Permutations & Other Counting Problems

1) How many permutations of the letters of the word “MOUSE” are there?

In the word "Mouse" five distinct letters are present.
Therefore, number of ways of permutation =5! = 120

2) Eight people line up for a photograph, four in a front row and four in a back row. How many different arrangements are possible?

4 people in front row 4!

4 people in back row 4!

So total number of ways = 4! X 4! = 576

3) A basketball team has 12 players. Five of these players will start the game. How many different ways can the starting line of 5 players be chosen?

Number of students = 12

Number of students in straight line = 5

Number of ways = ${}_{12}C_5 = \frac{12!}{5!\times 7!} = 792$

4) A class of 18 students lines up at the door to leave. Harris and Patrick refuse to stand together. How many different arrangements are still possible?

Total number of sequence = 18!

Case when harry and patrick are together = 17 x 16!

When harry and patrick are not together = 18! – 17 x 16! = 6.04e+15

5) How many permutations are there of the words “GO”, “TO”, and “SLEEP”?

For letter “GO” number of distinguishable permutations = 2! =2

For letter “TO” number of distinguishable permutations = 2! =2

For letter “SLEEP” number of distinguishable permutations = $\frac{5!}{2!} = 60$ =2

6) How many different 3-letter permutations are there of letters from “MATH”?

There are 4 ways to choose your first letter. There are 3 ways to choose your second letter. There are 2 ways to choose your third letter.

The answer will be a product = 4 x 3 x 2 = 24

7) Gmail ignores periods placed in email addresses. For example, hey.there@gmail.com is interpreted the same as heythere@gmail.com or he.yt.here@gmail.com. How many versions of the Gmail address no.one@gmail.com can be formed using periods in this way?

8) Harris and Patrick both want to stand next to Cameron. How many permutations are possible?

Harry , patric and Cameron can be arranged in 3!

And remaining students can be shuffled in 15!

Total number of ways = 15! X 3! = 7.86e+12

9) A new guitar player knows 4 chords. How many 4-chord sequences are possible if she plays each chord only once? How many sequences are possible if non-consecutive chords can be the same?

If chord played only once then $4! = 24$

If non- consecutive can be same $4^4 = 256$

10) A box of candy contains 8 blue candies, 4 red candies, and 3 green candies. If the candies are removed from the box one at a time, how many sequences of colours are possible?

Total number of candies = 8+4+3 = 15

Number of sequence = 15! = 1.307e+12

11) How many 5-digit numbers are there between 10000 and 99999 which have unique digits?

There are two cases

Case 1 if the last digit is 0

then first digit can be any of 1,2,…,9 = 9 ways

second digit will be any other 8 digits and so on

$$\rightarrow 9 \times 8 \times 7 \times 6 = 3024$$

Case 2 if the last digit is not 0

Then first digit can be selected in 8 ways

And second digit can be selected in 8 ways

And third digit can be selected in 7 ways

$$8 \times 8 \times 7 \times 6 \times 5 = 13440$$

So total number of ways = 13440+3024 = 16464

12) How many 5-digit numbers are there between 10000 and 99999 which have *non-unique* digits? (at least one digit repeats)

First digit can't be 0 hence 9 ways

Rest numbers can repeat hence 10 ways

Number of ways = 9 x 10 x 10 x 10 x 10 = 90000

13) How many 5-digit numbers are there between 10000 and 99999 which have at least one pair of identical consecutive digits?

For a base 10 number, each digit can be from 0 to 9

However, for a five digit number first digit can't be 0. Thus, there are 9 options from 1 to 9 for the first digit. There are 10 options from 0 to 9 for each of the second, third, fourth and fifth digit.

So, number of base 10 numbers having 5 digits $= 9 * 10 * 10 * 10 * 10 = \mathbf{90000}$

A five digit number to have no consecutive digit equal :

The first digit can have any of the 9 options from 1 to 9. The second digit can't be the same as the first digit.Thus, there are 9 options for the second digit.The third digit can't be same as the second digit. So, there are 9 options for the third digit. So, all the digits have five options where the first digit can be from 1 to 9 and any other digit can't be equal to the previous digit.

So, number of 5 digit numbers having no consecutive digits equal

$= 9 * 9 * 9 * 9 * 9 = \mathbf{59049}$

Number of 5 digit numbers having at least one pair of consecutive digits equal = Total count of base 10 numbers having 5 digits − Number of 5 digit numbers having no consecutive digits equal $= 90000 - 59049 = \mathbf{30951}$