

INDEFINITE INTEGRALS

12.42

a) $\int (24x^5 + 35x^4 - 64x^3) dx$

Solution

$$\int (24x^5 + 35x^4 - 64x^3) dx = \int 24x^5 + \int 35x^4 - \int 64x^3 dx$$

$$= 24 \int x^5 + 35 \int x^4 - 64 \int x^3$$

$$= 4x^6 + 7x^5 - 16x^4 + C$$

b) $\int (8x^{-5} - x^{-3} + 5x^{-2}) dx = \int 8x^{-5} - \int x^{-3} + \int 5x^{-2} dx$

$$= 8 \int x^{-5} - \int x^{-3} + 5 \int x^{-2} dx$$

$$= -\frac{2}{x^4} - \left(-\frac{1}{2x^2}\right) - \frac{5}{x}$$

$$= -\frac{2}{x^4} + \frac{1}{2x^2} - \frac{5}{x} + C$$

c) $\int 21x^{3/4} dx = 21 \int x^{3/4} dx$

$$= \underline{\underline{12x^{7/4} + C}}$$

d) $\int 8x^{-1/3} dx = 8 \int x^{-1/3} dx = \underline{\underline{12x^{2/3} + C}}$

e) $\int 6\sqrt{x} dx = 6 \int \sqrt{x} dx$

Applying rule: $\sqrt{a} = a^{1/2}$

$$= 6 \int x^{1/2} dx$$

$$= 4x^{3/2} + C$$

f) $\int 9\sqrt{x-13} dx = 9 \int \sqrt{x-13} dx$

substitute $x-13$ by u , i.e. $u = x-13$

$$= 9 \int u^{1/2} du$$

$$= 9 \frac{u^{1/2+1}}{1/2+1} = 9 \frac{(x-13)^{3/2}}{3/2} + C$$

$$= \underline{\underline{6(x-13)^{3/2} + C}}$$

g) $\int 28e^{-1.75t} dt = 28 \int e^{-1.75t} dt$
 Substitute $-1.75t$ by u

$$= 28 \int -\frac{1}{1.75} e^u du$$

$$= 28 \int -\frac{1}{1.75} e^u du$$

$$= 28 \left(-\frac{1}{1.75} \int e^u du \right)$$

$$= 28 (-0.57142 \int e^u du)$$

Now, $\int e^u du = e^u$

$$= 28 (-0.57142 e^u)$$

But $u = -1.75t$

$$= 28 (-0.57142 \times e^{-1.75t}) + C$$

$$= \underline{\underline{-15.99976 e^{-1.75t} + C}}$$

h) $\int \frac{5}{x-6} dx = 5 \int \frac{1}{x-6} dx$

Use u -substitution: $u = x-6$

$$= 5 \int \frac{1}{u} du$$

And $\int \frac{1}{u} du = \ln(|u|)$

$$= 5 \ln|u|$$

But $u = x-6$

$$\therefore = \underline{\underline{5 \ln|x-6| + C}}$$

12.43 g) $\int (16x^3 - 21x^{7/5}) dx$, given $F(0) = 13$

$$\int 16x^3 - 21x^{7/5} dx = \int 16x^3 dx - \int 21x^{7/5} dx$$

$$= 16 \int x^3 dx - 21 \int x^{7/5} dx$$

$$= 16 \left(\frac{x^4}{4} \right) - 21 \left(\frac{x^{7/5}}{7/5} \right) + C$$

$$= 4x^4 - 15x^{7/5} + C$$

Applying Initial conditions,

$$F(0) = 13.$$

$$13 = 4(0)^4 - 15(0)^{7/5} + C$$

$$13 = 0 - 0 + C$$

$$13 = 0 + 0 + C$$

$$C = \underline{13}$$

b) $\int (18x^5 + 4x^{-3}) dx$, given $F(1) = 8$

$$\begin{aligned} f(x) &= \int (18x^5 + 4x^{-3}) dx = 18 \int x^5 dx + 4 \int x^{-3} dx \\ &= 18 \left(\frac{x^6}{6} \right) + 4 \left(\frac{x^{-2}}{-2} \right) \\ &= 3x^6 - \frac{2}{x^2} + C \end{aligned}$$

Applying boundary/initial conditions:

$$F(1) = 8, \quad f(1) = 8$$

$$\therefore 8 = 3(1)^6 - \frac{2}{(1)^2} + C$$

$$8 = 3 - 2 + C$$

$$C = \underline{8}$$

c) $\int (3x^8 - 2x^4) dx$, given $F(1) = -4$

$$\begin{aligned} f(x) &= \int (3x^8 - 2x^4) dx = \int 3x^8 dx - \int 2x^4 dx \\ &= 3 \int x^8 dx - 2 \int x^4 dx \\ &= 3 \left(\frac{x^9}{9} \right) - 2 \left(\frac{x^5}{5} \right) + C \\ &= \frac{x^9}{3} - \frac{2x^5}{5} + C \end{aligned}$$

Applying initial conditions, $F(1) = -4$.

$$-4 = \frac{(1)^9}{3} - \frac{2(1)^5}{5} + C$$

$$-4 = \frac{1}{3} - \frac{2}{5} + C$$

$$\therefore C = \frac{1}{3} - \frac{2}{5} - 4 = \frac{5}{15} - \frac{12}{15} - \frac{24}{15} = \frac{-29}{15}$$

$$d) \int (28e^{4t} - 18e^{-6t}) dt, \quad F(0) = 2$$

$$f(t) = 28 \int e^{4t} dt - 18 \int e^{-6t} dt$$

$$28 \int e^{4t} dt = 28 \int e^u \cdot \frac{1}{4} du; \quad u = 4t$$

$$= 28 \times \frac{1}{4} \int e^u du$$

$$= 7e^u = 7e^{4t} + C$$

$$18 \int e^{-6t} dt = 18 \int -\frac{1}{6} e^u du; \quad e^u = \int e^u du$$

$$= 18 \left(-\frac{1}{6} e^{-6t} \right) + C$$

$$= -3e^{-6t} + C$$

$$\therefore f(t) = 7e^{4t} - 3e^{-6t} + C$$

Initial Conditions: $F(0) = 2$

$$2 = 7e^{4(0)} - 3e^{-6(0)} + C$$

$$2 = 7 \cdot 1 - 3 \cdot 1 + C$$

$$2 = 7 - 3 + C$$

$$C = -8$$

DEFINITE INTEGRALS

12.44

$$a) \int_1^4 9x^2 dx = 9 \int_1^4 x^2 dx = 9 \left[\frac{x^{2+1}}{2+1} \right]_1^4 = 9 \left[\frac{x^3}{3} \right]_1^4$$

$$= \frac{64}{3} - \frac{1}{3}$$

$$= \frac{63}{3} = 21$$

$$b) \int_5^{15} -60x^{-2} dx = -60 \int_5^{15} x^{-2} dx = -60 \left[-\frac{1}{x} \right]_5^{15}$$

$$= \left(-\frac{1}{15} - \left(-\frac{1}{5} \right) \right) \cdot 60$$

$$= \left(-\frac{1}{15} + \frac{1}{5} \right) \cdot 60$$

$$= \left(\frac{-1 + 3}{15} \right) \cdot 60$$

$$= \frac{2}{15} \cdot 60$$

$$= 8$$

$$c) \int_4^{64} 2.5 x^{-1/2} dx = 2.5 \int_4^{64} x^{-1/2} dx = 2.5 \left[2x^{1/2} \right]_4^{64} = 16 - 4 = 30$$

$$d) \int_1^7 8x^{-1} dx = 8 \int_1^7 x^{-1} dx = 8 [\ln x]_1^7 = 8 \left[\frac{1}{x} \right]$$

$$= 8 (\ln 7 - \ln 1);$$

$$\ln 1 = 0$$

$$\lim_{x \rightarrow 1} (\ln x) = \ln 1.$$

$$\text{Hence, } 8 (\ln 7 - 0) = 8 \ln 7.$$

$$\therefore f(x) = \underline{8 \ln 7}$$

$$e) \int_8^{27} \frac{dx}{\sqrt[3]{x}} = \int_8^{27} \frac{1}{x^{1/3}} dx = \int_8^{27} x^{-1/3} dx$$

$$= \left[\frac{3}{2} x^{2/3} \right]_8^{27}$$

$$= \frac{27}{2} - 6$$

$$= \underline{15/2}$$

$$f) \int_0^2 3e^{t/5} dt = 3 \int_0^2 e^{t/5} dt = 3 \times 5 \int_0^{2/5} e^u du \quad (\text{let } u = t/5)$$

$$= 3 \times 5 \times \int_0^{2/5} 1 du$$

$$= 15 \int_0^{2/5} e^u du$$

$$= 15 [e^u]_0^{2/5}$$

$$\text{Since } u = t/5$$

$$\text{Then, } f(x) = 15 (e^{2/5} - 1)$$

$$g) \int_{-0.5}^0 -42e^{-6t} dt = -42 \int_{-0.5}^0 e^{-6t} dt$$

$$\text{Use } u \text{ substitution, } u = -6t$$

$$= -42 \int_3^0 -1/6 e^u du$$

$$= -42 \left(- \int_0^3 \frac{1}{6} e^u du \right)$$

$$= -42 \left(- \left(-\frac{1}{6} [e^u]_0^3 \right) \right)$$

$$= -42 \times \frac{1}{6} [e^u]_0^3$$

$$= -7 [e^u]_0^3$$

$$\lim_{u \rightarrow 0^+} (e^u) = 1$$

$$\lim_{u \rightarrow 3^-} (e^u) = e^3$$

$$\text{Hence } f(x) = -7(e^3 - 1)$$

$$\text{11) } \int_{16}^{36} 15\sqrt{x} dx = 15 \int_{16}^{36} \sqrt{x} dx = 15 \int_{16}^{36} x^{1/2} dx$$

$$= 15 \left[\frac{2}{3} x^{3/2} \right]_{16}^{36}$$

$$= 15 \left(144 - 128 \frac{2}{3} \right)$$

$$= 15 \left(30 \frac{2}{3} \right)$$

$$= \underline{\underline{1520}}$$

$$\underline{\underline{12.45}} \quad 9) \int_2^5 (8x+7) dx = \int_2^5 8x dx + \int_2^5 7 dx$$

$$= 8 \int_2^5 x dx + 7 \int_2^5 dx$$

$$8 \int_2^5 x dx = 8 \left[\frac{x^2}{2} \right]_2^5 = 84$$

$$7 \int_2^5 dx = 7 [x]_2^5 = 21$$

$$\therefore f(x) = 84 + 21$$

$$= \underline{\underline{105}}$$

$$b) \int_2^4 (15x^2 - 6x) dx = 15 \int_2^4 x^2 dx - 6 \int_2^4 x dx$$

$$15 \int_2^4 (x^2) dx = 15 \left[\frac{x^3}{3} \right]_2^4 = 280$$

$$6 \int_2^4 x dx = 6 \left[\frac{x^2}{2} \right]_2^4 = 36$$

$$f(x) = 280 - 36 = 244$$

$$c) \int_8^{125} (4x^{-1/3} - x^{-2/3}) dx = 4 \int_8^{125} x^{-1/3} dx - \int_8^{125} x^{-2/3} dx$$

$$= 4 \left[\frac{3}{2} x^{2/3} \right]_8^{125} - \left[\frac{3}{2} x^{1/3} \right]_8^{125}$$

$$= 126 - 9 = 117$$

$$d) \int_8^{12} 20(x-7)^3 dx = 20 \int_8^{12} (x-7)^3 dx = 20 \int_1^5 u^3 du ; \text{ let } u = x-7$$

$$= 20 \left[\frac{u^4}{4} \right]_1^5 = 20 \times 156$$

$$= 3120$$

$$e) \int_{-2}^2 (6x^2 + 10x + 3) dx = 6 \int_{-2}^2 x^2 dx + 10 \int_{-2}^2 x dx + 3 \int_{-2}^2 dx$$

$$= 6 \left[\frac{x^3}{3} \right]_{-2}^2 + 10 \left[\frac{x^2}{2} \right]_{-2}^2 + [3x]_{-2}^2$$

$$= 32 + 0 + 12$$

$$= 44$$

$$f) \int_0^{0.25} (12e^{-3t} - 8e^{-2t}) dt = \int_0^{0.25} 12e^{-3t} dt - \int_0^{0.25} 8e^{-2t} dt$$

$$= 12 \int_0^{0.25} e^{-3t} dt - 8 \int_0^{0.25} e^{-2t} dt$$

~~let $u = -3t$~~

$$12 \int_0^{0.25} e^{-3t} dt = 12 \int_{0.75}^{0} -\frac{1}{3} e^u du \quad \text{let } u = -3t$$

$$= 12 \left(- \left(-\frac{1}{3} [e^u] \right)_{0.75}^0 \right)$$

$$= 4 [e^u]_{0.75}^0 = (1 - 0.47236) 4$$

$$= 4 \times 0.52764$$

$$= 2.11056$$

$$8 \int_0^{0.25} e^{-2t} dt = 8 \int_0^{-0.5} -\frac{1}{2} e^u du = 8 \left(- \int_{-0.5}^0 -\frac{1}{2} e^u du \right)$$

$$= 8 \left(- \left(-\frac{1}{2} [e^u]_{-0.5}^0 \right) \right)$$

$$= 4 [e^u]_{-0.5}^0$$

$$= (1 - 0.60653) 4$$

$$= 1.57388$$

$$f(x) = 2.11056 - 1.57388$$

$$= \underline{\underline{0.53668}}$$

AREA BETWEEN CURVES

12.46

1) $y_1 = 2x^2 - 8$ and $y_2 = -2x + 4$ from $x = -3$ to $x = 2$

$$f(x) = 2x^2 - 8$$

$$g(x) = -2x + 4$$

$$a = -3, b = 2$$

$$\begin{aligned}
 &= \int_{-3}^2 |2x^2 - 8 - (-2x + 4)| dx \\
 &= \int_{-3}^2 (-2x^2 - 2x + 12) dx \\
 &= -2 \int_{-3}^2 x^2 dx - 2 \int_{-3}^2 x dx + 12 \int_{-3}^2 1 dx \\
 &= -2 \left[\frac{x^3}{3} \right]_{-3}^2 - 2 \left[\frac{x^2}{2} \right]_{-3}^2 + 12 \left[x \right]_{-3}^2 \\
 &= -\frac{70}{3} - (-5) + 60 \\
 &= \frac{125}{3} \\
 &= \underline{\underline{41.667}}
 \end{aligned}$$

b) $y_1 = -4x^2 + 64$ and $y_2 = 4x + 16$ from $x = -4$ to $x = 3$

$$f(x) = -4x^2 + 64$$

$$g(x) = 4x + 16$$

$$a = -4, b = 3$$

$$A(x) = \int_{-4}^3 (-4x^2 + 64 - (4x + 16)) dx$$

$$= \int_{-4}^3 -4x^2 - 4x + 48 dx$$

$$= -4 \int_{-4}^3 x^2 dx - 4 \int_{-4}^3 x dx + 48 \int_{-4}^3 1 dx$$

$$= -4 \left[\frac{x^3}{3} \right]_{-4}^3 - 4 \left[\frac{x^2}{2} \right]_{-4}^3 + 48 \left[x \right]_{-4}^3$$

$$= \frac{364}{3} - (-14) + 336$$

$$= \frac{686}{3}$$

$$= \underline{\underline{228.667}}$$

c) $y_1 = 7x^2 - 14x + 13$ and $y_2 = -3x^2 + 6x - 2$ from $x=0$ to $x=2$.

$$f(x) = 7x^2 - 14x + 13$$

$$g(x) = -3x^2 + 6x - 2$$

$$a=0, b=2$$

$$= \int_0^2 (7x^2 - 14x + 13 - (-3x^2 + 6x - 2)) dx$$

$$= \int_0^2 10x^2 - 20x + 15 dx$$

$$= 10 \int_0^2 x^2 dx - 20 \int_0^2 x dx + 15 \int_0^2 dx$$

$$= 10 \left[\frac{x^3}{3} \right]_0^2 - 20 \left[\frac{x^2}{2} \right]_0^2 + 15 \left[x \right]_0^2$$

$$= \frac{80}{3} - 40 + 30$$

$$= \frac{50}{3}$$

$$= 16.6667$$

d) $y_1 = -3x^2 + 24x - 2$ and $y_2 = -17x + 68$ from $x=0$ to $x=4$

$$f(x) = -3x^2 + 24x - 2$$

$$g(x) = -17x + 68$$

$$\left. \begin{array}{l} f(x) = -3x^2 + 24x - 2 \\ g(x) = -17x + 68 \end{array} \right\} a=0, b=4$$

$$= \int_0^4 (-3x^2 + 24x - 2 - (-17x + 68)) dx$$

$$= \int_0^2 (-3x^2 - 4x + 70) dx + \int_2^4 (-3x^2 + 24x - 2 - (-17x + 68)) dx$$

$$= \int_0^2 (-3x^2 - 4x + 70) dx = 3 \int_0^2 x^2 dx - 4 \int_0^2 x dx + 70 \int_0^2 dx$$

$$= 3 \left[\frac{x^3}{3} \right]_0^2 - 4 \left[\frac{x^2}{2} \right]_0^2 + 70 \left[x \right]_0^2$$

$$= 8 - 82 + 140$$

$$= 66$$

$$= 66 + \int_2^4 (-3x^2 + 24x - 2 - (-17x + 68)) dx = 66 + \int_2^4 (-3x^2 + 41x - 70) dx$$

$$= 66 + -3 \int_2^4 x^2 dx + 41 \int_2^6 x^0 dx + 70 \int_2^4 dx$$

$$= 66 + -3 \left[\frac{x^3}{3} \right]_2^4 + 41 \left[\frac{x^1}{1} \right]_2^6 + 70 \left[x \right]_2^4$$

$$= 66 + (-56 + 246 - 140)$$

$$= 66 + 50$$

$$= \underline{\underline{116}}$$