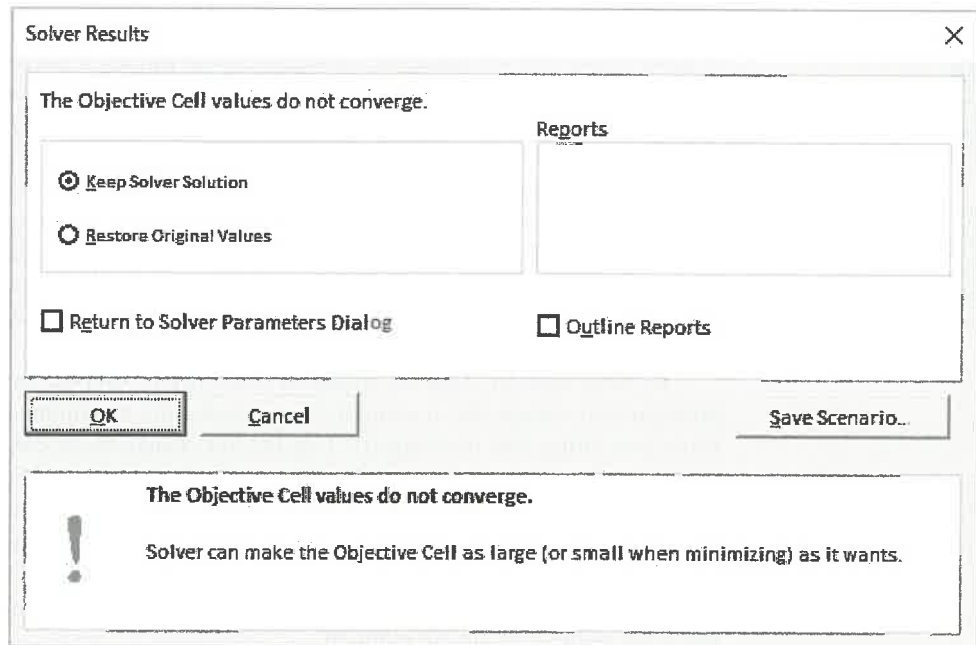


wrong input or forgotten some constraints. To see how this could occur in the product mix problem, suppose that you change *all* constraints to be $\leq$ instead of $\geq$. Now there is no upper bound on available labor hours or the number of PCs the company can sell. If you make these changes in the model and then use Solver, the message in Figure 13.19 appears, stating that the objective cell does not converge. In other words, the total net profit can grow without bound.

Figure 13.19
Unbounded
Solution Message



Except in very rare situations, if Solver informs you that your model is unbounded, you have made an error.

Infeasibility and unboundedness are quite different in a practical sense. It is quite possible for a reasonable model to have no feasible solutions. For example, the marketing department might impose several constraints, the production department might add some more, the engineering department might add even more, and so on. Together, they might constrain the problem so much that there are no feasible solutions left. The only way out is to change or eliminate some of the constraints. An unboundedness problem is quite different. There is no way a realistic model can have an unbounded solution. If you get the message shown in Figure 13.19, then you must have made a mistake: You entered an input incorrectly, you omitted one or more constraints, or there is a logical error in your model.

PROBLEMS

Note: Student solutions for problems whose numbers appear within a colored box are available for purchase at www.cengagebrain.com.

Level A

1. Other sensitivity analyses besides those discussed could be performed on the product mix model. Use SolverTable to perform each of the following. In each case keep track of the values in the decision variable cells and the objective cell, and discuss your findings.
 - a. Let the selling price for Basics vary from \$220 to \$350 in increments of \$10.
 - b. Let the labor cost per hour for assembling vary from \$5 to \$20 in increments of \$1.
 - c. Let the labor hours for testing a Basic vary from 0.5 to 3.0 in increments of 0.5.
 - d. Let the labor hours for assembling and testing an XP vary independently, the first from 4.5 to 8.0 and the second from 1.5 to 3.0, both in increments of 0.5.
2. In PC Tech's product mix problem, assume there is another PC model, the VXP, that the company can produce in addition to Basics and XPs. Each VXP requires eight hours for assembling, three hours for

testing, \$275 for component parts, and sells for \$560. At most 50 VXP's can be sold.

- a. Modify the spreadsheet model to include this new product, and use Solver to find the optimal product mix.
 - b. You should find that the optimal solution is *not* integer-valued. If you round the values in the decision variable cells to the nearest integers, is the resulting solution still feasible? If not, how might you obtain a feasible solution that is at least close to optimal?
3. Continuing the previous problem, perform a sensitivity analysis on the selling price of VXP's. Let this price vary from \$500 to \$650 in increments of \$10, and keep track of the values in the decision variable cells and the objective cell. Discuss your findings.
 4. Again continuing problem 2, suppose that you want to force the optimal solution to be integers. Do this in Solver by adding a new constraint. Select the decision variable cells for the left side of the constraint, and in the middle dropdown list, select the "int" option (for "integer"). How does the optimal integer solution compare to the optimal noninteger solution in problem 2? Are the decision variable cell values rounded versions of those in problem 2? Is the objective value more or less than in problem 2?
 5. If all of the inputs in PC Tech's product mix problem are nonnegative (as they should be for any realistic version of the problem), are there any input values such that the resulting model has no feasible solutions? (Refer to the graphical solution.)
 6. There are five corner points in the feasible region for the product mix problem. We identified the coordinates of one of them: (560, 1200). Identify the coordinates of the others.

- a. Only one of these other corner points has positive values for both decision variable cells. Discuss the changes in the selling prices of either or both models that would be necessary to make this corner point optimal.
- b. Two of the other corner points have one decision variable cell value positive and the other zero. Discuss the changes in the selling prices of either or both models that would be necessary to make either of these corner points optimal.

Level B

7. Using the graphical solution of the product mix model as a guide, suppose there are only 2800 testing hours available. How do the answers to the previous problem change? (Is the previous solution still optimal? Is it still feasible?)
8. Again continuing problem 2, perform a sensitivity analysis where the selling prices of Basics and XP's simultaneously change by the same percentage, but the selling price of VXP's remains at its original value. Let the percentage change vary from -25% to 50% in increments of 5%, and keep track of the values in the decision variable cells and the total profit. Discuss your findings.
9. Consider the graphical solution to the product mix problem. Now imagine that another constraint—*any* constraint—is added. Which of the following three things are possible: (1) the feasible region shrinks; (2) the feasible region stays the same; (3) the feasible region expands? Which of the following three things are possible: (1) the optimal value in the objective cell decreases; (2) the optimal value in the objective cell stays the same; (3) the optimal value in the objective cell increases? Explain your answers. Do they hold just for this particular model, or do they hold in general?

13-7 A LARGER PRODUCT MIX MODEL

The problem we examine in this section is a direct extension of the product mix model from the previous section. There are two modifications. First, the company makes eight computer models, not just two. Second, testing can be done on either of two lines, and the two lines have different characteristics.

EXAMPLE

13.2 PRODUCING COMPUTERS AT PC TECH

As in the previous example, PC Tech must decide how many of each of its computer models to assemble and test, but there are now eight available models, not just two. Each computer must be assembled and then tested, but there are now two lines for testing. The first line tends to test faster, but its labor costs are slightly higher, and each line has a certain number of hours available for testing. Any computer can be tested on either line. The inputs for the model are same as before: (1) the hourly labor costs for assembling

PROBLEMS

Level A

Note: All references to the product mix model in the following problems are to the larger product mix model in this section.

10. Modify PC Tech's product mix model so that there is no maximum sales constraint. (This is easy to do in the Solver dialog box. Just highlight the constraint and click on the Delete button.) Does this make the problem unbounded? Does it change the optimal solution at all? Explain its effect.
11. In the product mix model it makes sense to change the maximum sales constraint to a "minimum sales" constraint, simply by changing the direction of the inequality. Then the input values in row 23 can be considered customer demands that must be met. Make this change and rerun Solver. What do you find? What do you find if you run Solver again, this time making the values in row 23 one-quarter of their current values?
12. Use SolverTable to run a sensitivity analysis on the cost per assembling labor hour, letting it vary from \$5 to \$20 in increments of \$1. Keep track of the computers produced in row 21, the hours used in the range B26:B28, and the total profit. Discuss your findings. Are they intuitively what you expected?
13. Create a two-way SolverTable for the product mix model, where total profit is the only output and the two inputs are the testing line 1 hours and testing line 2 hours available. Let the former vary from 4000 to 6000 in increments of 500, and let the latter vary from 3000 to 5000 in increments of 500. Discuss the changes in profit you see as you look across the various rows of the table. Discuss the changes in profit you see as you look down the various columns of the table.
14. Model 8 has fairly high profit margins, but it isn't included at all in the optimal mix. Use SolverTable, along with some experimentation on the correct range,

to find the (approximate) selling price required for model 8 before it enters the optimal product mix.

Level B

15. Suppose that you want to increase *all three* of the resource availabilities in the product mix model simultaneously by the same percentage. You want this percentage to vary from -25% to 50% in increments of 5% . Modify the spreadsheet model slightly so that this sensitivity analysis can be performed with a *one-way* SolverTable, using the percentage change as the single input. Keep track of the computers produced in row 21, the hours used in the range B26:B28, and the total profit. Discuss the results.
16. Some analysts complain that spreadsheet models are difficult to resize. You can be the judge of this. Suppose the current product mix problem is changed so that there is an extra resource, packaging labor hours, and two additional PC models, 9 and 10. What additional input data are required? What modifications are necessary in the spreadsheet model (including range name changes)? Make up values for any extra required input data and incorporate these into a modified spreadsheet model. Then optimize with Solver. Do you conclude that it is easy to resize a spreadsheet model? (By the way, it turns out that algebraic models are typically *much* easier to resize.)
17. In Solver's sensitivity report for the product mix model, the allowable decrease for available assembling hours is 2375. This means that something happens when assembling hours fall to $20,000 - 2375 = 17,625$. See what this means by first running Solver with 17,626 available hours and then again with 17,624 available hours. Explain how the two solutions compare to the original solution and to each other.

13-8 A MULTIPERIOD PRODUCTION MODEL

The product mix examples illustrate a very important type of LP model. However, LP models come in many forms. For variety, we now present a quite different type of model that can also be solved with LP. (In the next chapter we provide other examples, linear and otherwise.) The distinguishing feature of the following model is that it relates decisions made during several time periods. This type of problem occurs when a company must make a decision now that will have ramifications in the future. The company does not want to focus completely on the short run and forget about the long run.

Modeling Issues

We assume that Pigskin uses a six-month planning horizon. Why six months? In multiple models such as this, the company has to make forecasts about the future, such as the level of customer demand. Therefore, the length of the planning horizon is usually the length of time which the company can make reasonably accurate forecasts. Here, Pigskin evidently believes that it can forecast up to six months from now, so it uses a six-month planning horizon. ■

PROBLEMS

Level A

18. Can you guess the results of a sensitivity analysis on the initial inventory in the Pigskin model? See if your guess is correct by using SolverTable and allowing the initial inventory to vary from 0 to 10,000 in increments of 1000. Keep track of the values in the decision variable cells and the objective cell.
19. Modify the Pigskin model so that there are eight months in the planning horizon. You can make up reasonable values for any extra required data. Don't forget to modify range names. Then modify the model again so that there are only four months in the planning horizon. Do either of these modifications change the optimal production quantity in month 1?
20. As indicated by the algebraic formulation of the Pigskin model, there is no real need to calculate inventory on hand after production and constrain it to be greater than or equal to demand. An alternative is to calculate ending inventory directly and constrain it to be nonnegative. Modify the current spreadsheet model to do this. (Delete rows 16 and 17, and calculate ending inventory appropriately. Then add an *explicit* nonnegativity constraint on ending inventory.)
21. In one modification of the Pigskin problem, the maximum storage constraint and the holding cost are based on the *average* inventory (not ending inventory) for a given month, where the average inventory is defined as the sum of beginning inventory and ending inventory, divided by 2, and beginning inventory is *before* production or demand. Modify the Pigskin model with this new

assumption, and use Solver to find the optimal solution. How does this change the optimal production schedule? How does it change the optimal total cost?

Level B

22. Modify the Pigskin spreadsheet model so that except for month 6, demand does not have to be met on time. The only requirement is that all demand must be met eventually by the end of month 6. How does this change the optimal production schedule? How does it change the optimal total cost?
23. Modify the Pigskin spreadsheet model so that demand in any of the first five months must be met no later than a month late, whereas demand in month 6 must be met on time. For example, the demand in month 3 can be met partly in month 3 and partly in month 4. How does this change the optimal production schedule? How does it change the optimal total cost?
24. Modify the Pigskin spreadsheet model in the following way. Assume that the timing of demand and production are such that only 70% of the production in a given month can be used to satisfy the demand in that month. The other 30% occurs too late in that month and must be carried as inventory to help satisfy demand in later months. How does this change the optimal production schedule? How does it change the optimal total cost? Then use SolverTable to see how the optimal production schedule and optimal cost vary as the percentage of production usable for this month demand (now 70%) is allowed to vary from 20% to 100% in increments of 10%.

13-9 A COMPARISON OF ALGEBRAIC AND SPREADSHEET MODELS

To this point you have seen algebraic optimization models and corresponding spreadsheet models. How do they differ? If you review the two product mix examples in this chapter, we believe you will agree that (1) the algebraic models are quite straightforward and (2) spreadsheet models are almost direct translations into Excel of the algebraic models. In particular, each algebraic model has a set of x 's that corresponds to the decision variable