

Physics Lab Manual

Department of Physics and Astronomy

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Contents

1	Uniformly Accelerated Linear Motion	6
1.1	Introduction and Theory	6
1.1.1	Motion along a Straight Line	6
1.1.2	Newton's Laws	7
1.1.3	Learning Outcomes	7
1.2	Experimental Setup and Measurements	7
1.2.1	Apparatus	7
1.2.2	Watch the Videos	8
1.2.3	Experiment and Data Collection	8
1.2.4	Calculations and Data Analysis	9
1.2.5	Questions	10
1.3	Prepare your report	11
2	Static and Kinetic Friction	12
2.1	Introduction and Theory	12
2.1.1	Learning Outcomes	14
2.2	Experimental Setup and Measurements: Static Friction	14
2.2.1	Apparatus	14
2.2.2	Watch the Videos	14
2.2.3	Experiment and Data Collection	14
2.2.4	Calculations and Data Analysis	15
2.2.5	Questions	17
2.3	Experimental Setup and Measurements: Kinetic Friction	17
2.3.1	Watch the Videos	17
2.3.2	Experiment and Data Collection	18
2.3.3	Calculations and Data Analysis	19
2.3.4	Questions	21
2.4	Prepare your report	21
3	Energy and Its (non)conservation	22
3.1	Introduction and Theory	22
3.1.1	Conservation of energy in general and in gravitational case	22
3.1.2	The case of a bouncing ball	23

3.1.3	Learning Outcomes	26
3.2	Experimental Setup and Measurements	26
3.2.1	Apparatus	26
3.2.2	Watch the Videos	26
3.2.3	Experiment and Data Collection	26
3.2.4	Calculations and Data Analysis	27
3.2.5	Questions	30
3.3	Prepare your report	30
4	Oscillations	31
4.1	Introduction and Theory	31
4.1.1	Motion of a Pendulum	31
4.1.2	Learning Outcomes	33
4.2	Experimental Setup and Measurements: Measuring g	34
4.2.1	Apparatus	34
4.2.2	Watch the Videos	34
4.2.3	Experiment and Data Collection	34
4.2.4	Calculations and Data Analysis	35
4.2.5	Questions	36
4.3	Experimental Setup and Measurements: Dependence of T on L	36
4.3.1	Apparatus	37
4.3.2	Watch the Videos	37
4.3.3	Experiment and Data Collection	37
4.3.4	Calculations and Data Analysis	38
4.3.5	Questions	39
4.4	Prepare your report	39
5	Angular Motion	40
5.1	Introduction and Theory	40
5.1.1	Kinematics	40
5.1.2	Dynamics	41
5.1.3	System of a pulley and a mass	41
5.1.3.1	General dynamics	41
5.1.3.2	Analysis related to the lab experiment	43
5.1.4	Learning Outcomes	43
5.2	Experimental Setup and Measurements	43
5.2.1	Apparatus	44
5.2.2	Watch the Videos	44
5.2.3	Experiment and Data Collection	44
5.2.4	Calculations and Data Analysis	45
5.2.5	Questions	46
5.3	Prepare your report	47

6	Static Equilibrium	48
6.1	Introduction and Theory	48
6.1.1	Equilibrium in extended objects	48
6.1.2	Analysis related to the lab experiment: traditional scales	49
6.1.3	Learning Outcomes	50
6.2	Experimental Setup and Measurements: a primitive scale to measure masses	50
6.2.1	Apparatus	50
6.2.2	Watch the Videos	51
6.2.3	Experiment and Data Collection	51
6.2.4	Calculations and Data Analysis	52
6.2.5	Questions	53
6.3	Prepare your report	54
7	Elasticity	55
7.1	Introduction and Theory	55
7.1.1	Analysis related to the lab experiment	56
7.1.2	Learning Outcomes	57
7.2	Experimental Setup and Measurements	57
7.2.1	Apparatus	57
7.2.2	Watch the Videos	57
7.2.3	Experiment and Data Collection	58
7.2.4	Calculations and Data Analysis	59
7.2.5	Questions	60
7.3	Prepare your report	60
8	Collisions	61
8.1	Introduction and Theory	61
8.1.1	Analysis related to the lab experiment	62
8.1.2	Learning Outcomes	64
8.2	Experimental Setup and Measurements	64
8.2.1	Apparatus	64
8.2.2	Watch the Videos	64
8.2.3	Experiment and Data Collection	64
8.2.4	Calculations and Data Analysis	65
8.2.5	Questions	66
8.3	Prepare your report	67
A	Measuring Mass	68
A.1	Method 1: Archimedes principle	68
A.2	Method 2: constructing a simple scale	73
B	phyphox App	74
B.1	Using Acoustic Timer	74

C	Uncertainty Calculations using Differentials	76
C.1	Addition $p = x + y$	76
C.2	Subtraction $p = x - y$	77
C.3	Multiplication $p = xy$	77
C.4	Division $p = \frac{x}{y}$	78
C.5	Exponentiation $p = x^a$	78
C.6	More general cases and examples	79
C.6.1	Example 1	79
D	Some Functions of Excel	81
D.1	Changing the significant figures	81

Chapter 1

Uniformly Accelerated Linear Motion

1.1 Introduction and Theory

1.1.1 Motion along a Straight Line

Four quantities: displacement, velocity, acceleration, along with time, are sufficient to describe the motion of an object along a straight line. The displacement Δx is defined as the change in the position of the object:

$$\Delta x = x - x_0 \quad (1.1)$$

This is for motion in one dimension only. Remember the important role $\pm$ signs play here. The velocity v is the rate at which the position changes.

$$v = \frac{dx(t)}{dt} \quad (1.2)$$

Recall that taking the derivative of a function is equivalent to finding the slope of that function. The instantaneous velocity is the slope of the the line tangent to a position vs time curve. The acceleration a is defined as the rate of change of velocity.

$$a = \frac{dv(t)}{dt} \quad (1.3)$$

And the slope of the tangent line to the $v(t)$ graph gives the instantaneous acceleration. The second important geometrical interpretation of the $v(t)$ graphs is that the area under the graph for some time interval gives the displacement of the object in that time interval. In this experiment you will consider a motion with constant acceleration. For this kind of motion the acceleration is related to velocity by:

$$v_x = v_{0x} + a_x t \quad (1.4)$$

and

$$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2 \quad (1.5)$$

1.1.2 Newton's Laws

The relationship between motion and forces is described by Newton's Laws. According to Newton's First Law, if the net force acting on an object is zero, the velocity of an object will not change. Newton's Second Law states that when a net force is exerted on an object, the object is accelerated. Mathematically, the relation between the net force and acceleration is given by:

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \quad (1.6)$$

where $\mathbf{F}_{\text{net}}$ is the net force, $\mathbf{a}$ is the acceleration, and m is the mass of the object.

Newton's Third Law states that forces are always present in pairs when two different objects A and B interact with each other. One of these two forces is called action (exerted by A on B), and the other is reaction (exerted by B on A). These two forces are equal in magnitude and have opposite direction. In this lab you will analyze the motion of an object falling freely. In this case, the only force responsible for the motion (the net Force) is the force of gravity mg which is directed downwards. Thus, according to Newton's second law:

$$\left\{ \begin{array}{ll} F_{\text{net}} = ma & \text{(Newton's 2nd law)} \\ F_{\text{net}} = -mg & \text{(specific form of the force due to gravity on Earth)} \end{array} \right\} \Rightarrow a = -g \quad (1.7)$$

where $g = 9.81\text{m/s}^2$, is the acceleration due to Earth's gravity.

1.1.3 Learning Outcomes

- Learn how to use computer and smartphone hardware and software to record data
- Analyze graphs of position and velocity as a function of time for a straight-line motion
- Study the relationship between forces and motion

1.2 Experimental Setup and Measurements

In this experiment you will measure the Earth's gravitational acceleration g close to its surface. You are allowed to get someone, not necessarily in the class, to help you with measurements if needed.

1.2.1 Apparatus

See the video for more details.

- Smartphone with the app phyphox installed on it: <https://phyphox.org/>
- Computer with access to Microsoft Excel (Excel is part of Office365 which is available to York University students free of charge)

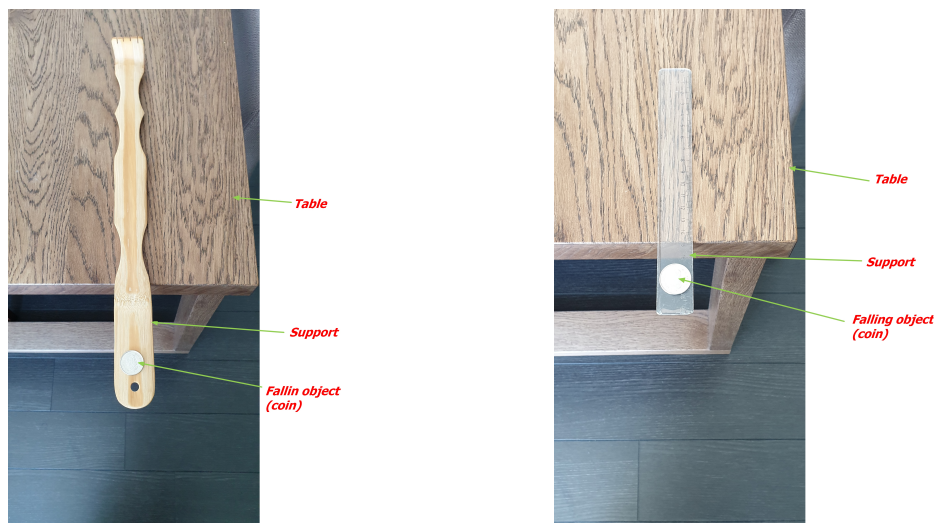


Figure 1.1: The falling object (here a coin) sitting on the end part of the support (here a wooden rectangular object or a plastic ruler).

- A metal coin. This will be referred to as the **falling object**.
- An object in shape of a rectangle to support the initial position of the coin (such as a ruler). It will be referred to as the **support**.
- Ruler. If you do not have one, you can install the the apps “Ruler” by Xalpha Lab or “Ruler” by NixGame on your smartphone.

1.2.2 Watch the Videos

Watch the video on how to use the **phyphox** app for this experiment and measurements.

1.2.3 Experiment and Data Collection

All the data should be recorded in SI units.

Step 1 Place the support on a table in your home such that a small part of the support is not on the surface of the table (Fig. 1.1)

Step 2 Measure the distance from the ground to the support. We will call this value h .

Step 3 Place the falling object near the end part of the support which is off the table’s surface (Fig. 1.1)

Step 4 Open the app on your **phyphox** smartphone, under **Timers**, click on **Acoustic Stopwatch** (Fig. B.1, left)

Step 5 Now (Fig. B.1, right) you will see a tab on top with options **SIMPLE**, **SEQUENCE**, **PARALLEL**, and **MANY**. For this activity we will use the **SIMPLE** option. Click on this option and then in the **Threshold** field, enter 0.1. Also, in the **Minimum Delay** field enter 0.1. These are the fields that can be changed based on your environment noise and the experiment. The lower the threshold, the more sensitive the app will be to starting and ending sound signals. If you find out that the starting and ending sound signals do not trigger the stopwatch, or other lower sounds undesirably trigger the stopwatch, change the **Threshold** value accordingly.

Step 6 Click on **Reset** (Fig. B.1, right).

Step 7 Click on the play button on top of the app screen (Fig. B.1, right).

Step 8 With a pen, hit the support side-wise such that it slips from below the falling object, and the falling object starts its downward motion. You should hit it with a force that makes a sound detectable for the stopwatch app, but not too strongly so that the coin does not move sideways (see the video).

Step 9 The app will be triggered by the initial sound signal (pen hitting the support) and the final sound signal (the falling object hitting the ground) and will record the time between these two events (Fig. B.1, right).

Step 10 Repeat this experiment for 10 times, and record the resulting times that the app gives you.

Step 11 Repeat the same activity for 10 more times but now using a different table or kitchen counter in your house with a different height compared to the previous table. Record the resulting new times that the app gives you.

1.2.4 Calculations and Data Analysis

In this activity, we take the vertical distance as y . The initial position of the falling object is $y_0 = h$ the height of the support from the ground, its initial velocity is $v_{0y} = 0$, and the acceleration comes with a negative sign, so equation (1.5) becomes

$$y - y_0 = -\frac{1}{2}a_y t^2 + v_{0y}t \Rightarrow -h = -\frac{1}{2}a_y t^2. \quad (1.8)$$

From this one gets

$$g = a_y = \frac{2h}{t^2}. \quad (1.9)$$

Using this relation and the data collected in the previous part, fill in the following table in the Excel sheet and determine the acceleration of this vertical linear motion. The table looks like the following

Height 1				
Trial	h (m)	t (s)	g (m/s ²)	$\frac{\delta g}{g_0}$
1				
2				
3				
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
10				
Mean				
Standard deviation				

- In the second column, enter the height of each measurement. For each table, all of the h of the 10 trial are the same.
- In the third column enter the measured values of time of free fall you recorded in **Acoustic Stopwatch** using **phyphox**.
- In the fourth column enter the computed value of g from equation 1.9. To compute it use the h and t values in the same row.
- Given that the acceleration due to gravity is $g_0 = 9.81 \text{ m/s}^2$ on the surface of Earth, find the fractional uncertainty

$$\frac{\delta g}{g_0} = \frac{g_{\text{measured}} - g_0}{g_0} \quad (1.10)$$

for each of your 10 measurements, and enter them in the last column.

- Finally you should also compute the mean and standard deviation of the values of the third and fourth columns and enter it in the appropriate place under their columns in the Excel file.
- Attach a clear photo of your experimental setting to the Excel file in the designated place. You can attach only one photo for one of the heights/tables.

Do this same experiment for the other table with a different height h .

The significant figures for the values of the measurements you get and enter in the tables should match the uncertainty in your measurement devices. For example if you ruler can only measure up to a millimeter, any SI measurements should have three significant figures, e.g., 0.250 m. To increase/decrease the significant figures in a cell in Excel, see Appendix D, particularly Fig. D.1.

1.2.5 Questions

Answer the following questions in the Excel template sheet.

1. Identify three errors that may happen in this activity and explain the source of each error.
2. What can be done to decrease each of the errors you mentioned above?

1.3 Prepare your report

Send the following documents as your lab report

- Answered pre-lab questions
- Filled Excel sheet

Chapter 2

Static and Kinetic Friction

2.1 Introduction and Theory

Forces of friction are of immense importance because up to $\frac{1}{3}$ of all energy produced in the world is wasted because of them. In this lab you will be investigating static and kinetic, dry forces of friction, as opposed to fluid friction (viscous drag). There are two types of friction forces that we investigate in this lab. One type, prevents objects from starting to move and thus is pertinent to the case where object is still at rest. This friction force is appropriately called the static friction force and is denoted by $\mathbf{f}_s$. The other type is a force that acts on a moving object from the environment and is directed against the direction of motion. This is called the kinetic friction force and is denoted by $\mathbf{f}_k$. Note that these are vectors since they are forces. Hence they have both magnitude and direction.

The direction of $\mathbf{f}_s$ is such that it tries to prevent an object at rest from starting to move. Its magnitude, denoted by f_s , can take any value from 0 to a maximum value

$$f_s^{\max} = \mu_s N, \quad (2.1)$$

which depends on two quantities. A scalar $\mu_s > 0$ which is called the coefficient of static friction and the magnitude of the normal force $\mathbf{N}$ on the object denoted by N . Here, the word “normal” means that $\mathbf{N}$ is perpendicular to the contact surface between the object and medium to which it is in contact. Thus $\mathbf{N}$ is also perpendicular to the the direction of $\mathbf{f}_s$ if $f_s \neq 0$. All of the above statements can be concisely written as an inequality

$$\boxed{f_s \leq \mu_s N}, \quad (2.2)$$

which implied that f_s is not constant, but is varying depending on the situation.

The direction of $\mathbf{f}_k$ is against the direction of motion. Its magnitude is constant and given by

$$\boxed{f_k = \mu_k N}. \quad (2.3)$$

Here the scalar $\mu_k > 0$ is the coefficient of kinetic friction.

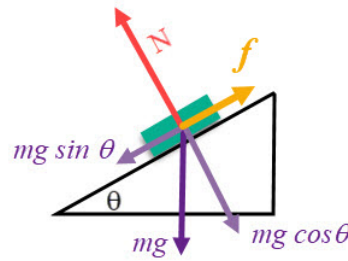


Figure 2.1: Free-body diagram of a block on an inclined surface. If the green block is stationary $f = f_s$, and if moving, $f = f_k$

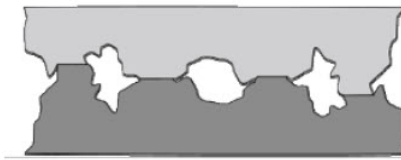


Figure 2.2: Contact between surfaces seen at a microscopic level.

As a classic example take the situation in Fig. 2.1. A green block is sitting on an inclined surface with the angle of inclination given by θ . If the green block is still not moving, then the friction force $f = f_s$. If the figure depicts a block that is moving then $f = f_k$.

As mentioned before, the kinetic friction is constant and does not depend on the speed of the object, although at very low and very high speeds this may not be true. Also, notice that in the case of a moving object, if the object is sliding, one uses the kinetic friction formula, while if the object is rolling, one must use the static friction formula.

Static and kinetic forces of friction do not depend on the apparent surface area, although they depend on the actual or real contact area between the two surfaces. The difference between the apparent and actual contact areas is depicted in the Fig. 2.2. At the microscopic level, surfaces are very rough. Therefore, two surfaces are never in total contact. The object that slides is supported by the surface at the top of roughness irregularities, called asperities. These asperities deform until the weight of the object is fully supported. Approximately only 1 part in 10,000 of the apparent area is usually in contact, which is the reason why friction is effectively independent of the apparent area of contact. For very smooth surfaces the actual contact area may be a significant fraction of the apparent contact area and in this case friction is dependent on the size of the surface.

The actual contact area depends on the magnitude of the normal force N . The larger the normal force the more the two surfaces push against each other, increasing the actual contact area and the force of friction. Because friction, from the microscopic point of view, occurs due to interactions between electrical charges, the larger the actual contact area between two surfaces, the larger the force of friction. The electromagnetic interactions that causes friction is quite complex and it is still the topic of current research.

2.1.1 Learning Outcomes

- Learn how to use computer and smartphone hardware and software to record data
- Understand the difference between static and kinetic friction and their origins

2.2 Experimental Setup and Measurements: Static Friction

In this experiment, you will study static friction with two different inclined surfaces and two slipping objects. You are allowed to get someone, not necessarily in the class, to help you with measurements if needed.

2.2.1 Apparatus

See the video for more details.

- Smartphone with the app **phyphox** installed on it: <https://phyphox.org/>
- Computer with access to Microsoft Excel (Excel is part of Office365 which is available to York University students free of charge)
- Two objects with stiff and flat surfaces (such as hardcover books or clean cutting boards), to be used as inclined surfaces. This is referred to as the **inclined surface**.
- Two small preferably cubic-shaped objects that will slip down the inclined surfaces. Such an object will be called the **sliding object**.
- Ruler. If you do not have one, you can install the the apps “Ruler” by Xalpha Lab or “Ruler” by NixGame on your smartphone.

2.2.2 Watch the Videos

Watch the video on how to use the **phyphox** app for this experiment and measurements.

2.2.3 Experiment and Data Collection

All the data should be recorded in SI units.

Step 1 Measure the mass of the sliding object. If you do not have a scale, you can use one of the methods mentioned in Appendix A.

Step 2 Put the inclined surface (book, etc.) on a flat surface such as a table.

Step 3 The inclined surface has a length, a width and a height. Measure the length of the inclined surface (Fig. 2.3) with a ruler and write it down. This value is called L .

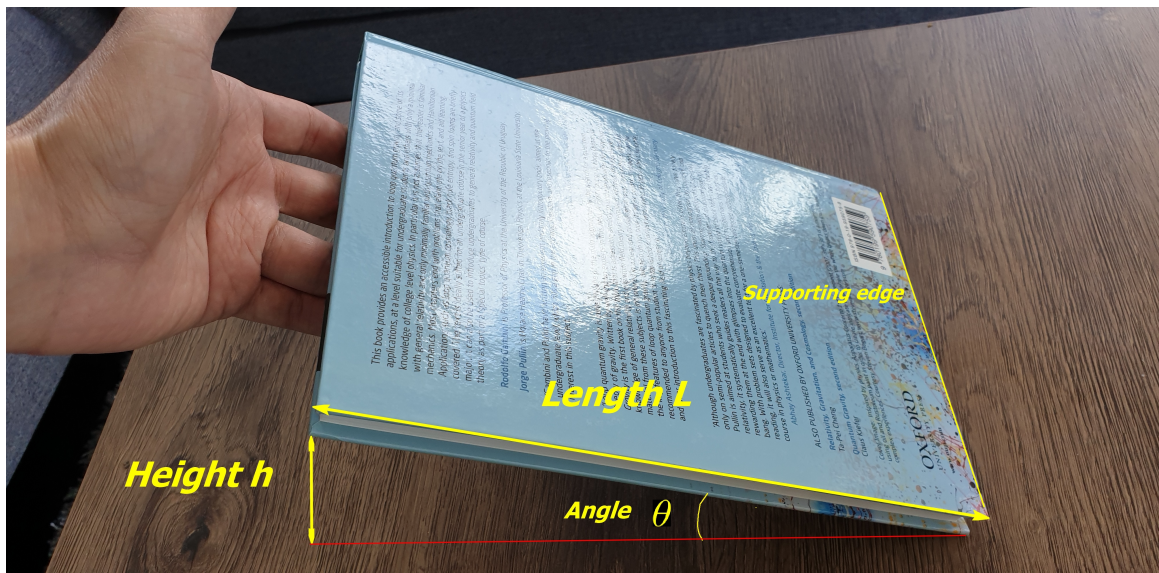


Figure 2.3: The variables L , h , θ and the supporting edge in a book used as an inclined surface.

Step 4 Place the sliding object on the inclined surface. Slowly increase the angle θ between the inclined surface and the table, keeping the supporting edge of the inclined surface attached to the table (Fig. 2.3), until the sliding object starts to move. You can consider bracing the object against a heavier object on the table.

Step 5 As soon as the sliding object starts to move, stop increasing the angle, and measure the height of the side of the book opposite to the supporting edge from the table and call it h (see Fig. 2.3). Write down this value for each trial in the table presented in the next subsection. Note that there will be 4 tables each corresponding to one of a sliding object 1 or 2 and the inclined surface 1 or 2.

Repeat this experiment 5 times for each sliding object and each surface. At the end you will have 20 test runs.

2.2.4 Calculations and Data Analysis

You should fill the following table in the Excel template sheet for each sliding object-inclined surface system (4 tables each with 5 test runs). Before filling the table, in appropriate cells provided in Excel, enter what did you use as inclined surfaces and as you sliding masses. Now fill the table as follows.

Object 1 - inclined surface 1						
Trial	m (kg)	h (m)	θ (deg)	N (N)	$f_s^{\max}$ (N)	μ_s
1						
2						
3						
4						
5						
Mean						
Standard Deviation						

- In the second column enter the mass of the sliding object 1 (2). This will be the same value for all the 5 trials.
- In the third column, enter the values of h in each trial.
- In the fourth column, the value of inclination angle θ for each trial is to be entered. Using the values of height h and length L that you measured in the previous subsection, you can find the inclination angle θ as

$$\sin(\theta) = \frac{h}{L} \Rightarrow \theta = \sin^{-1}\left(\frac{h}{L}\right). \quad (2.4)$$

Enter this value for each trial in the column under θ . Make sure the angle is in degrees. If your calculator gives you the value directly in degrees you are all set. Otherwise, if it gives you a value in radians, you can find the corresponding angle in degrees by using

$$\theta(\text{deg}) = \frac{180}{\pi}\theta(\text{rad}). \quad (2.5)$$

- In the fifth column enter the magnitude of the normal force. To find this value, note that using the free body diagram 2.1, one can see that

$$N - mg \cos(\theta) = 0 \Rightarrow N = mg \cos(\theta). \quad (2.6)$$

Enter this value for each trial in the rows under N .

- In the sixth column enter the maximum value of the static friction force $f_s^{\max}$. To find this, we see from Fig. 2.1 that as soon as the green block starts to move, the static friction force f_s should have increased to its maximum value that is equal to the gravitation force $mg \sin(\theta)$. This means that

$$f_s^{\max} - mg \sin(\theta) = 0 \Rightarrow f_s^{\max} = mg \sin(\theta) \quad (2.7)$$

Using this formula and the values of m , g , θ in the table, find the values of $f_s^{\max}$ for each trial and fill in the rows under $f_s^{\max}$.

- In the seventh column, computed value of μ_s for each trial should be entered. These values can be obtained using Eq. (2.1) and hence

$$\mu_s = \frac{f_s^{\max}}{N}, \quad (2.8)$$

where you need to use the values of N and $f_s^{\max}$ from the table.

- Finally find the mean and standard deviation of each column and write it below them in the table.
- Attach a clear photo of your experimental setting to the Excel file in the designated place.

The significant figures for the values of the measurements you get and enter in the tables should match the uncertainty in your measurement devices. For example if your ruler can only measure up to a millimeter, any SI measurements should have three significant figures, e.g., 0.250 m. To increase/decrease the significant figures in a cell in Excel, see Appendix D, particularly Fig. D.1

2.2.5 Questions

Answer the following questions in the Excel template sheet.

1. Identify three errors that may happen in this activity and explain the source of each error.
2. What can be done to decrease each of the errors you mentioned above?
3. From the data you have, does it seem that the values of the coefficient of friction μ_s depend on the mass of the slipping object? Explain your answer using your data.

2.3 Experimental Setup and Measurements: Kinetic Friction

In this experiment, you will study the kinetic friction with two different inclined surfaces and two slipping objects. You are allowed to get someone, not necessarily in the class, to help you with measurements if needed.

2.3.1 Watch the Videos

Watch the video on how to use the **phyphox** app for this experiment and measurements.

2.3.2 Experiment and Data Collection

All the data should be recorded in SI units.

Step 1 Record the mass of the sliding object. If you do not have a scale, you can use one of the methods mentioned in Appendix A.

Step 2 Put the inclined surface (book, etc.) on a flat surface such as a table.

Step 3 The inclined surface has a length, a width and a height. Measure the length of the inclined surface (Fig.) with a ruler and write it down. This value is called L .

Step 4 Open the app on your phyphox smartphone, under Timers, click on **Acoustic Stopwatch** (Fig. B.1, left)

Step 5 Now (Fig. B.1, right) you will see a tab on top with options **SIMPLE**, **SEQUENCE**, **PARALLEL**, and **MANY**. For this activity we will use the **SIMPLE** option. Click on this option and then in the **Threshold** field, enter 0.1. Also, in the **Minimum Delay** field enter 0.1. These are the fields that can be changed based on your environment noise and the experiment. The lower the threshold, the more sensitive the app will be to starting and ending sound signals. If you find out that the starting and ending sound signals do not trigger the stopwatch, or other lower sounds undesirably trigger the stopwatch, change the **Threshold** value accordingly.

Step 6 Click on **Reset** (Fig. B.1, right). Keep this screen on on your smartphone, as we will need it in Step 10.

Step 7 Later, we will support the inclined surface only on one of its sides that is shorter (that is, a width). Let us call this the supporting edge of the inclined surface (Fig. 2.3). While the inclined surface is still lying flat on the table, put the sliding object close to the other edge of the inclined surface (not the supporting edge), and measure the distance from the supporting edge of the inclined surface to the side of the sliding object that is closer to the supporting edge. This distance is called d . Write down this distance. Do not change the position of the sliding object.

Step 8 Keep the sliding object in place (e.g., by pressing down on it with your finger) and at the same time change the angle of the inclined surface to an angle that is larger than any of the angles you measured in the previous activity. Measure the height of the bottom side of the inclined surface opposite to the supporting edge from the table and call it h (see Fig. 2.3). Write down this value for each trial in the table presented in the next subsection. Note that there will be 4 tables each corresponding to one of a sliding object 1 or 2 and the inclined surface 1 or 2.

Step 9 When the sliding object leaves the inclined surface, it must make a noise loud enough to trigger the stopwatch. If necessary, you can put a metal object or ceramic cup on the table right next to the supporting edge of the inclined surface for the sliding object to hit and make a noise.

Step 10 Click on the play button on top of the **phyphox** app screen (Fig. B.1, right).

Step 11 Release the sliding object, and at the time of release, make a sound (e.g., loudly say “go” or tap on a surface close to the smartphone) so that the first sound trigger of the **Acoustic Stopwatch** of the **phyphox** app is activated. The sliding object will slide down the inclined surface and will hit the table or cup at the bottom of this surface, activating the second sound trigger on **Acoustic Stopwatch**. Record the time that the **Acoustic Stopwatch** registers for each trial. Also watch out for early and premature triggering of the stopwatch due to noise produced by the motion of the object on the inclined surface. If this happens before the object hits the cup at the bottom of the inclined surface, change the **Threshold** in the **Acoustic Stopwatch** accordingly until the app is only triggered by initial and final sounds.

Repeat this experiment 5 times for each sliding object and each surface. At the end you will have 20 test runs.

2.3.3 Calculations and Data Analysis

You should fill the following table in the Excel template sheet for each sliding object-inclined surface system (4 tables each with 5 test runs). Before filling the table, in appropriate cells provided in Excel, enter what did you use as inclined surfaces and as you sliding masses. Now fill the table as follows.

Object 1 - inclined surface 1							
Trial	m (kg)	h (m)	t (s)	θ (deg)	N (N)	f_k (N)	μ_k
1							
2							
3							
4							
5							
Mean							
Standard Deviation							

- In the second column enter the mass of the sliding object 1 (2). This will be the same value for all the 5 trials.
- In the third column, enter the values of h in each trial.
- In the fourth column enter the value of the time intervals that you recorded using in **Acoustic Stopwatch** each trial.
- In the fifth column, the value of inclination angle θ for each trial is to be entered. Using the values of height h and length L that you measured in the previous subsection, you can find the inclination angle θ as

$$\sin(\theta) = \frac{h}{L} \Rightarrow \theta = \sin^{-1}\left(\frac{h}{L}\right). \quad (2.9)$$

Enter this value for each trial in the column under θ . Make sure the angle is in degrees. If your calculator gives you the value directly in degrees you are all set. Otherwise, if it gives you a value in radians, you can find the corresponding angle in degrees by using

$$\theta(\text{deg}) = \frac{180}{\pi}\theta(\text{rad}). \quad (2.10)$$

- In the sixth column enter the magnitude of the normal force. To find this value, note that using the free body diagram 2.1, one can see that

$$N - mg \cos(\theta) = 0 \Rightarrow \boxed{N = mg \cos(\theta)}. \quad (2.11)$$

Enter this value for each trial in the rows under N .

- In the seventh column enter the value of the kinetic friction force f_k . To find this, we see from Fig. 2.1 that (taking the left direction as the positive direction)

$$mg \sin(\theta) - f_k = ma, \quad (2.12)$$

and thus

$$f_k = m[g \sin(\theta) - a]. \quad (2.13)$$

But we do not have a yet. To find it, we use the equation of motion

$$x - x_0 = \frac{1}{2}at^2 + v_{0x}t, \quad (2.14)$$

with $x_0 = 0 = v_{0x}$ and $x = d$, hence

$$a = \frac{2d}{t^2}. \quad (2.15)$$

Substituting this into equation (2.13) yields

$$\boxed{f_k = m \left[g \sin(\theta) - \frac{2d}{t^2} \right]}. \quad (2.16)$$

Using this formula and the values of m , g , θ , d and t in the table, find the values of f_k for each trial and fill in the rows under f_k .

- In the eighth column, computed value of μ_k for each trial should be entered. These values can be obtained using Eq. (2.3) and hence

$$\boxed{\mu_k = \frac{f_k}{N}}, \quad (2.17)$$

where you need to use the values of N and f_k from the table.

- Finally find the mean and standard deviation of each column and write it below them in the table.
- Attach a clear photo of your experimental setting to the Excel file in the designated place.

The significant figures for the values of the measurements you get and enter in the tables should match the uncertainty in your measurement devices. For example if your ruler can only measure up to a millimeter, any SI measurements should have three significant figures, e.g., 0.250 m. To increase/decrease the significant figures in a cell in Excel, see Appendix D, particularly Fig. D.1.

2.3.4 Questions

Answer the following questions in the Excel template sheet.

1. Identify three errors that may happen in this activity and explain the source of each error.
2. What can be done to decrease each of the errors you mentioned above?
3. From the data you have, does it seem that the values of the coefficient of friction μ_k depend on the mass of the slipping object? Explain your answer using your data.
4. Does μ_k depend on the velocity of the object? What evidence do you have?

2.4 Prepare your report

Send the following documents as your lab report

- Answered pre-lab questions
- Filled Excel sheet

Chapter 3

Energy and Its (non)conservation

3.1 Introduction and Theory

3.1.1 Conservation of energy in general and in gravitational case

Energy is the capability of systems to do work, i.e., to move objects, change their shapes, etc. In one type of categorization, energy can be divided into either kinetic or potential energy. Kinetic energy K as its name suggests, is related to the state of the motion, i.e., the speed of the object. In mathematical terms, for a single particle it is defined as

$$K = \frac{1}{2}mv^2, \quad (3.1)$$

where m is the mass of the object and $v = |\mathbf{v}|$ is its speed. Note that this energy can be defined for one or more particles. For a system of more than one particle, the total kinetic energy is just the sum of the kinetic energies of each particle. This is because energy is a scalar. Also note that if $v = 0$ then $K = 0$.

The potential energy is related to the configuration of the system, i.e., how the constituents of the system are positioned with respect to each other and what are their distances respectively. Thus it cannot be defined just for one particle. One needs at least a system of two particles so that “configuration” or “position with respect to each other” makes sense and is well-defined. Potential energy comes in various forms: gravitational potential energy, spring potential energy, electromagnetic potential energy, etc. In this experiment we focus on gravitational potential energy.

The gravitational potential energy for the system of Earth and any small object close to the surface of Earth is defined as

$$U = mgh, \quad (3.2)$$

where m is the mass of the object, $g = 9.81 \text{ m/s}^2$ is the acceleration of objects close to Earth due to its gravity, and h is the distance of the object from the surface of the Earth. As you can see, the variable that determines “positioning” or “configuration” of the constituents of the system (Earth and the object) is h .

The law of conservation of energy states that energy can be converted from kinetic to potential and vice versa, or from one potential energy form to another potential energy form, but it cannot be created or destroyed (This is only true “locally” in a small region in space and in time. In the whole expanding Universe as in ours, the energy is in fact not conserved!). More precisely it states that the total mechanical energy of a system

$$E = K + U \quad (3.3)$$

is conserved, i.e., its value now is the same as its value in any time in the future or in the past. Mathematically this is written as

$$E_1 = E_2 \Rightarrow \boxed{K_1 + U_1 = K_2 + U_2} \quad (3.4)$$

where index “1” refers to any initial configuration of the system while index “2” mean any final configuration. In case of gravitational potential energy, equation (3.4) becomes

$$\frac{1}{2}mv_1^2 + mgh_1 = \frac{1}{2}mv_2^2 + mgh_2. \quad (3.5)$$

In real world, some of the energy is always “wasted”, meaning that it is absorbed into the environment in form of heat or change of the geometrical shape of the objects (think of the change in the shape of cars after an accident, or the heat released when a meteorite hits the Earth). This is due to friction and in general electromagnetic forces between the particles that made up the macroscopic objects. In such cases, we have to take into account the “wasted” or “lost” energy into the equation (3.4) and write

$$E_1 + Q = E_2, \quad (3.6)$$

or

$$\boxed{K_1 + U_1 + Q = K_2 + U_2}, \quad (3.7)$$

where as we can see from equation (3.6),

$$Q = E_2 - E_1, \quad (3.8)$$

is the wasted or lost energy that has gone into the environment as heat, or is used to change the shape of the constituents of the system, and is the difference between the total mechanical energy of configuration 1 and configuration 2. In case of gravitational potential energy, equation (3.7) becomes

$$\boxed{\frac{1}{2}mv_1^2 + mgh_1 + Q = \frac{1}{2}mv_2^2 + mgh_2}. \quad (3.9)$$

3.1.2 The case of a bouncing ball

Let us consider the familiar situation of a ball released from a certain height h_0 and bouncing several times until it rests on the ground (Fig. 3.1). If mechanical energy was fully conserved

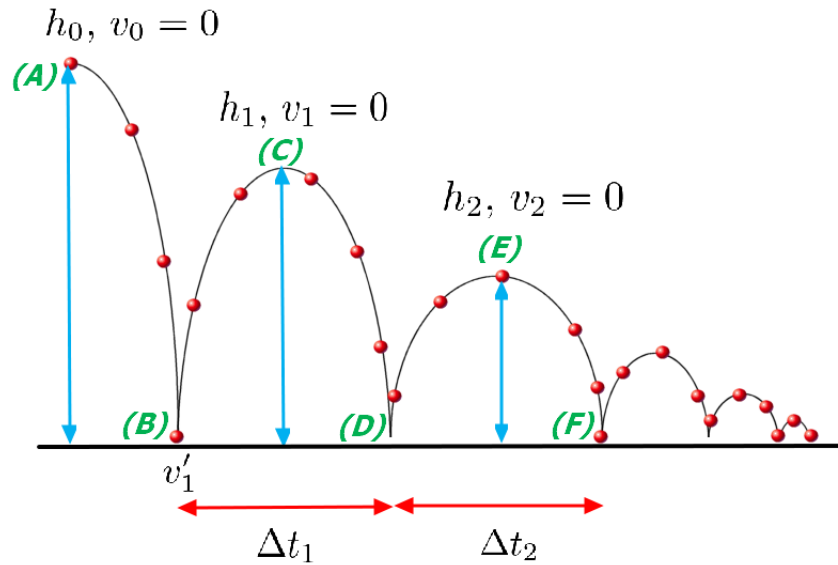


Figure 3.1: The bouncing ball in case where at each bounce, some energy is lost.

and no energy would be lost, then the ball would bounce up and down infinite times and each time it would reach the same height. However, in real world this does not happen since every time that the ball hits the ground it loses some mechanical energy (see questions).

Let us take the heights after each bouncing h_1, h_2 as shown in Fig. 3.1. For the two configuration points (A) and (C), we can write from Eq. (3.9)

$$\frac{1}{2}mv_0^2 + mgh_0 + Q_1 = \frac{1}{2}mv_1^2 + mgh_1, \quad (3.10)$$

where Q_1 is the energy lost when the ball hits the ground for the first time. Given that $v_0 = 0 = v_1$ we get

$$Q_1 = mg(h_1 - h_0). \quad (3.11)$$

Note that this is a negative number since $h_1 < h_0$, hence the energy is lost. If we had h_1 , could have computed Q_1 . Since we do not have the means to measure it experimentally in our lab, we need to compute it using the data we have access to via experiment. That piece of data is Δt_1 which as can be seen from Fig. 3.1 is the time between the first time the ball hits the ground and the second time it does.

Using the conservation of energy between points (B) and (C), where there is no energy loss (we neglect air drag), the height h_1 can be computed as

$$\frac{1}{2}mv_1'^2 = mgh_1 \Rightarrow h_1 = \frac{v_1'^2}{2g}. \quad (3.12)$$

Here if there were no energy loss when the ball hits the ground, then v_1' would have been equal to v_1 (and their directions would be inverse of each other), but now some energy is lost

when the ball hits the ground and thus $v'_1 < v_1$. On the other hand, by using equations of motion between points (B) and (C), and assuming that the ball bounces back up with an angle close to 90° , we can write

$$v_1 - v'_1 = -g\Delta t \quad (3.13)$$

where

$$\Delta t = \frac{\Delta t_1}{2} \quad (3.14)$$

is the time in which the ball goes from point (B) to point (C). Using the above formula and also since $v_1 = 0$, equation (3.13) becomes

$$0 - v'_1 = -g\frac{\Delta t_1}{2} \Rightarrow v'_1 = g\frac{\Delta t_1}{2}. \quad (3.15)$$

Substituting this value into (3.12) we get

$$\boxed{h_1 = \frac{1}{8}g(\Delta t_1)^2}. \quad (3.16)$$

So we have used the quantity Δt_1 we measured experimentally, to deduce h_1 theoretically. Using this value of h_1 from (3.16) in (3.11) yields

$$\boxed{Q_1 = mg \left[\frac{1}{8}g(\Delta t_1)^2 - h_0 \right]}. \quad (3.17)$$

The same line of reasoning for points (C) and (E) complemented by equations of motion between (D) and (E) yields

$$\boxed{h_2 = \frac{1}{8}g(\Delta t_2)^2}, \quad (3.18)$$

and

$$\boxed{Q_2 = mg \left[\frac{1}{8}g(\Delta t_2)^2 - h_1 \right] = \frac{1}{8}mg^2 [(\Delta t_2)^2 - (\Delta t_1)^2]}, \quad (3.19)$$

which will be negative as expected since $\Delta t_2 < \Delta t_1$.

The ratio of the lost energy after the first bounce (point(C)) to the initial energy at point (A) is

$$\boxed{\frac{Q_1}{U_1} = \frac{mg \left[\frac{1}{8}g(\Delta t_1)^2 - h_0 \right]}{mgh_0} = \frac{\frac{1}{8}g(\Delta t_1)^2 - h_0}{h_0}}. \quad (3.20)$$

In the same way, the ratio of the lost energy after the second bounce (point(E)) to the energy at the first bounce point (C) is

$$\boxed{\frac{Q_2}{U_2} = \frac{\frac{1}{8}mg^2 [(\Delta t_2)^2 - (\Delta t_1)^2]}{mgh_1} = \frac{\frac{1}{8}mg^2 [(\Delta t_2)^2 - (\Delta t_1)^2]}{\frac{1}{8}mg^2 (\Delta t_1)^2} = \frac{(\Delta t_2)^2 - (\Delta t_1)^2}{(\Delta t_1)^2}}. \quad (3.21)$$

3.1.3 Learning Outcomes

- Learn how to use computer and smartphone hardware and software to record data
- Understand the concept of energy and its (non)conservation
- Understand the concept of lost energy
- Do a typical real-world experiment and find the lost energy

3.2 Experimental Setup and Measurements

In this experiment, you will study the non-conservation of energy, and find the amount of lost energy in a real life situation. You are allowed to get someone, not necessarily in the class, to help you with measurements if needed.

3.2.1 Apparatus

See the video for more details.

- Smartphone with the app **phyphox** installed on it: <https://phyphox.org/>
- Computer with access to Microsoft Excel (Excel is part of Office365 which is available to York University students free of charge)
- A ping pong ball, basketball or a similar object. This is referred to as the **object**. You can also use a toothpaste cap or possibly a coin if you do not have access to a spherical object that can bounce such as a ball, but the results will have more error. As a general guideline, use an object that when dropped, you can get at least 3 distinct bounces before the sound starts getting more continuous.
- Ruler. If you do not have one, you can install the the apps “Ruler” by Xalpha Lab or “Ruler” by NixGame on your smartphone.

3.2.2 Watch the Videos

Watch the video on how to use the **phyphox** app for this experiment and measurements.

3.2.3 Experiment and Data Collection

All the data should be recorded in SI units. For the mass of the ball/coin, either measure them using a household scale, or search the web for “typical mass of a ping pong ball” etc. and use it in your formulas

- Step 1** Find a table in your house and measure its height. This is not a necessary part of the experiment but it will help you decrease the error in find the initial height of the object (ball or coin).
- Step 2** Open the app on your **phyphox** smartphone, under **Timers**, click on **Acoustic Stopwatch** (Fig. B.1, left)
- Step 3** Now (Fig. B.1, right) you will see a tab on top with options **SIMPLE**, **SEQUENCE**, **PARALLEL**, and **MANY**. For this activity we will use the **SEQUENCE** option. But first click on **SIMPLE** option and then in the **Threshold** field, enter 0.1. Also, in the **Minimum Delay** field enter 0.1. These are the fields that can be changed based on your environment noise and the experiment. The lower the threshold, the more sensitive the app will be to starting and ending sound signals. If you find out that the starting and ending sound signals do not trigger the stopwatch, or other lower sounds undesirably trigger the stopwatch, change the **Threshold** value accordingly.
- Step 4** Now click on **SEQUENCE** option (Fig. B.1, right)
- Step 5** Click on **Reset** (Fig. B.1, right). Keep this screen on on your smartphone; we need it in Step 7.
- Step 6** We will release the ball (step 8) but before that and in order to make sure you know the initial height of the ball, bring the ball close to the surface table (a few centimeters away from the table). This way the initial height of the ball will be the same as the initial height of the table. Call this height h_0 and write its value down.
- Step 7** Click on the play button on top of the **phyphox** app screen (Fig. B.1, right) while you are in the **SEQUENCE** option.
- Step 8** Release the ball. Once the ball hits the ground for the first time, it make a sound and the **Acoustic Stopwatch** will record that as its first number in the sequence. The second time that ball hits the ground, another time is recorded in **Acoustic Stopwatch** and this number is the total time between the first time the ball hit the ground and the send time it did. This will repeat and the **Acoustic Stopwatch** will record the time between two events where the ball hits the ground (Fig. 3.2). We will need Δt_1 and Δt_2 which are seen in this figure.

Repeat this experiment 10 times and record the needed data that is mentioned in the next subsection.

3.2.4 Calculations and Data Analysis

You should fill the following table in the Excel template sheet for each run. Also enter the mass you used in the cell dedicated to it. There is another cell in which you should enter the type of the object you used as the bouncing object.

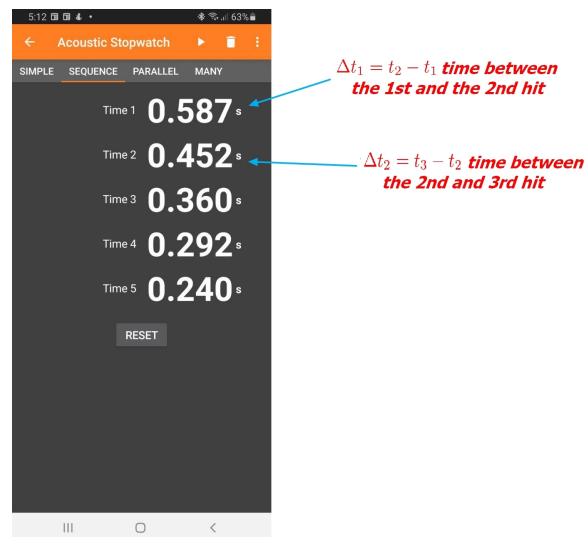


Figure 3.2: Using the **SEQUENCE** option in **Acoustic Stopwatch** in **phyphox** app. It records various times each of which the time between the bounces of the ball when it hits the ground.

Trial	h_0 (m)	Δt_1 (s)	Δt_2 (s)	h_1 (m)	h_2 (m)	Q_1 (J)	Q_2 (J)	$\frac{Q_1}{U_1} \times 100$	$\frac{Q_2}{U_2} \times 100$
1									
2									
3									
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
10									
Mean									
Standard Deviation									

- In the second column enter the initial height h_0 . This will be the same value for all the trials.
- In the third column enter Δt_1 , the time between the first time the ball hits the ground and the second time it does so (Fig. 3.2)
- In the fourth column enter Δt_2 , the time between the second time the ball hits the ground and the third time it does so (Fig. 3.2)
- In the fifth column enter h_1 , the computed maximum height that the ball reaches after it hits the ground for the first time. This value is (see Sec. 3.1.2)

$$h_1 = \frac{1}{8}g(\Delta t_1)^2 \quad (3.22)$$

for which you need to use the value of Δt_1 from the third column.

- In the sixth column enter h_2 , the computed maximum height that the ball reaches after it hits the ground for the second time. This value is (see Sec. 3.1.2)

$$h_2 = \frac{1}{8}g(\Delta t_2)^2 \quad (3.23)$$

for which you need to use the value of Δt_2 from the fourth column.

- In the seventh column enter Q_1 , which is the lost energy after the ball hits the ground for the first time. This value is (see Sec. 3.1.2)

$$Q_1 = mg \left[\frac{1}{8}g(\Delta t_1)^2 - h_0 \right] \quad (3.24)$$

for which you need to use the value of Δt_1 from the third column.

- In the eighth column enter Q_2 , which is the lost energy after the ball hits the ground for the second time. This value is (see Sec. 3.1.2)

$$Q_2 = \frac{1}{8}mg^2 [(\Delta t_2)^2 - (\Delta t_1)^2] \quad (3.25)$$

for which you need to use the values of Δt_1 and Δt_2 from the third and fourth columns.

- In the ninth column enter the ratio of the lost energy to the total energy in the first bounce (see Sec. 3.1.2) multiplied by 100

$$\frac{Q_1}{U_1} \times 100 = \frac{\frac{1}{8}g(\Delta t_1)^2 - h_0}{h_0} \times 100 \quad (3.26)$$

for which you need to use the value of Δt_1 from the third column.

- In the tenth column enter the ratio of the lost energy to the total energy in the second bounce (see Sec. 3.1.2)

$$\frac{Q_2}{U_2} \times 100 = \frac{(\Delta t_2)^2 - (\Delta t_1)^2}{(\Delta t_1)^2} \times 100 \quad (3.27)$$

for which you need to use the values of Δt_1 and Δt_2 from the third and fourth columns.

- Finally find the mean and standard deviation of each column and write it below them in the table.
- Attach a clear photo of your experimental setting to the Excel file in the designated place.

The significant figures for the values of the measurements you get and enter in the tables should match the uncertainty in your measurement devices. For example if your ruler can only measure up to a millimeter, any SI measurements should have three significant figures, e.g., 0.250 m. To increase/decrease the significant figures in a cell in Excel, see Appendix D, particularly Fig. D.1.

3.2.5 Questions

Answer the following questions in the Excel template sheet.

1. Identify two errors that may happen in this activity and explain the source of each error.
2. What can be done to decrease each of the errors you mentioned above?
3. Where does the energy of the ball go each time it hits the ground? Answer in detail.
4. What type of ball or floor covering is to be used to decrease the lost energy for each time the ball hits the ground?
5. Show that the dimensions (or units) of Q_1 and Q_2 in Eqs. (3.24) and (3.25) are indeed the dimensions (or units) of energy. Use math to do this. You can attach an additional PDF, Word, or scanned image of your computations to your report if needed.

3.3 Prepare your report

Send the following documents as your lab report

- Answered pre-lab questions
- Filled Excel sheet
- If needed, additional file showing your work for questions 5

Chapter 4

Oscillations

4.1 Introduction and Theory

Oscillations are the most common types of motion in the Universe. In fact they are so common that we try to explain most of phenomenon in the universe as some kind of superposition of oscillatory motion.

Many of such oscillations can be considered as *small oscillations* which are much simpler to analyze mathematically. In this type of motion, an object moves back and fourth around a position called the *equilibrium* under a *restoring force* F . The equilibrium point is where no restoring force acts on the object, i.e., $F = 0$. The force itself is called restoring since it always pulls (pushes) the object back towards the equilibrium. Examples of such motion are the oscillations of a mass connected to a spring, which at its initial configuration, is pushed or pulled away from its equilibrium position, and then released. Another example which we study in this experiment is the motion of a pendulum pulled away from its equilibrium vertical configuration (Fig. 4.1).

4.1.1 Motion of a Pendulum

A (idealized) pendulum consists of a bob of mass m and a massless string or rod connecting it to a fixed point (e.g., ceiling). Initially the string is vertical and the bob is at rest at its equilibrium position (Fig. 4.1). The bob is then pulled to one side to an initial angle θ_0 . This angle is the maximum angle that the bob will gain with respect to the vertical line and is called the *amplitude* of the oscillation. Note that in this case the amplitude is not a length, but an angle. At any angle θ from the vertical called *displacement* (note the difference between displacement θ and the amplitude θ_0 ; amplitude is the *maximum* displacement), a restoring force

$$F = mg \sin(\theta), \quad (4.1)$$

due to gravity acts on the bob. This force is always directed towards the equilibrium position and pulls the bob back towards that point. From this equation you can see that F is maximum at the maximum displacements on both sides where $\theta = \theta_0$, and is zero at the equilibrium where $\theta = 0$.

Looking at Fig. 4.1, we see that Newton's second law

$$F = ma, \quad (4.2)$$

in the direction tangent to the bobs trajectory can be written as

$$-mg \sin(\theta) = m \frac{d^2 s}{dt^2}, \quad (4.3)$$

where

$$s = L\theta, \quad (4.4)$$

is the arc length of the distance of the bob from the equilibrium point (see figure) and L is the length of the string. Also the negative sign on F is due to its restoring nature; if the bob is on the right side of the equilibrium point, the force is towards left and vice versa. Note that in equation (4.4), the angle θ should be written in radians. Combining the last two equations we get

$$\frac{d^2(L\theta)}{dt^2} = -g \sin(\theta). \quad (4.5)$$

Since L is constant it can be taken out of the derivative. Also, since we are considering small oscillation angles θ , we can set $\sin(\theta) \approx \theta$. Using these information into the above equation we get

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta. \quad (4.6)$$

We can also define

$$\omega^2 = \frac{g}{L}, \quad (4.7)$$

for which the equation (4.6) becomes the famous equation of small oscillations

$$\boxed{\frac{d^2\theta}{dt^2} = -\omega^2\theta}. \quad (4.8)$$

The solution to this equation is called the *equation of motion* of the pendulum

$$\boxed{\theta = \theta_0 \cos(\omega t + \phi)}, \quad (4.9)$$

where θ_0 is the amplitude discussed above and ϕ is an angle called the phase which depends on initial position and velocity. If the pendulum starts with zero initial velocity, we can take $\phi = 0$ and hence

$$\theta = \theta_0 \cos(\omega t). \quad (4.10)$$

Remember that all the angles should be in radians.

The quantity

$$\boxed{\omega = \sqrt{\frac{g}{L}}}, \quad (4.11)$$

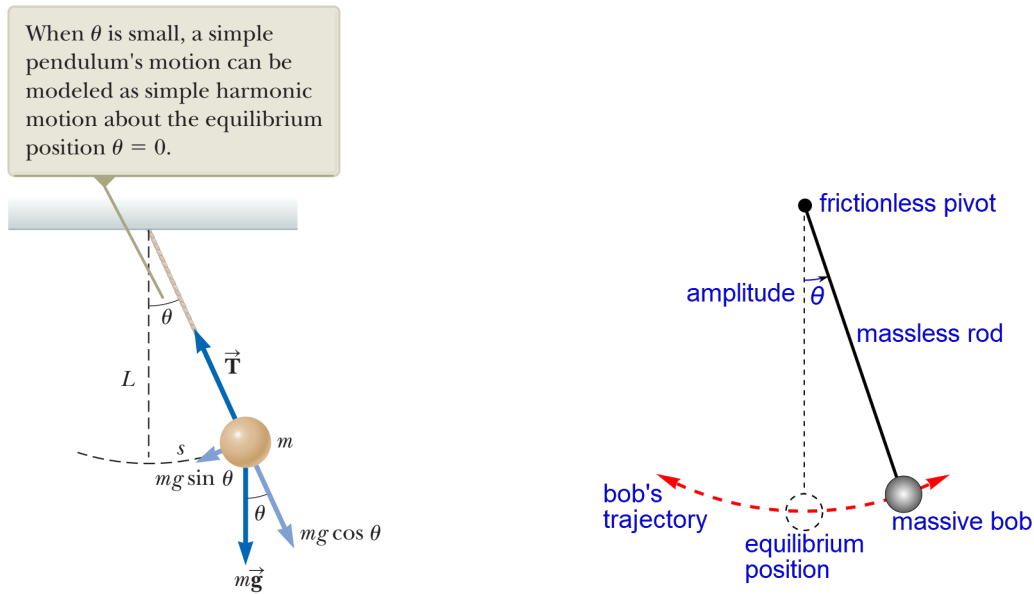


Figure 4.1: A typical pendulum. Courtesy of *Wikipedia* and *Physics for Scientists and Engineers With Modern Physics*, Serway and Jewett

is called the *angular speed* of the pendulum and measures how much angle per second in rad/s the pendulum sweeps in its motion. Since we also know that in general

$$\omega = \frac{2\pi}{T}, \quad (4.12)$$

with T being the *period* of the motion, we can write from the two last equations

$$\sqrt{\frac{g}{L}} = \frac{2\pi}{T} \Rightarrow T = 2\pi \sqrt{\frac{L}{g}}. \quad (4.13)$$

Also since we know that the *frequency* f is the inverse of T

$$f = \frac{1}{T}, \quad (4.14)$$

we can write

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}. \quad (4.15)$$

4.1.2 Learning Outcomes

- Learn how to use computer and smartphone hardware and software to record data
- Understand the concept of small oscillation via a pendulum
- Understand the dependence of period of the pendulum on its mass and length
- Use pendulum as a device to find g

4.2 Experimental Setup and Measurements: Measuring g

In this experiment, you will measure the gravitational acceleration near the earth surface g using a pendulum. You are allowed to get someone, not necessarily in the class, to help you with measurements if needed.

4.2.1 Apparatus

See the video for more details.

- A chronometer. You can use a smartphone with a timer app, or the app **phyphox**: <https://phyphox.org/>
- Computer with access to Microsoft Excel (Excel is part of Office365 which is available to York University students free of charge)
- One (non-elastic) string with a suitable length, e.g., between 20 - 80 cm.
- Two different masses m_1 and m_2 , preferable small, dense and not very light, can be tied to the string as the pendulum bob.
- Ruler. If you do not have one, you can install the the apps “Ruler” by Xalpha Lab or “Ruler” by NixGame on your smartphone.

4.2.2 Watch the Videos

Watch the video on how to perform this experiment and measurements.

4.2.3 Experiment and Data Collection

All the data should be recorded in SI units.

Step 1 Connect/tie the mass m_1 (or m_2) to the string as the pendulum bob. Make sure it is tied tightly and will not slip out of the knot in the middle of the experiment.

Step 2 Connect/tie the other side of the string to a place (pivot point, see Fig 4.1) such as a door knob, hangar in your closets, etc., such that pendulum can oscillate freely with least possible friction at the pivot point.

Step 3 Slowly leave the bob in it vertical equilibrium position and make sure it does not move. Now measure the length of the string from the pivot point to the bob. call this the string length L and write it down. This value will be the same for all the 10 runs of this experiment.

Step 4 Slowly pull the bob towards right or left of its vertical position while maintaining the string completely stretched, to an angle less than 30° from the vertical line.

Step 5 Release the pendulum and at the same time start the stopwatch.

Step 6 Watch the pendulum closely. Let the bob complete n complete oscillations where the suggested number of oscillations is $n = 4$ or $n = 5$. Then stop the stopwatch. Divide the time you recorded by n and the resulting value is the period T of the pendulum. Record this T . Note: one oscillation is complete when the bob returns back to its initial position. For example, if you have initially pulled the bob to the right of the equilibrium position, you should wait for it to go to the maximum displacement on the left and then come back to the maximum displacement to the right again.

Step 7 Do this experiment 10 times and write down your T 's in the table in the next subsection.

Do the same procedure again, but now for mass m_2 . Record your T 's for this experiment too and write them in the table in the next subsection.

4.2.4 Calculations and Data Analysis

You should fill the following table in the Excel template sheet for each run. There should be 10 runs for mass m_1 and 10 runs for mass m_2 . There will be two tables one for each mass.

Mass m_1				
Trial	L (m)	T (s)	g (m/s ²)	$\frac{\delta g}{g_0}$
1				
2				
3				
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
10				
Mean				
Standard Deviation				

- In the second column enter the length of the pendulum L . This will be the same value for all the trials for this mass.
- In the third column enter the periods T that you measured.
- In the fourth column enter the computed value of g . This is done using the equation (4.13) from which we can get

$$g = 4\pi^2 \frac{L}{T^2}. \quad (4.16)$$

- In the fifth column enter the fractional uncertainty in finding g experimentally compared to its known value $g_0 = 9.81 \text{ m/s}^2$

$$\boxed{\frac{\delta g}{g_0} = \frac{g - g_0}{g_0}}. \quad (4.17)$$

- Finally find the mean and standard deviation of each column and write it below them in the table.
- Attach a clear photo of your experimental setting to the Excel file in the designated place.

The table for m_2 should also be filled similarly.

The significant figures for the values of the measurements you get and enter in the tables should match the uncertainty in your measurement devices. For example if you ruler can only measure up to a millimeter, any SI measurements should have three significant figures, e.g., 0.250 m. To increase/decrease the significant figures in a cell in Excel, see Appendix D, particularly Fig. D.1.

4.2.5 Questions

Answer the following questions in the Excel template sheet.

1. Identify three errors that may happen in this activity and explain the source of each error.
2. What can be done to decrease each of the errors you mentioned above?
3. Does it seem from your data that g depends on masses of the bobs? Explain using your data, and argue why from the theoretical formulas.
4. A good clock is one that shows the same period of time between two events, irrespective of where on Earth it is used (assuming that the Earth has the same density and radius everywhere). Can a pendulum be used as a good clock? why? Is the bob mass important in this type of clock? Why? Explain all of these using the theoretical part. You can also complement it with the results from your data.
5. Is the value of the g you obtained within the 10% error of the expected value of 9.81 m/s^2 ? If not, what factors do you think contributed to the larger error?

4.3 Experimental Setup and Measurements: Dependence of T on L

In this experiment, you will study dependence of the period of a pendulum on its length. You are allowed to get someone, not necessarily in the class, to help you with measurements if needed.

4.3.1 Apparatus

See the video for more details.

- A chronometer. You can use a smartphone with a timer app, or the app **phyphox**: <https://phyphox.org/>
- Computer with access to Microsoft Excel (Excel is part of Office365 which is available to York University students free of charge)
- Two (non-elastic) string with a suitable length, e.g., between 20 - 80 cm with length having the ration 1:2, e.g., 20 cm and 40 cm. Alternatively you can use the same string but change its length by doubling it, or rolling it around the pivot point of the pendulum, etc., such that you get a new pendulum with a length half of the original one.
- Any object, preferable small, dense and not very light, that can be used as a pendulum bob.
- Ruler. If you do not have one, you can install the the apps “Ruler” by Xalpha Lab or “Ruler” by NixGame on your smartphone.

4.3.2 Watch the Videos

Watch the video on how to perform this experiment and measurements.

4.3.3 Experiment and Data Collection

All the data should be recorded in SI units.

Step 1 Connect/tie the bob to the first string. Make sure it is tied tightly and will not slip out of the knot in the middle of the experiment.

Step 2 Connect/tie the other side of the string to a place (pivot point, see Fig 4.1) such as a door knob, hangar in your closets, etc., such that pendulum can oscillate freely with least possible friction at the pivot point.

Step 3 Slowly leave the bob in it vertical equilibrium position and make sure it does not move. Now measure the length of the string from the pivot point to the bob. call this the string length L_1 (L_2 of you are doing the second 10 run) and write it down. This value will be the same for all the 10 runs of this experiment.

Step 4 Slowly pull the bob towards right or left of its vertical position while maintaining the string completely stretched, to an angle less than 30° from the vertical line.

Step 5 Release the pendulum and at the same time start the stopwatch.

Step 6 Watch the pendulum closely. Let the bob complete n complete oscillations where the suggested number of oscillations is $n = 4$ or $n = 5$. Then stop the stopwatch. Divide the time you recorded by n and the resulting value is the period T of the pendulum. Record this T . Note: one oscillation is complete when the bob returns back to its initial position. For example, if you have initially pulled the bob to the right of the equilibrium position, you should wait for it to go to the maximum displacement on the left and then come back to the maximum displacement to the right again.

Step 7 Do this experiment 10 times and write down your T 's in the table in the next subsection.

Do the same procedure again, but now for the new string length L_2 . Record your T 's for this experiment too and write them in the table in the next subsection.

4.3.4 Calculations and Data Analysis

You should fill the following table in the Excel template sheet for each run. There should be 10 runs for mass L_1 and 10 runs for mass L_2 . There will be two tables one for each mass.

Length L_1				
Trial	L_1 (m)	T (s)	$\frac{T^2}{L_1}$ (s ² /m)	$\frac{T^2}{L_1} - \frac{4\pi^2}{g}$ (s ² /m)
1				
2				
3				
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
10				
Mean				
Standard Deviation				

- In the second column enter the length of the pendulum L_1 (L_2 if you are doing the second 10 run). This will be the same value for all the trials for this mass.
- In the third column enter the periods T that you measured.
- In the fourth column enter the ratio $\frac{T^2}{L_1}$ ($\frac{T^2}{L_2}$ if you are doing the second 10 run).
- In the fifth column enter the difference

$$\frac{T^2}{L_1} - \frac{4\pi^2}{g}. \quad (4.18)$$

As you can see from equation (4.16) this value should ideally be zero. With the errors involved it should be very close to zero.

- Finally find the mean and standard deviation of each column and write it below them in the table.

- Attach a clear photo of your experimental setting to the Excel file in the designated place.

Fill a similar table for L_2 in the Excel sheet.

The significant figures for the values of the measurements you get and enter in the tables should match the uncertainty in your measurement devices. For example if your ruler can only measure up to a millimeter, any SI measurements should have three significant figures, e.g., 0.250 m. To increase/decrease the significant figures in a cell in Excel, see Appendix D, particularly Fig. D.1.

4.3.5 Questions

Answer the following questions in the Excel template sheet.

1. If the period T becomes 3 times its original value, i.e., $T \rightarrow 3T$ after changing the length, by what factor has the length increased/decreased? Explain mathematically. You can add an additional file (PDF) to your report if you need.
2. If we had several experiments with several length of the string L , and plotted T versus different L , with T represented on the x axis and L on the y axis, what type of graph would have we obtained? Why?
3. If we had several experiments with several length of the string L , and plotted T^2 versus different L , with T^2 represented on the y axis and L on the x axis, what type of graph would have we obtained? Why?

4.4 Prepare your report

Send the following documents as your lab report

- Answered pre-lab questions
- Filled Excel sheet
- If needed, additional file showing your work for questions 1 of the second activity

Chapter 5

Angular Motion

5.1 Introduction and Theory

5.1.1 Kinematics

Simple angular motion happens when an object rotates around a fixed axis such as in a Ferris Wheel, etc. These angular motions can be described using three quantities: angular position θ measured with respect to a fixed initial position θ_0 , angular velocity ω , and angular acceleration α . The angular velocity is the rate of change of angular position

$$\omega = \frac{d\theta}{dt}, \quad (5.1)$$

and the angular acceleration is the rate of change of angular velocity,

$$\alpha = \frac{d\omega}{dt}. \quad (5.2)$$

The three angular quantities θ , ω , α are respectively analogous to the three translational quantities position, velocity and acceleration, x , v , a . Hence, as expected analogous equations of motion also holds for the angular quantities, e.g.,

$$y - y_0 = \frac{1}{2}at^2 + v_0t \quad (\text{Translational}), \quad (5.3)$$

$$\theta - \theta_0 = \frac{1}{2}\alpha t^2 + \omega_0 t \quad (\text{Rotational}), \quad (5.4)$$

and

$$v^2 - v_0^2 = 2a(y - y_0) \quad (\text{Translational}), \quad (5.5)$$

$$\omega^2 - \omega_0^2 = 2\alpha(\omega - \omega_0) \quad (\text{Rotational}), \quad (5.6)$$

and

$$v - v_0 = at \quad (\text{Translational}), \quad (5.7)$$

$$\omega - \omega_0 = \alpha t \quad (\text{Rotational}). \quad (5.8)$$

If an object is rotating with respect to an axis and the radius of rotation (distance between the axis and object) is r , then the relation between translational and tangential angular quantities are

$$s = r\theta \quad (5.9)$$

$$v_T = r\omega \quad (5.10)$$

$$a_T = r\alpha \quad (5.11)$$

where s is the length of arc of the circle of the trajectory of the object opposite to the angle θ . Here the index T refers to the component of the quantity tangent to the trajectory. Note that the tangential acceleration only exists if $\alpha \neq 0$, i.e., if the angular velocity changes with time. On the other hand, for any circular motion, whether with or without α , there is also a radial acceleration a_r , called the centripetal acceleration which is

$$a_r = r\omega^2 = \frac{v_T^2}{r}. \quad (5.12)$$

As evident from its name, the direction of $\mathbf{a}_r$ is always towards the center of the circular trajectory of rotation.

5.1.2 Dynamics

Dynamics is about how forces affect the motion. In translational mechanics we have translational forces denoted by F that change the translational velocity v of the object. The equivalent of this quantity is called torque

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}, \quad (5.13)$$

which changes the angular velocity ω of the object. If an object has a distance $\mathbf{r}$ from the center of the rotation and a translational force acts on it and changes its ω , the amount of torque on it is defined as above. The magnitude of torque is

$$\tau = rF \sin(\theta_{\mathbf{r}, \mathbf{F}}), \quad (5.14)$$

where $\theta_{\mathbf{r}, \mathbf{F}}$ is the angle between the vectors $\mathbf{r}, \mathbf{F}$ if we draw them from the same origin. The analogous angular version of Newton's law $\mathbf{F} = m\mathbf{a}$ is

$$\boldsymbol{\tau} = I\boldsymbol{\alpha} \quad (5.15)$$

where I , being analogous to m , is called the moment of inertia of the object.

5.1.3 System of a pulley and a mass

5.1.3.1 General dynamics

A common system in which the rotational concepts is used is the system in which a mass m is connected to a massless string that has been wrapped around a pulley of mass M , radius

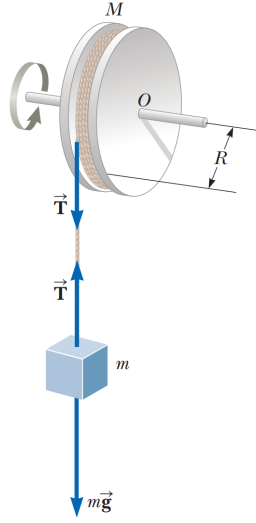


Figure 5.1: A pulley. Courtesy of *Physics for Scientists and Engineers With Modern Physics*, Serway and Jewett

$r = R$, and moment of inertia I (Fig. 5.1). When the system starts, the mass m has zero initial velocity $v_0 = 0$. The pulley also starts with zero angular velocity $\omega_0 = 0$. Using the Newton's second law for m we get

$$T - mg = -ma \Rightarrow T = m(g - a), \quad (5.16)$$

where T is the tension of the rope. The tension also produces a torque on pulley which from (5.14) becomes

$$\tau = RT \sin(\theta_{\mathbf{R}, \mathbf{T}}) = RT \sin\left(\frac{\pi}{2}\right) \Rightarrow \tau = RT, \quad (5.17)$$

where R is the radius of the pulley as mentioned above. Replacing (the magnitude of) τ from (5.15) into (5.17) yields

$$I\alpha = RT. \quad (5.18)$$

The translational acceleration of the mass m and that of the pulley ($R\alpha$) are the same. Using that and since we know the relation between α and a from (5.11), equation (5.18) becomes

$$I \frac{a}{R} = RT \Rightarrow T = I \frac{a}{R^2}. \quad (5.19)$$

Combining (5.16) and (5.19) we get

$$\left\{ \begin{array}{l} T = m(g - a) \\ T = I \frac{a}{R^2} \end{array} \right\} \Rightarrow a = \frac{g}{\frac{I}{mR^2} + 1}. \quad (5.20)$$

From this formula and (5.11), we can also obtain the angular acceleration.

5.1.3.2 Analysis related to the lab experiment

Suppose that we only know the mass of the falling object m and the time it took for it to fall down a height of h . We want to compute various quantities from this data. First, from equation (5.3) and with $v_0 = 0$, $y = 0$, $y_0 = h$ and noting that $a \rightarrow -a$, we obtain

$$\boxed{a = \frac{2h}{t^2}}. \quad (5.21)$$

On the other hand, noting that the translational acceleration of the mass m and that of the pulley ($R\alpha$) are the same, and using (5.18) we get

$$\left\{ \begin{array}{l} a = \frac{2h}{t^2} \\ a = R\alpha \end{array} \right\} \Rightarrow \boxed{\alpha = \frac{2h}{Rt^2}}. \quad (5.22)$$

We do not know the moment of inertia of the pulley but we would like to compute that too. Using equation (5.20) we get

$$I = mR^2 \left(\frac{g}{a} - 1 \right) \quad (5.23)$$

and using a from (5.21) in this, we obtain

$$\boxed{I = mR^2 \left(\frac{gt^2}{2h} - 1 \right)}. \quad (5.24)$$

Finally the torque acting on the pulley, using (5.24) and (5.22) is

$$\tau = I\alpha = \left[mR^2 \left(\frac{gt^2}{2h} - 1 \right) \right] \left[\frac{2h}{Rt^2} \right] \Rightarrow \boxed{\tau = mR \left(g - \frac{2h}{t^2} \right)} \quad (5.25)$$

5.1.4 Learning Outcomes

- Learn how to use computer and smartphone hardware and software to record data
- Understand the concept of angular kinematics and dynamics in experiments

5.2 Experimental Setup and Measurements

In this experiment, you will study angular kinematics and dynamics related to a homemade pulley. You are allowed to get someone, not necessarily in the class, to help you with measurements if needed.

5.2.1 Apparatus

See the video for more details.

- A chronometer. You can use a smartphone with a timer app, or the app **phyphox**: <https://phyphox.org/>
- Computer with access to Microsoft Excel (Excel is part of Office365 which is available to York University students free of charge)
- An object, in the shape of a hollow cylinder (e.g., the support of an adhesive tape) to be used as a **pulley**.
- One (non-elastic) string with a suitable length, preferable 60 cm or larger.
- An object that can be connected/tied to the end of the string that is attached to the pulley. We call this the **falling object**.
- A pen or a similar object to be used as the axis of the pulley.
- Ruler. If you do not have one, you can install the the apps “Ruler” by Xalpha Lab or “Ruler” by NixGame on your smartphone.

5.2.2 Watch the Videos

Watch the video on how to perform this experiment and measurements.

5.2.3 Experiment and Data Collection

All the data should be recorded in SI units.

Step 1 Measure the mass of the falling object m and write it down. Also measure the radius of the pulley R and write it down. If you do not have a scale, you can use one of the methods mentioned in Appendix A.

Step 2 Wrap the string one time around the pulley and tie it tightly. Then connect/tie the other end of the string to the falling object. Measure the length of the string

Step 3 Measure the length of the string between the pulley and the falling object and call it h . Write this value down.

Step 4 Wrap the string connecting between the pulley and the falling object around the pulley. Keep the system in this configuration and bring the system to a height that is larger (or equal to) h . This way you make sure that when the string unwinds, it does not hit the floor before it is completely unwrapped.

Step 5 Still keeping the system in the same stationary configuration, use the pen as the axis of the pulley.

Step 6 Now you should prepare your chronometer for the experiment. You can also use the **Acoustic Stopwatch** on the **phyphox** app. In that case refer to Appendix B on how to prepare it.

Step 7 Release the falling object and let the pulley rotate and the string unwrap. At the same time start your chronometer. If you are using **Acoustic Stopwatch** on **phyphox** app, make a sound (e.g., loudly say “go” or tap on a surface close to the smartphone) when releasing the falling object so that stopwatch gets triggered. The object will fall down until all of the string around the pulley is unwrapped. At that moment stop the chronometer. Again if you are using the **Acoustic Stopwatch** on **phyphox** app, make another sound so it gets triggered for the second time. Record the time t that the chronometer or the **Acoustic Stopwatch** registers for each trial. Also watch out for early and premature triggering of the stopwatch due to noise produced by the motion of the system. If this happens before the object reaches the bottom point, change the **Threshold** in the **Acoustic Stopwatch** accordingly until the app is only triggered by initial and final sounds.

5.2.4 Calculations and Data Analysis

You should fill the following table in the Excel template sheet for each run. There should be 10 runs for this experiment. Before filling the table, fill in three cells that are related to the mass of the falling object, the height of the fall, and the radius of the pulley. Then follow these instructions to fill the table.

Trial	t (s)	a (m/s ²)	α (rad/s ²)	ω (rad/s)	I (kg · m ²)	τ (N · m)
1						
2						
3						
⋮	⋮	⋮	⋮	⋮	⋮	⋮
10						
Mean						
Standard deviation						

- In the second column enter the time t you measured for each run.
- In the third column enter the translational acceleration of the pulley which is the same as the acceleration of the falling object, as you saw in the theory section

$$a = \frac{2h}{t^2}. \quad (5.26)$$

- In the fourth column enter the rotational acceleration of the pulley, as you saw in the theory section

$$\alpha = \frac{2h}{Rt^2}. \quad (5.27)$$

- In the fifth column enter the final angular velocity of the pulley that can be computed using the last equation and equation (5.8) as (with $\omega_0 = 0$)

$$\omega = \alpha t = \left(\frac{2h}{Rt^2} \right) t \Rightarrow \boxed{\omega = \frac{2h}{Rt}} \quad (5.28)$$

- In the sixth column enter your computed value of the moment of inertia as in the theory part

$$I = mR^2 \left(\frac{gt^2}{2h} - 1 \right). \quad (5.29)$$

- In the seventh column enter the amount of torque as you saw in the theory section

$$\tau = mR \left(g - \frac{2h}{t^2} \right). \quad (5.30)$$

- Finally find the mean and standard deviation of each column and write it below them in the table.
- Attach a clear photo of your experimental setting to the Excel file in the designated place.

The significant figures for the values of the measurements you get and enter in the tables should match the uncertainty in your measurement devices. For example if you ruler can only measure up to a millimeter, any SI measurements should have three significant figures, e.g., 0.250 m. To increase/decrease the significant figures in a cell in Excel, see Appendix D, particularly Fig. D.1.

5.2.5 Questions

Answer the following questions in the Excel template sheet.

1. Identify three errors that may happen in this activity and explain the source of each error.
2. What can be done to decrease each of the errors you mentioned above?
3. Consider the mean value of the final ω in your table. Turn that into translational velocity of the falling object using one of the formulas in theory part. Does your value make sense? Explain by comparing it with free fall speed of the object.
4. Can we safely assume that the acceleration of the object a and the angular acceleration of the pulley α are constant? Explain your answer.

5.3 Prepare your report

Send the following documents as your lab report

- Answered pre-lab questions
- Filled Excel sheet

Chapter 6

Static Equilibrium

6.1 Introduction and Theory

6.1.1 Equilibrium in extended objects

Most of human-made structures are built to stay put and do not move or rotate under small perturbations. In other words they should be in a state of equilibrium and remain in that state. This includes houses, skyscrapers, bridges, and even moving objects such as cars and trains, etc. The property of being and remaining in equilibrium is extremely crucial since a loss of equilibrium in most cases mean serious injuries, loss of life and loss of valuable resources.

In order for an object to be in equilibrium, it should not move or (in case of an extended object) rotate. More precisely its translational and rotational velocity vectors should be zero

$$\mathbf{v} = 0, \quad \boldsymbol{\omega} = 0. \quad (6.1)$$

While being in equilibrium (the above conditions), it should also remain in equilibrium. This means that the object should keep its state of motion, i.e., keep the above conditions. In the language of Newton's second law, this means that the net force on it should vanish in order for its velocity to not change from $\mathbf{v} = 0$, i.e.,

$$\sum_{i=1}^N \mathbf{F}_i = 0, \quad (6.2)$$

where N is the number of forces acting on the object. In case of an extended object, it also means in order for the object to keep its state of non-rotating, the net torque on it should vanish

$$\sum_{i=1}^N \boldsymbol{\tau}_i = 0. \quad (6.3)$$

The last equation can be expanded as

$$\boldsymbol{\tau}_1 + \dots + \boldsymbol{\tau}_N = 0 \Rightarrow \mathbf{r}_1 \times \mathbf{F}_1 + \dots + \mathbf{r}_N \times \mathbf{F}_N = 0, \quad (6.4)$$

where $\mathbf{r}_i$ is the distance between the point of action of force $\mathbf{F}_i$ and any arbitrary point in the object. This point can be thought of the center or axis of rotation. All the $\mathbf{r}_i$ in this formula should be measured from the same arbitrary point (or center of rotation). Note that $\boldsymbol{\tau}_i$ are vectors. Also both $\mathbf{r}_i$ and $\mathbf{F}_i$ in this formula are vectors, and the symbol $\times$ represents the cross product between them.

If the forces that act on an extended object are in a plane, like the force of your two hands on sides of the steering wheel of a car, they either rotate it clockwise or counterclockwise inside that plane. In this case, we can just work with the magnitude τ_i of $\boldsymbol{\tau}_i$

$$\tau_i = r_i F_i \sin(\theta), \quad (6.5)$$

and their sign which signifies if they are rotating clockwise or counterclockwise. Here θ is the angle between $\mathbf{r}_i$ and $\mathbf{F}_i$. By convention, a counterclockwise torque will have a positive sign while a clockwise torque will be negative.

6.1.2 Analysis related to the lab experiment: traditional scales

Consider the system in Fig. 6.1. There are two forces that act on the pen on its ends and they are the tensions T_1 and T_2 of the two strings connected to it and attached to masses m_1 and m_2 . Since the system is in equilibrium, for the pen we have

$$\sum_{i=1}^2 \boldsymbol{\tau}_i = 0 \Rightarrow \mathbf{d}_1 \times \mathbf{T}_1 + \mathbf{d}_2 \times \mathbf{T}_2 = 0. \quad (6.6)$$

Given that both tensions are in the same plane, we can write

$$-\tau_1 + \tau_2 = 0 \Rightarrow -d_1 T_1 \sin(\theta_1) + d_2 T_2 \sin(\theta_2) = 0. \quad (6.7)$$

Note that since τ_1 is clockwise we have written it with a negative sign. Now, since both tensions are perpendicular to the pen ($\theta_1 = \theta_2 = 90^\circ$) we get

$$d_1 T_1 = d_2 T_2. \quad (6.8)$$

On the other hand since each of the masses m_1 and m_2 are also in equilibrium, we can write for them

$$T_1 - m_1 g = 0 \Rightarrow T_1 = m_1 g \quad (6.9)$$

$$T_2 - m_2 g = 0 \Rightarrow T_2 = m_2 g \quad (6.10)$$

and using them in (6.8) yields

$$m_1 d_1 = m_2 d_2. \quad (6.11)$$

Hence by measuring the ratio of the distances we should be able to determine the ratio of masses

$$\frac{d_1}{d_2} = \frac{m_2}{m_1}. \quad (6.12)$$

Now, suppose that we know the mass of m_1 . Then we can find the mass of m_2 just by measuring the ratio of distances

$$m_2 = \frac{d_1}{d_2} m_1. \quad (6.13)$$

This is the concept behind old (and some new!) scales.

6.1.3 Learning Outcomes

- Learn how to use computer and smartphone hardware and software to record data
- Understand the concept equilibrium particularly the vanishing of net torque in simple systems

6.2 Experimental Setup and Measurements: a primitive scale to measure masses

In this experiment, you will study the static equilibrium in a simple scale that operates under two torques.

6.2.1 Apparatus

See the video for more details.

- Computer with access to Microsoft Excel (Excel is part of Office365 which is available to York University students free of charge)
- Three objects with different masses that will be set to equilibrium. They will be called the **objects** or the **masses** (m_1 and m_2 in Fig. 6.1. There is another mass m_3 which you should use later)
- Two similar (non-elastic) string with a suitable length based on the design of your system.
- A pen or a similar object, preferably with the cross section in the form of a hexagon, so that it has some flat sides instead of being round. We will call this the **balancing rod** (the pen in Fig. 6.1)
- Optional: an object which can act as the **pivot**. You can also use your finger as the pivot as in Fig. 6.1.
- Ruler. If you do not have one, you can install the the apps “Ruler” by Xalpha Lab or “Ruler” by NixGame on your smartphone.

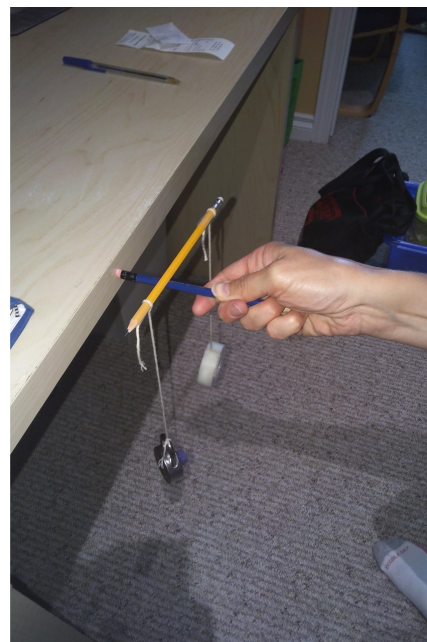
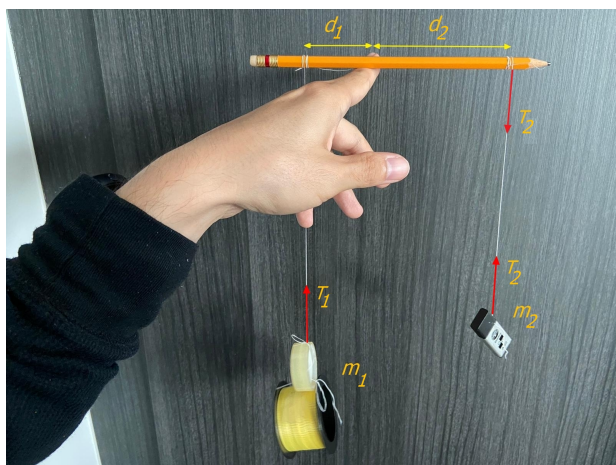


Figure 6.1: Two suggested ways of designing the system. The masses of the objects connected to the left and right end points of the pen should be different. In the left figure, you can see the relevant forces. Note that there is a vector $\mathbf{T}_1$ in the left string where it is attached to the pen but it is not drawn in the picture.

6.2.2 Watch the Videos

Watch the video on how to perform this experiment and measurements.

6.2.3 Experiment and Data Collection

All the data should be recorded in SI units. You should start with pair of masses m_1 and m_2 . Then repeat the experiment with the pair m_1 and m_3 . Finally, repeat the experiment with the pair m_2 and m_3 .

Step 1 Measure the masses m_1 and m_2 of the two objects to be set in equilibrium and write them down. If you do not have a scale, you can use one of the methods mentioned in Appendix A.

Step 2 Tie the two masses m_1 and m_2 , each to one end of the pen. Try to have the same length of the string from the masses to the end of the pen, for both objects (Fig. 6.1).

Step 3 Try to set the system in equilibrium on a solid pivot point (either on your finger or on some other solid thin object) as in Fig. 6.1. Do this by sliding the pen and thereby changing d_1 and d_2 until the system does not rotate or fall.

Step 4 Measure the distances d_1 and d_2 from the pivot point to the points where the strings are attached to the pen and write them down (Fig. 6.1).

After step 4, remove the system from the pivot point, replace the masses with the pair m_1 and m_3 and redo the steps. After that, replace the masses with the pair m_2 and m_3 and redo the steps again.

6.2.4 Calculations and Data Analysis

You should fill the following table in the Excel template sheet for each run. Before filling the table, fill in the cells that ask you about the objects you used for the pivot, the balancing rod, the strings, and the masses. Then follow these instructions to fill the table.

Here, M represents the larger mass of the pair of masses you are using (it can be m_1, m_2 or m_3 depending on the pair) and m is the smaller mass of the pair. Also d_M is the distance from the pivot to the larger mass M , and d_m is the distance from the pivot to the smaller mass m . As an example, suppose that we are experimenting with the pair m_1, m_2 such that $m_1 > m_2$. Then we have $M = m_1$, $m = m_2$, $d_M = d_1$, $d_m = d_2$, $m_{(\text{comp})} = m_{2(\text{comp})}$, etc.

Trial	$M(\text{kg})$	$m(\text{kg})$	$d_M(\text{m})$	$d_m(\text{m})$	$\frac{d_M}{d_m}$	$m_{(\text{comp})} (\text{kg})$	$m_{(\text{comp})} - m (\text{kg})$	$\frac{m_{(\text{comp})} - m}{m}$
m_1, m_2								
m_1, m_3								
m_2, m_3								
Mean	N/A	N/A	N/A	N/A	N/A	N/A		
Stand. dev.	N/A	N/A	N/A	N/A	N/A	N/A		

- In the second column enter the mass of the more massive object, M .
- In the third column enter the mass of the less massive object, m .
- In the fourth column enter the distance d_M from the string that connects M to the balancing rod to the pivot point (Fig. 6.1).
- In the fifth column enter the distance d_m from the string that connects m to the balancing rod to the pivot point (Fig. 6.1).
- In the sixth column enter the ratio of distances

$$\frac{d_M}{d_m}. \quad (6.14)$$

- In the seventh column enter the computed value of m

$$m_{(\text{comp})} = \frac{d_M}{d_m} M. \quad (6.15)$$

- In the eighth column enter the difference between the computed and measured value of m

$$m_{(\text{comp})} - m. \quad (6.16)$$

- In the ninth column enter the fractional uncertainty of m

$$\frac{m_{(\text{comp})} - m}{m}. \quad (6.17)$$

- Finally find the mean and standard deviation of the column without N/A (only the blue cells in the Excel file) and write them in the table.
- Attach a clear photo of your experimental setting to the Excel file in the designated place.

Repeat the experiment as mentioned for other pairs of masses.

The significant figures for the values of the measurements you get and enter in the tables should match the uncertainty in your measurement devices. For example if your ruler can only measure up to a millimeter, any SI measurements should have three significant figures, e.g., 0.250 m. To increase/decrease the significant figures in a cell in Excel, see Appendix D, particularly Fig. D.1.

6.2.5 Questions

Answer the following questions in the Excel template sheet.

1. Identify three errors that may happen in this activity and explain the source of each error.
2. What can be done to decrease each of the errors you mentioned above?
3. Do you think that the tensions of the strings are the same or are different? Explain using mathematics. You can attach a separate scanned document (PDF) or a Word file to show your computations.
4. We did not take into account the mass of the balancing rod (here, the pen). It may not affect the results very much, why? If instead of a pen, we have used a very long (say 1 meter) rod made of steel, would that also not affect the results too much? Explain your answer.
5. In a perfect experiment, the ratio $\frac{m_1}{m_2}$ should be the same as the ratio $\frac{d_2}{d_1}$. Is that so in your experiment? Explain why?
6. Does the scale you built in this experiment work in another planet? Explain why and mention the crucial step in the calculations in the theory part that makes the scale useful/useless in another planet.

6.3 Prepare your report

Send the following documents as your lab report

- Answered pre-lab questions
- Filled Excel sheet
- If needed, additional file showing your work for questions 3

Chapter 7

Elasticity

7.1 Introduction and Theory

No material is perfectly rigid. All materials deform (what we will call strain) when stretched or squeezed. The amount by which a material deforms is a fundamental property of that material. When acted on by a force, a material such as rubber will deform a lot, and a material like stainless steel will deform a little.

Rather than referring to the force $\mathbf{F}$ applied to an object, we will refer to the magnitude of the force $F = |\mathbf{F}|$ per unit area A , called *stress*, defined as

$$\sigma = \frac{F}{A}. \quad (7.1)$$

The Greek letter σ is “sigma”. Stress is a scalar, since $F = |\mathbf{F}|$ and its SI units is Pascal, $\text{Pa} = \text{N}/\text{m}^2$.

Strain ϵ (Greek letter epsilon) is a measure of the deformation caused by the stress σ . In simple terms, stress leads to strain. Stress can be divided into longitudinal and transverse types. In longitudinal case (Fig. 7.1) two opposing forces usually with the same magnitude F act on a long solid cylinder of length L and cross section A , along (or parallel to) its

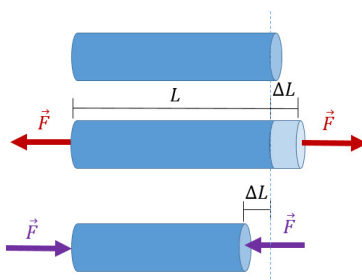


Figure 7.1: A rod, a rod being stretched and being squeezed under longitudinal forces.

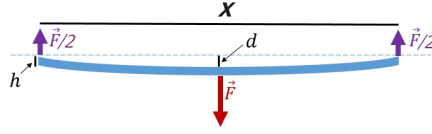


Figure 7.2: A beam being deflected (straining) when under a load (stress). Note that the total length of the beam *before* deflection is L which is not shown in figure, but after the deflection, the straight distance between the two ends of the beam is X .

length such that it is elongated by ΔL under stress $\sigma = \frac{F}{A}$. In this case, strain is given by

$$\epsilon = \frac{\Delta L}{L} \quad (7.2)$$

In the transverse case, usually a force $\mathbf{F}$ which has a component perpendicular to the beam creates strain and changes the shape of the beam (Fig. 7.2). The amount of deformation depends on a number of factors: the width of the beam w (the dimension into the page of Fig. 7.2, the height of the beam h , the length of the beam L , the magnitude of the applied forces, and the material from which the beam was constructed.

In any type of strain, the stretching, squeezing and bending of an object is much like the stretching and squeezing of a spring. A spring with a spring constant k will deform by amount Δx when acted on by a force of magnitude F , and this is mathematically expressed by the Hooke's law

$$F = -k\Delta x, \quad (7.3)$$

where k is the proportionality constant. In a solid object such as a beam, rather than thinking about a force resulting in a linear deformation (like a spring), we can more generally think about a force per unit area resulting in a strain. So, the stress σ (force per area) leads to strain ϵ , which is expressed as

$$\sigma = Y\epsilon \quad (7.4)$$

where now Y is the constant of proportionality which is called *Young's Modulus*. The SI units of Young's Modulus, as can be seen from (7.1), is Pa. Young's Modulus is a property of the material from which the object is made.

7.1.1 Analysis related to the lab experiment

Consider a transverse stress as in Fig. 7.2 or Fig. 7.3. In such a case, the Young's Modulus turns out to be

$$Y = \frac{mgL^3}{4wh^3d}, \quad (7.5)$$

where d is the amount of deflection as seen in these figures and $F = mg$ is the weight of the object that is creating the deflection. Using this formula we are able to compute the Young's

Modulus of the object in our experiment. Note that if the cross section of your object is circular with radius r , you can take

$$\boxed{w = h = 1.75r}. \quad (7.6)$$

In reality for a circular cross section (a cylindrical beam) the Young's Modulus is

$$Y = \frac{mgL^3}{12\pi r^4 d}, \quad (7.7)$$

but for this lab you can just use (7.5) together with (7.6) if you work with a cylindrical beam.

7.1.2 Learning Outcomes

- Learn how to use computer and smartphone hardware and software to record data
- Understanding stress, strain and Young's Modulus of solid beams and finding them

7.2 Experimental Setup and Measurements

In this experiment, you will find the Young's Modulus of a solid beam using its geometric properties.

7.2.1 Apparatus

See the video for more details.

- Computer with access to Microsoft Excel (Excel is part of Office365 which is available to York University students free of charge).
- A solid long object which we call the **beam** that will be bent under the weight of an object hanging from its center. They will be called the **objects** or the **masses** (Fig. 7.3). In Fig. 7.3 we have used the inner plastic tube of a pen that contains the ink but you can use any other suitable beam that may have a rectangular or circular cross section.
- Three objects with different masses m_1, m_2, m_3 that each will separately be hung from the center of the beam (Fig. 7.3).
- A string to connect the object to center of the beam.
- Ruler. If you do not have one, you can install the the apps "Ruler" by Xalpha Lab or "Ruler" by NixGame on your smartphone.

7.2.2 Watch the Videos

Watch the video on how to perform this experiment and measurements.

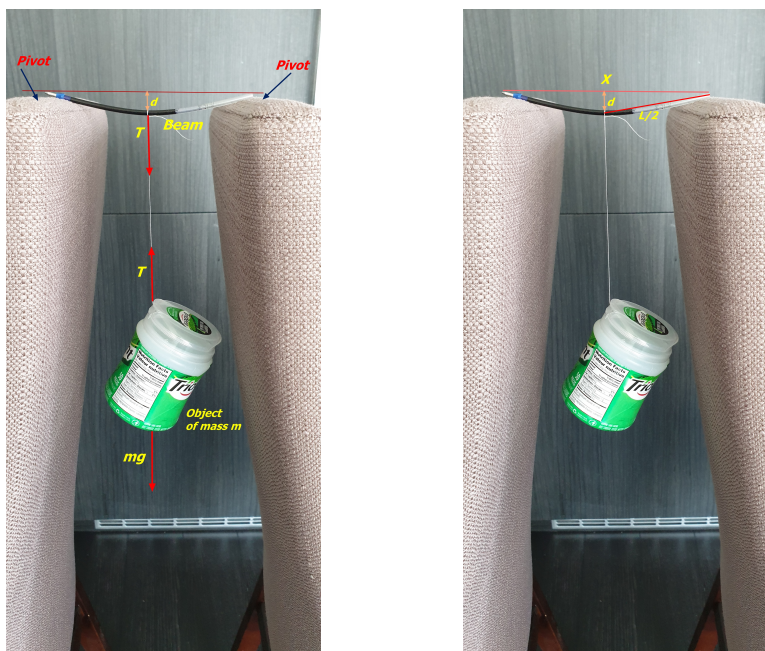


Figure 7.3: Left: A suggested configuration and the forces at work. Right: if you cannot measure d directly, you can make an approximation using Pythagorean theorem.

7.2.3 Experiment and Data Collection

All the data should be recorded in SI units.

Step 1 Choose beam and a mass m_1 to be hung from the center of the beam. Measure the length of the beam L and write it down. Also, measure the mass m_1 of the object and write it down. If you do not have a scale, you can use one of the methods mentioned in Appendix A.

Step 2 Tie one end of the string to the center of the beam and the other end to the object.

Step 3 Find two pivots with the same height that can hold the beam parallel to the ground and slowly put the end of the beam on these pivots such that the object hangs from the center of the beam (Fig. 7.3).

Step 4 Measure the deflection d at the center of the beam. You may either measure it directly, or, as shown in the picture in the right in Fig. 7.3, you can measure the distance X from one end to the other end of the beam while it is bent, and then using what is shown in figure and the Pythagorean theorem find d approximately as

$$\left(\frac{X}{2}\right)^2 + d^2 \approx \left(\frac{L}{2}\right)^2 \Rightarrow \boxed{d = \frac{1}{2}\sqrt{L^2 - X^2}} \quad (7.8)$$

Note that the mass of the object should be such that it creates enough deflection in the beam that can be measured, yet it does not break the beam.

After step 4, remove the system from the pivot point, change the object with a different one with mass m_2 and repeat the steps. You should repeat the experiment with three different masses m_1, m_2, m_3 .

7.2.4 Calculations and Data Analysis

You should fill the following table in the Excel template sheet for each run. There should be 3 runs for this experiment, one for each mass m_1, m_2, m_3 . Before filling the table, fill in the cells that ask you about the object you used for the pivot, the beam, the string, and the masses. Then follow these instructions to fill the table.

Trial	$m(\text{kg})$	$w(\text{m})$	$h(\text{m})$	$L(\text{m})$	$d(\text{m})$	Y (Pa)
m_1						
m_2						
m_3						
Mean	N/A	N/A	N/A	N/A		
Stand. dev.	N/A	N/A	N/A	N/A	N/A	

- In the second column enter the mass of the hanging object.
- In the third column enter the width of the beam w (Fig. 7.2). In case of a beam with circular cross section with radius r , we can set $w = 2r$.
- In the fourth column enter the height of the beam h (Fig. 7.2). In case of a beam with circular cross section with radius r , we can set $h = 2r$.
- In the fifth column enter the length L of the beam (before deflection).
- In the sixth column enter the deflection d from the string that connects object 2 to the balancing rod to the pivot point at the center of the beam (Figs. 7.2 and 7.3). Note that you can also find this using Pythagorean theorem as mentioned in Eq. (7.8).
- In the seventh column enter your computed Young's Modulus

$$Y = \frac{mgL^3}{4wh^3d}. \quad (7.9)$$

- Finally find the mean and standard deviation of the column without N/A (only the blue cells in the Excel file) and write them in the table.
- Attach a clear photo of your experimental setting to the Excel file in the designated place.

The significant figures for the values of the measurements you get and enter in the tables should match the uncertainty in your measurement devices. For example if you ruler can only measure up to a millimeter, any SI measurements should have three significant figures, e.g., 0.250 m. To increase/decrease the significant figures in a cell in Excel, see Appendix D, particularly Fig. D.1.

7.2.5 Questions

Answer the following questions in the Excel template sheet.

1. Identify two errors that may happen in this activity and explain the source of each error.
2. What can be done to decrease each of the errors you mentioned above?
3. Using one of the formulas in the theory part, argue that if the length of a beam remains constant but it becomes thicker, i.e., its width and/or height increases, then it deflects less, i.e., d becomes smaller. Does that make sense intuitively?
4. Using one of the formulas in the theory part, argue that if the thickness of a beam remains constant but it becomes longer, i.e., its length increases, then it deflects more, i.e., d becomes larger. Does that make sense intuitively?
5. In which case the use of Pythagorean theorem (7.8) gives a more precise value for d : when d is small or when d is large? Present your argument.
6. Using Eq. (7.5) find d for yourself (no need to present it). Based on this formula, if a material has a larger Y , is it stronger or weaker? Why? How about a material with smaller Y ? Is it stronger or weaker?

7.3 Prepare your report

Send the following documents as your lab report

- Answered pre-lab questions
- Filled Excel sheet

Chapter 8

Collisions

8.1 Introduction and Theory

Collisions are very common in the physical world. Essentially whenever two objects come in contact where the velocity of one of them is non-zero, we are dealing with a collision. Collisions can happen between two or more objects and can be one- two- or three-dimensional in the sense that the initial and final trajectories of particles may lie in one or more dimensions. A classic example of two dimensional collision is the collision between billiard balls. Other examples are car accidents, collision of comets and asteroids with planets, etc.

There are two important concepts that we should consider in a collision. The first one is the **linear momentum**

$$\mathbf{p} = m\mathbf{v} \quad (8.1)$$

where m is the mass of an object and $\mathbf{v}$ is its velocity. The second one is the energy. In this case, since for this course we only consider collisions between objects that do not exert any force on each other from a distance and only affect each other when they come into contact, we neglect the potential energy and only consider the **kinetic energy**

$$K = \frac{1}{2}mv^2. \quad (8.2)$$

In what follows we see why these two concepts are important.

Suppose that we have N objects, each with a mass m_i , where $i = 1, \dots, N$, and they are involved in a collisions. If we ignore the friction and drag forces of surfaces, air, water, etc., the total linear momentum of the system in any collision is always conserved. It means that the sum of the initial linear momenta of all objects $\sum_{i=1}^N p_i$ is the same as the sum of their final linear momenta, i.e., $\sum_{i=1}^N p'_i$

$$\boxed{\sum_{i=1}^N \mathbf{p}_i = \sum_{i=1}^N \mathbf{p}'_i} \Rightarrow \boxed{\sum_{i=1}^N m_i \mathbf{v}_i = \sum_{i=1}^N m_i \mathbf{v}'_i}. \quad (8.3)$$

Here non-primed quantities are initial ones and primed ones are the final values.

On the other hand kinetic energy is not always conserved. In cases where the particles can be considered so small and without internal structure, i.e., point particles, or when their shapes does not change during the collision, they do not stick together or no heat is lost/created during the collision (of course usually this is an approximation), the kinetic energy is conserved, i.e.,

$$\boxed{\sum_{i=1}^N K_i = \sum_{i=1}^N K'_i} \Rightarrow \boxed{\sum_{i=1}^N \frac{1}{2} m_i v_i^2 = \sum_{i=1}^N \frac{1}{2} m_i v_i'^2} \quad (8.4)$$

and this kind of collision is called an **elastic collision**.

If during the collision, objects change their shapes (such as cars that deform during an accident), or stick together, part of the initial kinetic energy of the system is used in deforming the objects and is “wasted” in thermodynamic processes where energy is transferred to internal degrees of freedom in the form of heat and molecular motion. This type of collision where the kinetic energy is not conserved is called an **inelastic collision**, for which we have

$$\boxed{\sum_{i=1}^N K_i = \sum_{i=1}^N K'_i + Q} \quad (8.5)$$

where Q (positive or negative) includes the conversion of mechanical energy into other forms such as heat. A certain type of inelastic collision is when after the collision, the objects all stick together and move together.

8.1.1 Analysis related to the lab experiment

Suppose we want to study a collision in one dimension in which (Fig. 8.1):

- Only two objects with the same mass $m_1 = m_2 = m$ are involved,
- The “target”, i.e. the mass m_2 is sitting still before the collision, so $v_2 = 0$, and object m_1 comes towards it with a velocity $\mathbf{v}_1$.
- It is a perfect inelastic collision, i.e., the two masses stick together after the collision and move as one body of mass $2m$ with the same velocity $\mathbf{v}'_1 = \mathbf{v}'_2 = \mathbf{v}'$.

Given these conditions, the conservation of linear momentum (8.3) becomes (no vectors since we are in one dimensions)

$$mv_1 = 2mv' \Rightarrow v_1 = 2v'. \quad (8.6)$$

Because the collision is perfectly inelastic we also have from (8.5),

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + Q \Rightarrow mv_1^2 = 2mv'^2 + Q, \quad (8.7)$$

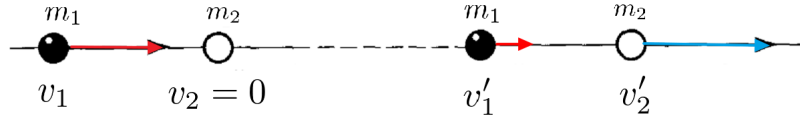


Figure 8.1: A one-dimensional (head on) collision between two objects.

So we have a system of equations

$$\begin{cases} v_1 = 2v' \\ mv_1^2 = 2mv'^2 + Q \end{cases} \quad (8.8)$$

and if we assuming we do know know the values of v_1 and Q but we know v' and m , the system can be solved for the unknown to give us

$$Q = 2mv'^2. \quad (8.9)$$

This is a positive quantity on the right hand side of (8.5), so it means the initial kinetic energy is now partially gone into the final kinetic energy and partially into $Q > 0$ and is thus partially lost.

In our experiment we cannot measure v' directly. However we can do as following. After the collision the objects move together on a surface. If there is no kinetic friction between the surface and the objects, the objects will move with the same final velocity v' forever. However, if there is, the the kinetic friction force

$$f_k = \mu_k N = 2m\mu_k g, \quad (8.10)$$

will produce a negative acceleration with a magnitude

$$a_k = \frac{f_k}{2m} = \mu_k g, \quad (8.11)$$

and slows down the bodies until a complete stop, over a distance Δx . This distance Δx can be found from the equation of motion

$$0^2 - v'^2 = -2a_k \Delta x \Rightarrow v'^2 = 2a_k \Delta x. \quad (8.12)$$

Using (8.11) in this equation we get

$$v'^2 = 2\mu_k g \Delta x \quad (8.13)$$

and replacing this value into (8.9) yields

$$\boxed{Q = 4m\mu_k g \Delta x}. \quad (8.14)$$

This way, if we have the coefficient of kinetic friction μ_k between the objects and the surface, we can compute the total lost energy.

8.1.2 Learning Outcomes

- Learn how to use computer and smartphone hardware and software to record data
- Understand the concepts of elastic and inelastic collision, and conservation of momentum in the processes
- Computing the lost energy during a perfectly inelastic collision

8.2 Experimental Setup and Measurements

In this experiment, you will find the lost energy during an inelastic collision.

8.2.1 Apparatus

See the video for more details.

- Computer with access to Microsoft Excel (Excel is part of Office365 which is available to York University students free of charge).
- Several (at least eleven) ice cubes with the same mass. For each run, one of the cubes will act as the **target** and another one as the **incident** particle. One other cube is only used to lubricate the surface to have less friction. In Fig. 8.1 the left object is the incident particle and the right one is the target.
- A sheet of aluminum foil that acts as the surface on which the ice cubes move and collide.
- Ruler. If you do not have one, you can install the the apps “Ruler” by Xalpha Lab or “Ruler” by NixGame on your smartphone.

8.2.2 Watch the Videos

Watch the video on how to perform this experiment and measurements.

8.2.3 Experiment and Data Collection

All the data should be recorded in SI units.

Step 1 Measure the masses m of the ice cubes and write it down. They should have the same mass. If you do not have a scale, you can use one of the methods mentioned in Appendix A.

Step 2 Cut a large rectangular sheet of aluminum foil and put it on a flat surface such as the kitchen counter, etc. (Fig. 8.2).

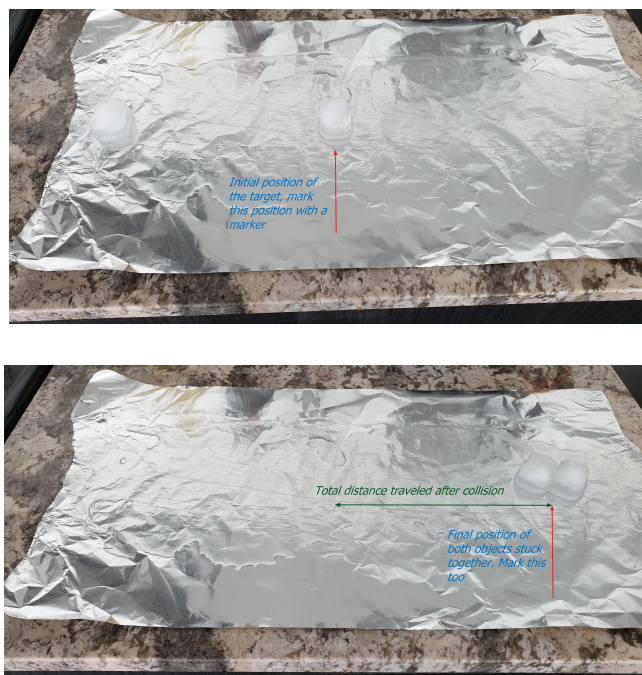


Figure 8.2: Top: Initial position of the target (ice cube in the center of the image) and the incident particle (ice cube near the left edge). Bottom: the final position of the two ice cube after colliding, sticking together, and stopping after moving a total distance Δx .

Step 3 Move one of the three ice cubes slowly on all parts of the aluminum foil to make all of the surface wet such that other ice cubes can later slip on it much easier.

Step 4 Place the target somewhere close to the center of the aluminum sheet. Mark this point with a pen or a marker (Fig. 8.2).

Step 5 Place the incident ice cube on one side of the sheet away from the target and push it so that it moves towards the target ice cube and collide with it in a line (Fig. 8.2). If the initial and final motions do not happen in a straight line, try again.

Step 6 The ice cubes will generally stick together after collision. If it does not happen try again. If they do stick together and move together, let them move until they stop. After that, mark the final position in which they stop (Fig. 8.2). Measure the distance between the initial position of the target and this final position. This will be your Δx .

After step 6, use two new ice cubes and repeat the experiment. You should perform 5 runs.

8.2.4 Calculations and Data Analysis

You should fill the following table in the Excel template sheet for each run. There should be 5 runs for this experiment, each with two new ice cubes. Follow instructions below to fill the table.

Trial	$m(\text{kg})$	$\Delta x(\text{m})$	$Q(\text{J})$	$\frac{Q}{\Delta x}(\text{J/m})$
1				
2				
3				
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
5				
Mean				
Standard deviation				

- In the second column enter the mass of the ice cubes.
- In the third column enter the total distance traveled after the collision (Fig. 8.2).
- In the fourth column enter the lost energy Q computing using the formula

$$Q = 4m\mu_k g \Delta x \quad (8.15)$$

where you can use

$$\boxed{\mu_k = 0.03} \quad (8.16)$$

- In the fifth column enter the ratio of Q to Δx for that run.
- Finally find the mean and standard deviation of each column and write it below them in the table.
- Attach a clear photo of your experimental setting to the Excel file in the designated place.
- Also a graph of Q vs Δx should be plotted automatically in your Excel file using your data. If it does not appear, talk to the TA to help you.

The significant figures for the values of the measurements you get and enter in the tables should match the uncertainty in your measurement devices. For example if your ruler can only measure up to a millimeter, any SI measurements should have three significant figures, e.g., 0.250 m. To increase/decrease the significant figures in a cell in Excel, see Appendix D, particularly Fig. D.1.

8.2.5 Questions

Answer the following questions in the Excel template sheet.

1. Identify two errors that may happen in this activity and explain the source of each error.
2. What can be done to decrease each of the errors you mentioned above?

3. The form of the fit line in your graph should be similar to a linear plot. Explain why it has this form using Eq. (8.15).
4. Is Q always constant even if mass, friction coefficient and g are kept constant? Using your graph and Eq. (8.15) explain why.
5. In your plot, an equation in the form of $y = mx + b$ appears. The value of m is the slope of the fit line. Explain what does this represent.

8.3 Prepare your report

Send the following documents as your lab report

- Answered pre-lab questions
- Filled Excel sheet

Appendix A

Measuring Mass

A.1 Method 1: Archimedes principle

The Archimedes principle states that a body immersed in a fluid experiences a buoyant force directed towards the surface equal to the weight of the fluid it displaces. Using this principle, it is possible to relate the weight of a floating object to the volume of liquid which has been displaced when it is floating. Using Newton's second law, we can write the following where m is the unknown mass, F_b is the buoyant force, M_d is the mass of the displaced liquid, and g is the gravitational constant.

$$mg = F_b = M_d g. \quad (\text{A.1})$$

Using this equation and the fact that 1 ml of water = 1 cm³ of volume and 1 ml of water has a mass of 1 g, we can derive a procedure to obtain the mass of an object using a basin of water and a floating vessel. In fact, canceling g on both sides yields

$$m = M_d. \quad (\text{A.2})$$

To illustrate this procedure, the mass of a nail clipper, illustrated in figure A.1 will be measured with Archimedes's principle.

Our goal is to obtain the volume of displaced water by this clipper when it is floating. However, its density is too large to float on its own. To mitigate this, we can use a cup shaped plastic object (such as a plastic cap on a bottle of spices) as a boat for this object. The initial setup for this process is illustrated in figure A.2.

The first step is to create a setup similar to figure A.2. To do this, fill a cup of water not all the way. Place a floating vessel inside without placing the unknown mass. You can then use a ruler to measure the water line in this configuration. In the figure, the ruler is taped onto the glass with the water level indicated at 0 cm. The next step is to put the object to be measured into the boat and log the change in water level as is illustrated in figure A.3. Here we observed that the water level increased to 0.3 cm.

In addition to the water level rising, the boat has also sunk a bit due to the increased mass within it. To obtain accurate mass, we must also keep track of this sinking. To do this, the boat is brought close to the edge of the cup so that its depth before and after can be



Figure A.1: The mass of the clipper which will be measured using Archimedes' principle. Its mass is 15g according to a kitchen scale.



Figure A.2: The setup of this experiment. A transparent cup is filled with water and a plastic cap is floating at the top. A ruler is used to indicate the water level, which sits initially at 0 cm.

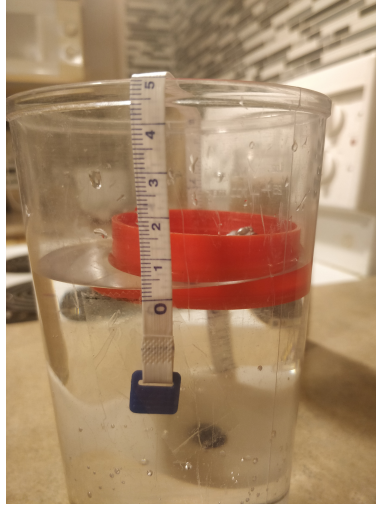


Figure A.3: The water level after the mass is placed into the boat. The level is now at 0.3 cm relative to where it was before adding the mass.

logged. This can be seen in figures A.4 and A.5. From this data, we can measure that the boat sinks by 0.25 cm in this example.

The final two parameters we need to complete our calculations are the diameters of the cup and the the boat. These are measured in figures A.6 and A.7 respectively.

To determine the volume of displaced fluid, We can use the fact that the increased water level is in the shape of the cup, which is a cylinder. However, since the boat has sunk and also displaced water that was initially there, we must remove the volume of water that was displaced by the cup in our calculations, which is also a cylinder shape. Using measured parameters d_c as the diameter of the cup and d_b as the diameter of the boat as well as L_c being the difference in water level in the cup and L_b being the depth the boat has sunk, we can calculate the mass of the unknown object with the following equation (A.3).

$$m = \pi \frac{d_c}{2} L_c - \pi \frac{d_b}{2} L_b \quad (\text{A.3})$$

It's important to note that equation (A.3) will yield a mass in grams only if all lengths are measured in centimeters! Using this formula, we can calculate a mass of 13.9g for the mass of the clipper used in the example. This is only 1.1g off of the value calculated with a commercial scale which is very reasonable for an estimate!



Figure A.4: The boat without mass is approached to the same side as the ruler to log its initial depth. We measure it to be 3.1 cm.



Figure A.5: After the mass is added to the boat, we log the new depth which falls to about 2.85 cm.



Figure A.6: Measuring the diameter of the cup. We find its diameter is 9.5 cm.



Figure A.7: Measuring the diameter of the boat. We find its diameter is 6.1 cm.

A.2 Method 2: constructing a simple scale

The essence of this method is presented in Chapter 6. For a fast usage of this method, you need to create a scale that is then set to equilibrium on a pivot as in Fig. 6.1. Find a solid beam that will act as the balancing rod (pen in Fig. 6.1), attach a known mass m_1 to one side of it using a string and attach the unknown mass m_2 to the other side of it using another string. Bring the system to equilibrium on a pivot (another pen, your finger, etc.). The unknown mass m_2 can then be found using the known mass m_1 , and the distances from each mass to the pivot point d_1 and d_2 (Fig. 6.1), as

$$m_2 = \frac{d_1}{d_2} m_1. \tag{A.4}$$

Appendix B

phyphox App

B.1 Using Acoustic Timer

This timer measures the time between two sound triggers. To access that

- Open the app on your **phyphox** smartphone, under **Timers**, click on **Acoustic Stopwatch** (Fig. B.1, left)
- Now (Fig. B.1, right) you will see a tab on top with options **SIMPLE**, **SEQUENCE**, **PARALLEL**, and **MANY**. For this activity we will use the **SIMPLE** option. Click on this option and then in the **Threshold** field, enter 0.1. Also, in the **Minimum Delay** field enter 0.1. These are the fields that can be changed based on your environment noise and the experiment. The lower the threshold, the more sensitive the app will be to starting and ending sound signals. If you find out that the starting and ending sound signals do not trigger the stopwatch, or other lower sounds undesirably trigger the stopwatch, change the **Threshold** value accordingly.
- Click on **Reset** (Fig. B.1, right).
- Click on the play button on top of the app screen (Fig. B.1, right).
- After this, any sound with that overcomes the threshold of the timer will trigger the timer to start, and the next sound will trigger its stop. So one can measure the time passed between the two triggers.

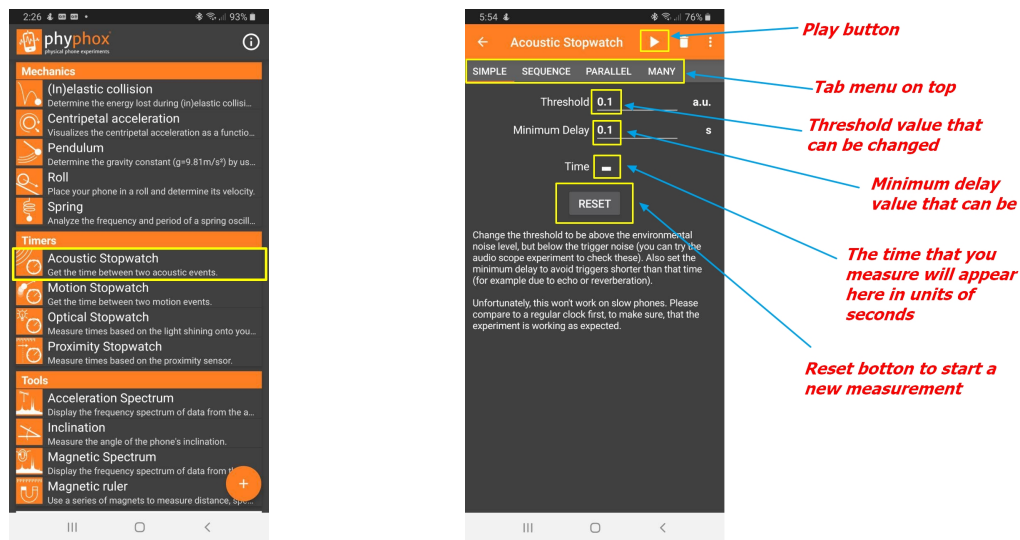


Figure B.1: Left: where to find the option Acoustic Stopwatch in the main menu of the phyphox app. Right: After clicking on Acoustic Stopwatch, several buttons and options are available.

Appendix C

Uncertainty Calculations using Differentials

If x, y etc., are measured quantities how does one calculate the resultant uncertainty in p where

1. $p = x + y$
2. $p = x - y$
3. $p = xy$
4. $p = \frac{x}{y}$
5. $p = x^a$

and combinations of all the above?

Uncertainties can be found using differentials. This method is described below for each of the above four cases.

C.1 Addition $p = x + y$

Suppose that we would like to find the uncertainty δp in p where

$$p = x + y, \tag{C.1}$$

given that the uncertainty in x is δx and uncertainty in y is δy . To do this take the differential of both sides of (C.1)

$$dp = dx + dy, \tag{C.2}$$

and replace the differential symbol d with the uncertainty symbol δ to get

$$\boxed{\delta p = \delta x + \delta y}. \tag{C.3}$$

This is the uncertainty in p if uncertainty in x, y are $\delta x, \delta y$.

C.2 Subtraction $p = x - y$

Here, we would like to find the uncertainty δp in p where

$$p = x - y, \quad (\text{C.4})$$

given that the uncertainty in x is δx and uncertainty in y is δy . To do this, again we take the differential of both sides of (C.4)

$$dp = dx - dy, \quad (\text{C.5})$$

and replace the differential symbol d with the uncertainty symbol δ to get

$$\delta p = \delta x - \delta y. \quad (\text{C.6})$$

However, this is not the uncertainty due to the negative sign. The reason is that uncertainties give us maximum (upper) or minimum (lower) bounds. But in the formula above we are subtracting two uncertainties in x, y and getting neither maximum nor minimum uncertainty. In this case we should have both $\delta x, \delta y$ have the same sign. Hence, in case of subtraction (C.4) the uncertainty is the same as addition

$$\boxed{\delta p = \delta x + \delta y}. \quad (\text{C.7})$$

C.3 Multiplication $p = xy$

In case of multiplication

$$p = xy \quad (\text{C.8})$$

we again follow the same method and take the differential of both sides

$$dp = xdy + ydx. \quad (\text{C.9})$$

As you see this is more complicated since not only dx, dy , but also x, y themselves appear in the formula. In such a case we can divide (C.9) by (C.8) to get

$$\frac{dp}{p} = \frac{dy}{y} + \frac{dx}{x}. \quad (\text{C.10})$$

Now if we replace the differential symbol d with the uncertainty symbol δ , we get

$$\boxed{\frac{\delta p}{p} = \frac{\delta y}{y} + \frac{\delta x}{x}}. \quad (\text{C.11})$$

The ratio of the uncertainty in p to the value of p itself is called the *relative uncertainty* in p . Hence, the above equation has a clear meaning: relative uncertainty in p is equal to the relative uncertainty in x plus relative uncertainty in y . From (C.11) we can find the *percentage uncertainty* of p by multiplying both side by 100 to get

$$\frac{\delta p}{p} \times 100 = \frac{\delta y}{y} \times 100 + \frac{\delta x}{x} \times 100. \quad (\text{C.12})$$

Each of the terms in the above equation are called a percentage uncertainty.

C.4 Division $p = \frac{x}{y}$

Following the same method as above for

$$p = \frac{x}{y}, \quad (\text{C.13})$$

yields

$$dp = \frac{ydx - xdy}{y^2}. \quad (\text{C.14})$$

Once again diving this by (C.13) yields

$$\begin{aligned} \frac{dp}{p} &= \frac{\frac{ydx - xdy}{y^2}}{\frac{x}{y}} \\ &= \frac{ydx - xdy}{y^2 x} y \\ \frac{dp}{p} &= \frac{dx}{x} - \frac{dy}{y}. \end{aligned} \quad (\text{C.15})$$

Replace the differential symbol d with the uncertainty symbol δ in this equation yields

$$\frac{\delta p}{p} = \frac{\delta x}{x} - \frac{\delta y}{y}. \quad (\text{C.16})$$

By the same reasoning as in section C.2, we should change the minus sign to a plus sign to finally get the correct relative uncertainty in p as

$$\boxed{\frac{\delta p}{p} = \frac{\delta y}{y} + \frac{\delta x}{x}}. \quad (\text{C.17})$$

This is similar to the multiplication case. Using this we can also compute the percentage uncertainty in this case as

$$\frac{\delta p}{p} \times 100 = \frac{\delta y}{y} \times 100 + \frac{\delta x}{x} \times 100. \quad (\text{C.18})$$

C.5 Exponentiation $p = x^a$

In the case

$$p = x^a \quad (\text{C.19})$$

we see that

$$dp = ax^{a-1}dx \quad (\text{C.20})$$

and diving (C.20) by (C.19) we get

$$\frac{dp}{p} = a \frac{dx}{x}. \quad (\text{C.21})$$

Once again replacing the differential symbol d with the uncertainty symbol δ we get

$$\boxed{\frac{\delta p}{p} = a \frac{\delta x}{x}}. \quad (\text{C.22})$$

C.6 More general cases and examples

When addition, subtraction, multiplication and division are all involved in one formula, things become more complicated. These may be treated by the differential process or by using the rules on one part of the function at a time. Below you can find a few simple examples of the usage of the uncertainty formulae.

C.6.1 Example 1

The dimensions of a room are

$$\begin{aligned}\text{length (l)} &: (3.04 \pm .02) \text{ m} \\ \text{width (w)} &: (3.66 \pm .02) \text{ m} \\ \text{height (h)} &: (2.74 \pm .02) \text{ m}\end{aligned}$$

What are the resultant uncertainties in

(a) the perimeter of the floor?

$$\text{Perimeter } P = 2 \times \text{length} + 2 \times \text{width} = 2l + 2w = 2(3.04) + 2(3.66) = 13.40 \text{ m.}$$

$$\text{Uncertainty in perimeter is } \delta P = 2\delta l + 2\delta w = 2(0.02) + 2(0.02) = 0.08 \text{ m.}$$

So the perimeter of the floor is therefore $(13.40 \pm 0.08) \text{ m}$.

(b) the floor area?

$$\text{Area } A_{\text{floor}} = \text{length} \times \text{width} = l \times w = (3.04 \text{ m}) \times (3.66 \text{ m}) = 11.13 \text{ m}^2$$

Uncertainty in Area is

$$\begin{aligned}\frac{\delta A_{\text{floor}}}{A_{\text{floor}}} &= \frac{dl}{l} + \frac{dw}{w} \\ \delta A_{\text{floor}} &= \left(\frac{dl}{l} + \frac{dw}{w} \right) A_{\text{floor}} \\ &= \left(\frac{0.02}{3.04} + \frac{0.02}{3.66} \right) 11.13 \\ &= 0.13 \text{ m}^2\end{aligned}$$

So the area of the floor is $(11.13 \pm 0.13) \text{ m}^2$

(c) the wall area?

$$\begin{aligned}A_{\text{walls}} &= 2(\text{length} \times \text{height}) + 2(\text{width} \times \text{height}) \\ &= 2(3.04)(2.74) + 2(3.66)(2.74) \\ &= 36.71 \text{ m}^2\end{aligned}$$

and

$$\begin{aligned}
 \delta A_{\text{walls}} &= \delta(2l \times h) + \delta(2w \times h) \\
 \delta(2l \times h) &= \left(\frac{\delta l}{l} + \frac{\delta h}{h} \right) 2lh \\
 \delta(2l \times h) &= \left(\frac{0.02}{3.04} + \frac{0.02}{2.74} \right) 2(3.04)(2.74) \\
 &= 0.23 \text{ m}^2 \\
 \delta(2w \times h) &= \left(\frac{\delta w}{w} + \frac{\delta h}{h} \right) 2lh \\
 \delta(2l \times h) &= \left(\frac{0.02}{3.04} + \frac{0.02}{2.74} \right) 2(3.04)(2.74) \\
 &= 0.26 \text{ m}^2 \\
 \delta A_{\text{walls}} &= (0.23 + 0.26) \text{ m}^2 \\
 &= 0.49 \text{ m}^2 = 0.5 \text{ m}^2
 \end{aligned}$$

So the area of the walls is $(36.7 \pm 0.5) \text{ m}^2$.

Appendix D

Some Functions of Excel

D.1 Changing the significant figures

If you want to increase or decrease the number of significant figures in any cell, select the cell and then click on one of the buttons shown in Fig. D.1 that can be found in the “Home” ribbon under “Numbers” section.

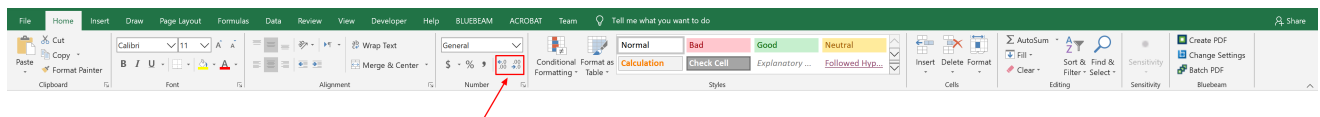


Figure D.1: Buttons used to increase/decrease the number of significant figures in a cell. Here a red box shows their position.