

3.

(a)

$$\sec \theta = \frac{29}{30}$$

$$\sec^2 \theta = \left(\frac{29}{30}\right)^2$$

$$\sec^2 \theta - 1 = \frac{841}{900} - 1$$

$$\sec^2 \theta - 1 = -\frac{59}{900}$$

$$[\sec^2 \theta - 1 = \tan^2 \theta]$$

$$\tan^2 \theta = -\frac{59}{900}$$

$$\tan \theta = \sqrt{-\frac{59}{900}}$$

$$\tan \theta = \frac{\sqrt{59}}{30} i$$

(b)

Based on the information given, $\cos \theta$ is negative, which happens in the 2nd and 3rd quadrants, and $\cot \theta$ is positive, which happens in 1st and 3rd quadrants. The only time both are true is in the 3rd quadrant, so that's where our answer will be.

$$\cos \theta = \frac{A}{H} = \frac{-3}{7}$$

$$O^2 = H^2 - A^2 \text{ [Pythagoras Theorem]}$$

$$O = \sqrt{7^2 - (-3)^2} = \sqrt{40} = 2\sqrt{10}$$

$$\sin \theta = \frac{O}{H} = \frac{2\sqrt{10}}{7}$$

Our answer is supposed to be in the 3rd quadrant, and $\sin \theta$ is negative in the 3rd quadrant, so:

$$\sin \theta = -\frac{2\sqrt{10}}{7}$$

4.

$$\sec(-2x) = -\sqrt{2}$$

$$\frac{1}{\cos(-2x)} = -\sqrt{2}$$

$$\cos(-2x) = -\frac{1}{\sqrt{2}}$$

$$\cos \alpha = \frac{1}{\sqrt{2}}$$

where α is acute

$$\alpha = \cos^{-1} \frac{1}{\sqrt{2}}$$

$$\alpha = \frac{\pi}{4}$$

Since $\cos$ is negative, the answer should be in the 2nd and 3rd quadrant

For 2nd quadrant

$$\pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

For 3rd quadrant

$$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$-2x = \frac{3\pi}{4}, \frac{5\pi}{4}$$

$$x = -\frac{3\pi}{8}, \frac{5\pi}{8}$$

5.

$$\cos 3x - \cos x = \sin 2x$$

$$\cos(2x + x) - \cos(2x - x) = \sin 2x$$

Using the cos addition and subtraction of angle formulae

$$[\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B]$$

$$(\cos 2x \cos x - \sin 2x \sin x) - (\cos 2x \cos x + \sin 2x \sin x) = \sin 2x$$

$$\cos 2x \cos x - \cos 2x \cos x - \sin 2x \sin x - \sin 2x \sin x = \sin 2x$$

$$-2 \sin 2x \sin x = \sin 2x$$

Canceling $\sin 2x$ on both sides

$$-2 \sin x = 1$$

$$\sin x = -\frac{1}{2}$$

$$\sin \alpha = \frac{1}{2}$$

where α is acute

$$\alpha = \sin^{-1} \frac{1}{2}$$

$$\alpha = \frac{\pi}{6}$$

Since $\sin$ is negative, it is in the 3rd and 4th quadrants

For 3rd quadrant

$$\frac{\pi}{6} + \pi = \frac{7\pi}{6}$$

For 4th quadrant

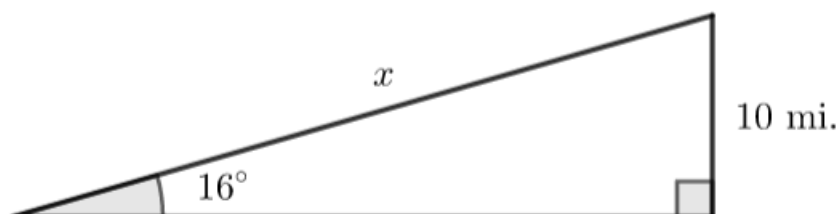
$$2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

6.

(a)

Let the actual distance covered by the airplane in climbing 10 miles vertically be x . Then we can draw a right-angled triangle with the hypotenuse x and 10 miles the opposite side to the angle 16° :



$$\sin 16^\circ = \frac{10}{x}$$

$$x = \frac{10}{\sin 16^\circ}$$

$$x \approx 36.28 \text{ miles}$$

The time taken to travel 36.28 miles at an average speed of 420 miles/hour is

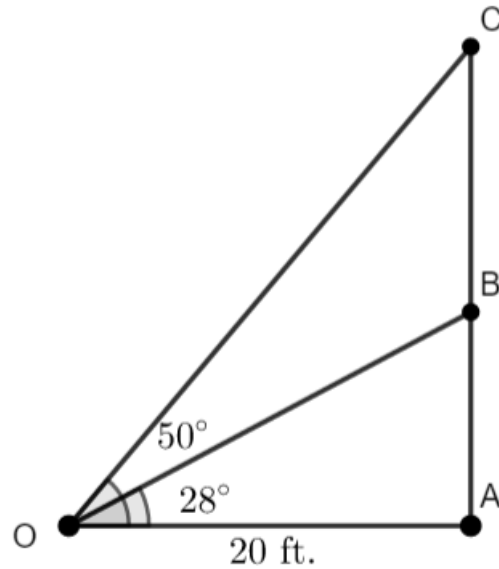
$$t = \frac{36.28}{420}$$

$$t = 0.08637 \text{ hours}$$

$$t = 5.18 \text{ minutes}$$

(b)

Let A be the base of the tree, B the bottom of the treehouse, C the top of the treehouse, and O the point from which Joaquin makes the measurement. Then we can make the following diagram



$$\tan 50^\circ = \frac{AC}{20}$$

$$AC = 20 \tan 50^\circ$$

$$AC \approx 23.835 \text{ ft.}$$

$$\tan 28^\circ = \frac{AB}{20}$$

$$AB = 20 \tan 28^\circ$$

$$AB \approx 10.634 \text{ ft.}$$

$$BC = AC - AB$$

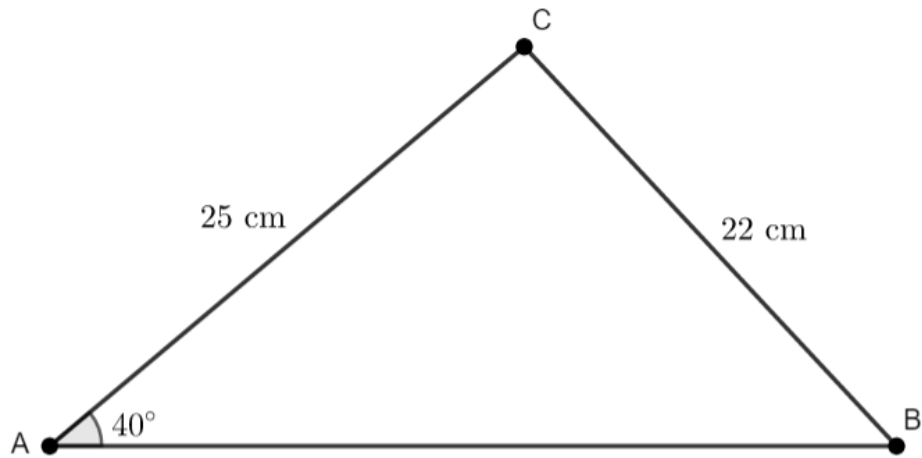
$$BC = 23.835 - 10.634$$

$$BC \approx 13.2 \text{ ft.}$$

The treehouse is 13.2 ft. tall.

7.

The triangle with the given information is shown below



We know $\hat{A} = 40^\circ$, $a = 22$ cm and $b = 25$ cm

We have to find $\hat{B}$, $\hat{C}$ and c

To find $\hat{B}$, we will use the Sine Rule

$$\frac{\sin \hat{B}}{b} = \frac{\sin \hat{A}}{a}$$

$$\frac{\sin \hat{B}}{25} = \frac{\sin 40^\circ}{22}$$

$$\sin \hat{B} = 0.7304$$

$$\hat{B} = \sin^{-1} 0.7304$$

$$\hat{B} \approx 46.9^\circ$$

We can find $\hat{C}$ by using the sum of a triangle's interior angles

$$\hat{A} + \hat{B} + \hat{C} = 180^\circ$$

$$40^\circ + 46.9^\circ + \hat{C} = 180^\circ$$

$$\hat{C} = 93.1^\circ$$

We can find c by using the Cosine Rule

$$c^2 = a^2 + b^2 - 2ab \cos \hat{C}$$

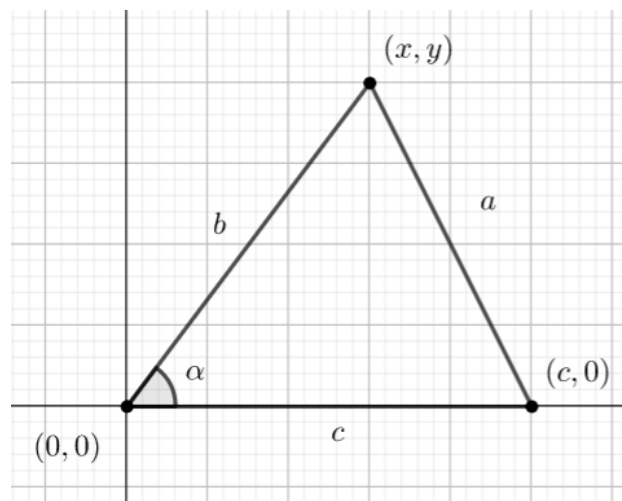
$$c^2 = 22^2 + 25^2 - 2(22)(25) \cos 93.1^\circ$$

$$c^2 \approx 1168.49$$

$$c \approx 34.2 \text{ cm}$$

8.

Triangle drawn as instructed



$$x = b \cos \alpha$$

$$y = b \sin \alpha$$

$$a^2 = (x - c)^2 + (y - 0)^2 \text{ [Distance between two points]}$$

$$a^2 = x^2 - 2xc + c^2 + y^2$$

$$a^2 = (b \cos \alpha)^2 - 2(b \cos \alpha)c + c^2 + (b \sin \alpha)^2$$

$$a^2 = b^2 \cos^2 \alpha + b^2 \sin^2 \alpha + c^2 - 2bc \cos \alpha$$

$$a^2 = b^2(\cos^2 \alpha + \sin^2 \alpha) + c^2 - 2bc \cos \alpha$$

$$\text{Using } \cos^2 \alpha + \sin^2 \alpha \equiv 1$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha \text{ [Cosine Rule]}$$

9.

$$r = 9 \sin \theta$$

Multiply both sides by r

$$r^2 = 9r \sin \theta$$

Use $r^2 = x^2 + y^2$ and $y = r \sin \theta$

$$x^2 + y^2 = 9y$$

Complete the square for y

$$x^2 + (y^2 - 9y) = 0$$

Add and subtract $\left(\frac{9}{2}\right)^2$

$$x^2 + \left(y^2 - 2\left(\frac{9}{2}\right)y + \left(\frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2\right) = 0$$

Rearrange terms

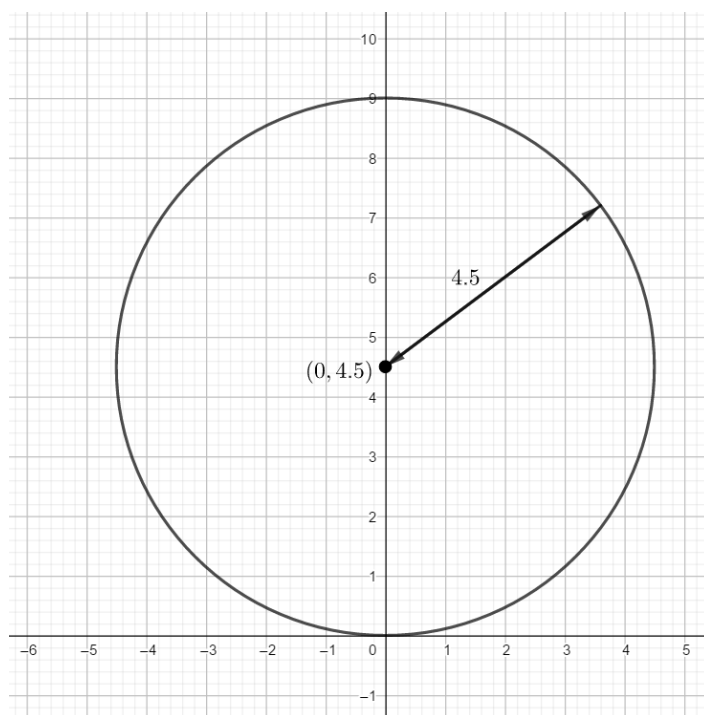
$$x^2 + \left(y^2 - 2\left(\frac{9}{2}\right)y + \left(\frac{9}{2}\right)^2\right) = \left(\frac{9}{2}\right)^2$$

The term in brackets fits the pattern $a^2 - 2ab + b^2$, so write it as $(a - b)^2$

$$x^2 + \left(y - \frac{9}{2}\right)^2 = \left(\frac{9}{2}\right)^2$$

$(x - a)^2 + (y - b)^2 = r^2$ is the equation for a circle with center (a, b) and radius r

Our equation is the equation for a circle with center $(0, \frac{9}{2})$ and radius $\frac{9}{2}$



10.

$$r = 1 + 4 \cos \theta$$

Substitute $(r, -\theta)$

$$r = 1 + 4 \cos(-\theta)$$

Using $\cos(-\theta) \equiv \cos \theta$

$$r = 1 + 4 \cos \theta$$

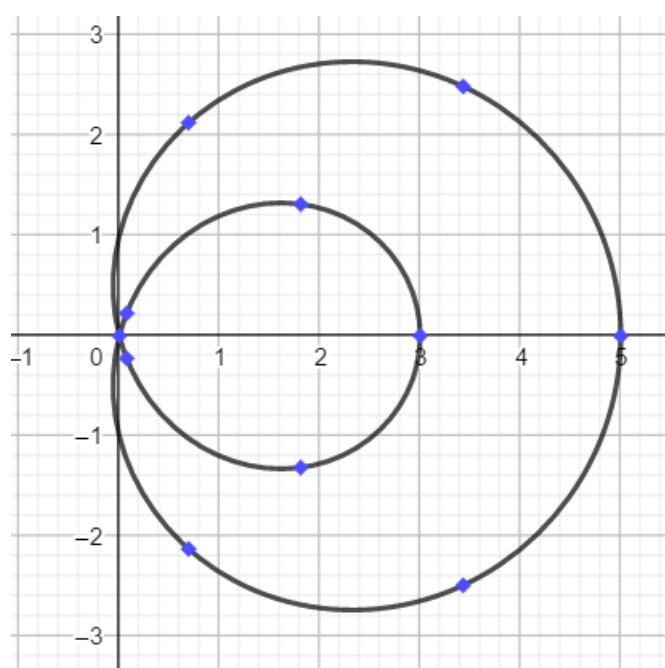
Since the equation didn't change when we substituted $(r, -\theta)$, the graph is symmetric about the x -axis

Use 11 evenly spaced values of θ in the interval $[0, 2\pi]$, substitute them in the equation to find r and make a table of polar coordinates

θ	0	0.628	1.257	1.885	2.513	3.142	3.770	4.398	5.027	5.655	6.283
r	5	4.236	2.236	-0.236	-2.236	-3	-2.236	-0.236	2.236	4.236	5

Plot the polar coordinates by putting a point r units away from the origin, at an anticlockwise angle of θ from the x -axis. Ignore negative sign of r

Draw a spiraling curve through the plotted points



11.

Convert $z = -32i$ to polar form

$$|z| = \sqrt{32^2 + 0^2}$$

$$|z| = 32$$

$$\arg z = \tan^{-1} -\frac{32}{0}$$

$$\arg z = -\frac{\pi}{2}$$

$$z = 32e^{-i\frac{\pi}{2}}$$

Express the 4th root of z using the Root Theorem

$$z^{\frac{1}{4}} = (32)^{\frac{1}{4}}e^{-i(\frac{1}{4}\frac{\pi}{2}+0)}, (32)^{\frac{1}{4}}e^{-i(\frac{1}{4}\frac{\pi}{2}+\frac{2\pi}{4})}, (32)^{\frac{1}{4}}e^{-i(\frac{1}{4}\frac{\pi}{2}+\frac{4\pi}{4})}, (32)^{\frac{1}{4}}e^{-i(\frac{1}{4}\frac{\pi}{2}+\frac{6\pi}{4})}$$

Simplify the angles

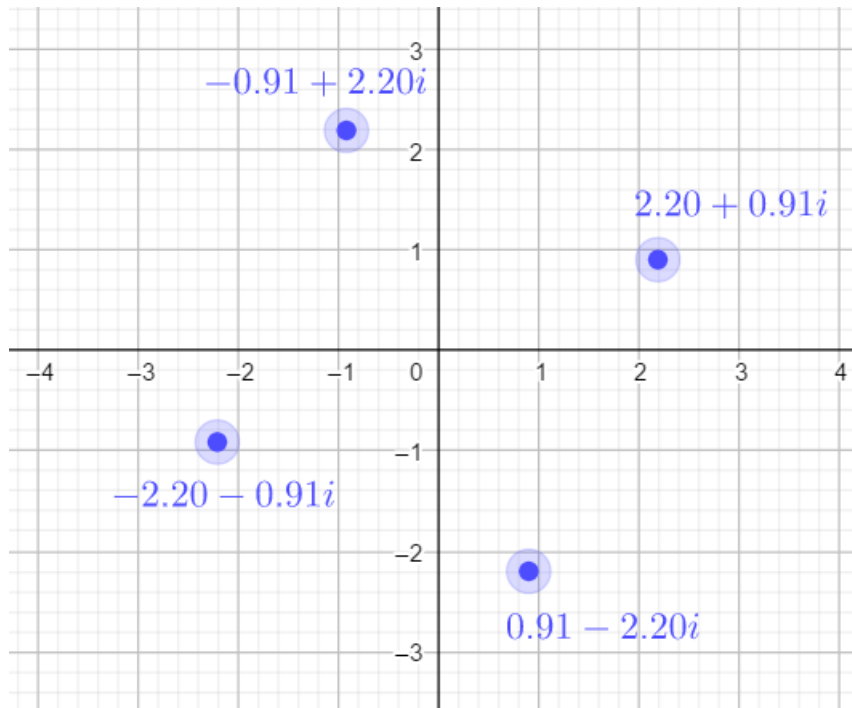
$$z^{\frac{1}{4}} = (32)^{\frac{1}{4}}e^{-i\frac{\pi}{8}}, (32)^{\frac{1}{4}}e^{-i\frac{5\pi}{8}}, (32)^{\frac{1}{4}}e^{-i\frac{9\pi}{8}}, (32)^{\frac{1}{4}}e^{-i\frac{13\pi}{8}}$$

Convert to cartesian form

$$z^{\frac{1}{4}} = (32)^{\frac{1}{4}}\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right), (32)^{\frac{1}{4}}\left(\cos\frac{5\pi}{8} + i\sin\frac{5\pi}{8}\right), (32)^{\frac{1}{4}}\left(\cos\frac{9\pi}{8} + i\sin\frac{9\pi}{8}\right),$$

$$= (32)^{\frac{1}{4}}\left(\cos\frac{13\pi}{8} + i\sin\frac{13\pi}{8}\right)$$

$$z^{\frac{1}{4}} = 2.20 + 0.91i, -0.91 + 2.20i, -2.20 - 0.91i, 0.91 - 2.20i$$



12.

$$\mathbf{v} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -3 \end{pmatrix} = (7)(0) + (0)(-3) = 0 \text{ [Orthogonal]}$$

$$\mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -6 \end{pmatrix} = (-2)(0) + (0)(-6) = 0 \text{ [Orthogonal]}$$

13.

(a)

$$x = h + a \cos \theta$$

$$a \cos \theta = x - h$$

$$a^2 \cos^2 \theta = (x - h)^2$$

$$\cos^2 \theta = \frac{(x - h)^2}{a^2}$$

$$y = k + b \sin \theta$$

$$b \sin \theta = y - k$$

$$b^2 \sin^2 \theta = (y - k)^2$$

$$\sin^2 \theta = \frac{(y - k)^2}{b^2}$$

$$\cos^2 \theta + \sin^2 \theta = \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2}$$

Use $\cos^2 \theta + \sin^2 \theta \equiv 1$ and swap sides

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

(b)

The major axis of the ellipse lies between the vertices $(-5,1)$ and $(1,1)$, so the major axis is horizontal, because both points have the same y -coordinate. The length of the major axis is

$$1 - (-5) = 6$$

$$a = 6$$

The minor axis of the ellipse lies between the endpoints $(-2,1)$ and $(-2,3)$, so the minor axis is vertical, because both points have the same x -coordinate. The length of the minor axis is

$$3 - 1 = 2$$

$$b = 2$$

The center of the ellipse is $(-2,1)$

$$(h, k) = (-2, 1)$$

Using the parametric pattern $x = h + a \cos t$, $y = k + b \sin t$

$$x = -2 + 6 \cos t$$

$$y = 1 + 2 \sin t$$