	86
91	90
95	93
97	100

- (a) Jakub considers displaying the data as a dotplot. Do you think this would be a good choice for representing this data? Give a reason for your answer. [2]
- (b) Jakub decides to create a boxplot for each class as displayed in Figure 2.

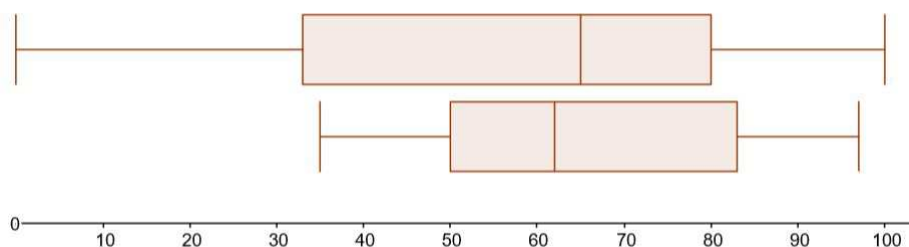


Figure 2

With reference to Subsection 1.2 of Unit 11, state **three** ways in which Jakub could improve the clarity of his presentation of the boxplots. [3]

- (c) Jakub can't remember which boxplot belongs to which class. State one way in which he could check by looking at the raw data for each class. [1]
- (d) Jakub realises that the top boxplot represents the data for Class B and the bottom boxplot represents the data for Class A. He tries to interpret what the data is telling him by making the statements below. For each one, state whether you agree or disagree with Jakub's interpretation, and explain why he is correct or incorrect.
- (i) 'On average, Class A performed better than Class B.' [2]
 - (ii) 'There was more variability in the marks from Class B than from Class A.' [2]
 - (iii) 'In Class A, more students scored less than the median mark than scored greater than the median mark.' [2]
- (e) Use the boxplot for Class B to say whether the data are symmetrical or skewed. If the data are skewed, then state whether they are skewed to the left or skewed to the right, explaining your reasoning briefly. [2]
- (f) Jakub creates histograms for each of the two classes, shown in Figure 3 and Figure 4 below.

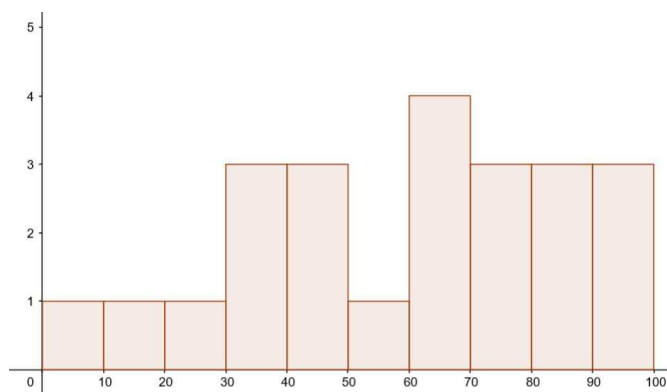


Figure 3

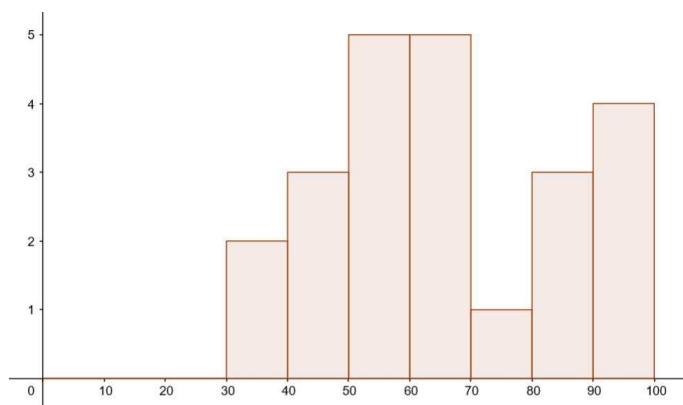


Figure 4

- (i) Which histogram belongs to Class A – Figure 3 or Figure 4? Give a reason for your answer. [2]
- (ii) Comment on one aspect of the performance of the two classes that can be seen more easily on the histograms than on the boxplots. [2]
- (iii) Comment on one aspect of the performance of the two classes that can be seen more easily on the boxplots than on the histograms. [2]

Question 4 – 19 marks

This question is based on your work on MU123 up to and including Unit 12.

- (a) Figure 5 shows a sketch of a triangle, where all side lengths are measured in centimetres. Find the length of the side marked x , giving your answer correct to the nearest centimetre. [3]

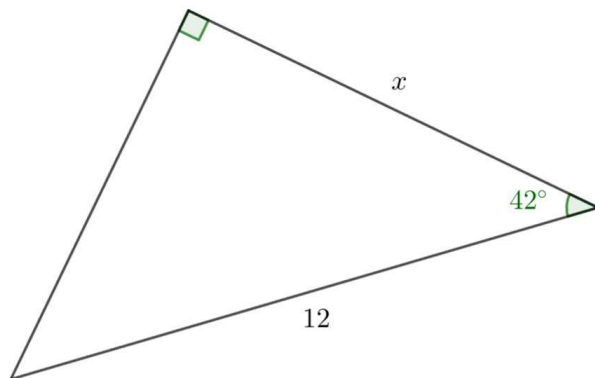


Figure 5

- (b) Triangle PQR has a right angle at R . The length of side QR is 6.9 cm, and the length of side RP is 4.7 cm. Find $\angle PQR$, giving your answer correct to the nearest degree. [3]
You may find it useful to sketch triangle PQR in your answer.

- (c) (i) Figure 6 shows the triangle ABC , where all side lengths are measured in centimetres. Find $\angle ABC$, giving your answer correct to the nearest degree. [5]

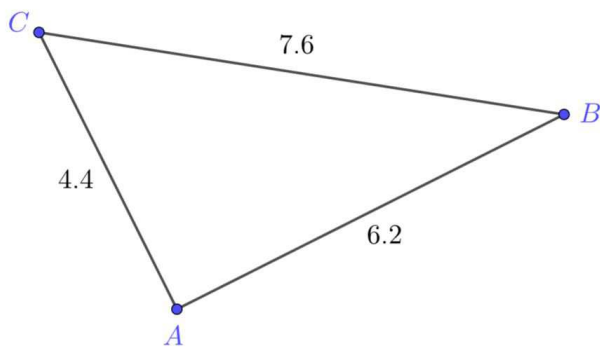


Figure 6

- (ii) Find the area of the triangle ABC in Figure 6, giving your answer correct to the nearest square centimetre. [3]
- (d) (i) Convert 33° to radians, leaving your answer in terms of π . [2]
- (ii) Use your answer from part (d)(i) to find the area of a sector of a circle of radius 4.7 centimetres and angle 33° , giving your answer correct to two significant figures. [3]

Question 5 – 11 marks

This question is based on your work on MU123 up to and including Unit 12.

You should use trigonometry, not scale drawings, to find your answers.

Caitlin is testing the strength of mobile phone signal she gets depending on how close she is to two transmitters. She knows that Transmitter A is exactly 5 kilometres due west of Transmitter B. Caitlin walks in a straight line in a roughly south-westerly direction from Transmitter A for exactly 2 kilometres. From this position, she can see both transmitters, and measures that the angle between her lines of sight from Transmitter A to Transmitter B is 56° .

- (a) Caitlin would like to sketch a diagram of the situation, using A to represent the position of Transmitter A, B for the position of Transmitter B, and C for the position of Caitlin herself. Sketch a diagram showing the points A , B and C , marking in the angle and the lengths that you are given. Join the three points with line segments to make the triangle ABC . [2]
- (b) Caitlin would like to calculate the distance between herself and Transmitter B. She realises that in triangle ABC she has two side lengths and an angle. She erroneously concludes that she can solve her problem with a single direct application of the Cosine Rule. Explain, as if directly to Caitlin, why she can't use the Cosine Rule directly to solve her problem. [2]
- (c) (i) Use the Sine Rule to find the angle at B . Give your answer correct to the nearest degree. [3]
- (ii) Use your answer to part (c)(i) to find the angle at A . Give your answer correct to the nearest degree. [1]
- (iii) Use the Cosine Rule and your answer to part (c)(ii) to find the distance between Caitlin and Transmitter B. Give your answer correct to one decimal place. [3]

Question 6 – 10 marks

This question is based on your work on MU123 up to and including Unit 12.

In this question, you are asked to comment on a student's attempt at answering the question detailed below. Read the question and the student's incorrect attempt first, then answer the questions below.

The Question

Jamie would like to buy a webcam for videoconferencing meetings which take place along the length of a long conference table which is 7 metres long. He is considering buying a webcam whose lens covers an angle of 100° . The webcam will be placed against the wall of the conference room at a distance of 3 metres from the conference table.

Jamie sketches the situation in Figure 7, where all lengths are measured in metres. The point C represents the camera's position against the wall. The point M represents the midpoint of the conference table. Points A and B are the furthest reaches of the camera's range at this distance. The length of BC is equal to the length of AC . Angle BCA is 100° , so angle BCM is 50° .

Will the camera be able to cover the entire length of the table? (You may assume that if the entire length of the table is visible, the people sitting behind it are also visible, and that no one will sit at either end of the table.)

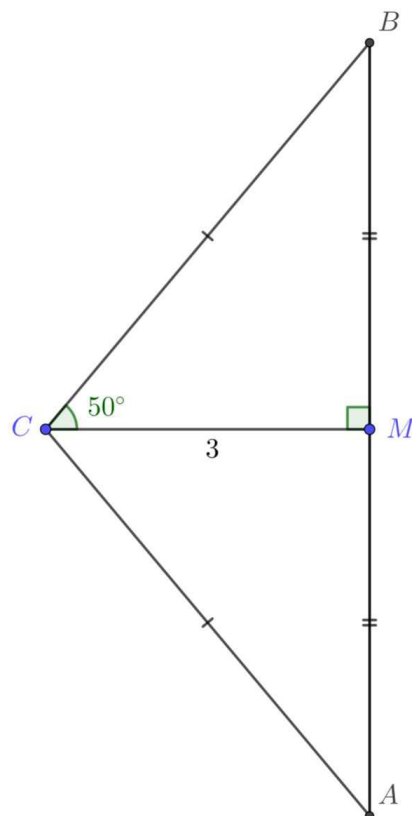


Figure 7

The student's incorrect attempt

Triangle CBM is a right angled triangle. We have that

$$\cos 50^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{3}{BC} \quad \text{so} \quad BC = 3 \cos 50^\circ = 1.928...$$

and so the length of BC is 1.93 m (to 2 d.p.)

using the Cosine rule in triangle ABC, and the fact that the length of BC is the same as the length of AC, we have that

$$AB^2 = BC^2 + AC^2 - 2 \times BC \times AC \times \cos C$$

$$AB^2 = 1.93^2 + 1.93^2 - 2 \times 1.93 \times 1.93 \times \cos 100^\circ$$

$$AB^2 = 8.743...$$

$$AB = \sqrt{8.743...}$$

$$AB = 2.956...$$

So the length of AB is 2.96 m (to 2 d.p.)

Since the conference table is 7 m long, the camera will not be able to film the whole length of the table

- (a) Using what you know about right-angled triangles, and without performing any calculations, explain how you know that the student's answer for the length BC must be wrong. [1]
- (b) There are two places in the student's attempt where a mistake has been made. Identify these mistakes, and explain, as if directly to the student, why, for each mistake, their working is incorrect. [4]
- (c) Do you think the student's approach to solving the problem is the most efficient method? Give a reason for your answer. [1]
- (d) Write out your own solution to the problem, explaining your working. [4]

Question 7 – 5 marks

In this question you are asked to reflect on how your studies so far on MU123 fit in with your wider study interests and future study plans.

- (a) Choose one topic from MU123 that you think is relevant to your wider study interests. Explain why you think your chosen topic is particularly interesting and/or relevant to your future study plans.

If you don't find any of the topics from MU123 interesting or relevant to your future study plans, then explain why. [2]

- (b) Do you feel confident in your chosen topic? If so, describe one step you could take to maintain your confidence in years to come. If not, describe one step you could take to help you work more confidently in that topic.

If you did not choose one specific topic in your answer to part (a), choose one topic from MU123 in order to answer this part. For example, you could choose a topic which you found particularly challenging. [3]

Question 8 – 5 marks

A score out of 5 marks for good mathematical communication over the entire TMA will be recorded under Question 8. [5]
