

Calculus III - Project 1 (Chapters 12 & 14)

Parabolic antenna

Project description

Parabolic antennas are popularly used in industries such as radio/TV and radar. The function of an antenna is to transmit and receive radio waves. In transmitting a radio wave into air, a point source emits the wave from a point in front of the paraboloidal reflector named the focal point. After being reflected by the reflector, the wave becomes a plane wave moving in the same direction as the antenna's axis. Conversely, an incoming plane wave parallel to the antenna's axis will be focused to the focal point.

Assume that the parabolic antenna is an elliptical paraboloid given by the equation $z = a(x^2 + y^2)$, $a > 0$.

1. Prove that all waves coming into the antenna in the direction parallel to the antenna's axis will pass through a common point named the focal point. Find the position of the focal point. (Make use of the Snell's law of refraction)

2. Show that the time it takes for a wave to travel from the focal point to the reflector and then to a given plane perpendicular to the antenna's axis does not depend on the point at which the wave is reflected (which means that a spherical wavefront emitted at the focal point will be reflected into a plane wave traveling in the same direction as the antenna's axis).

Theoretical aspects needed:

- Quadric surfaces
- Equation of tangent lines/tangent plane
- Snell's law