

Project 4 (001): The Koch Snowflake

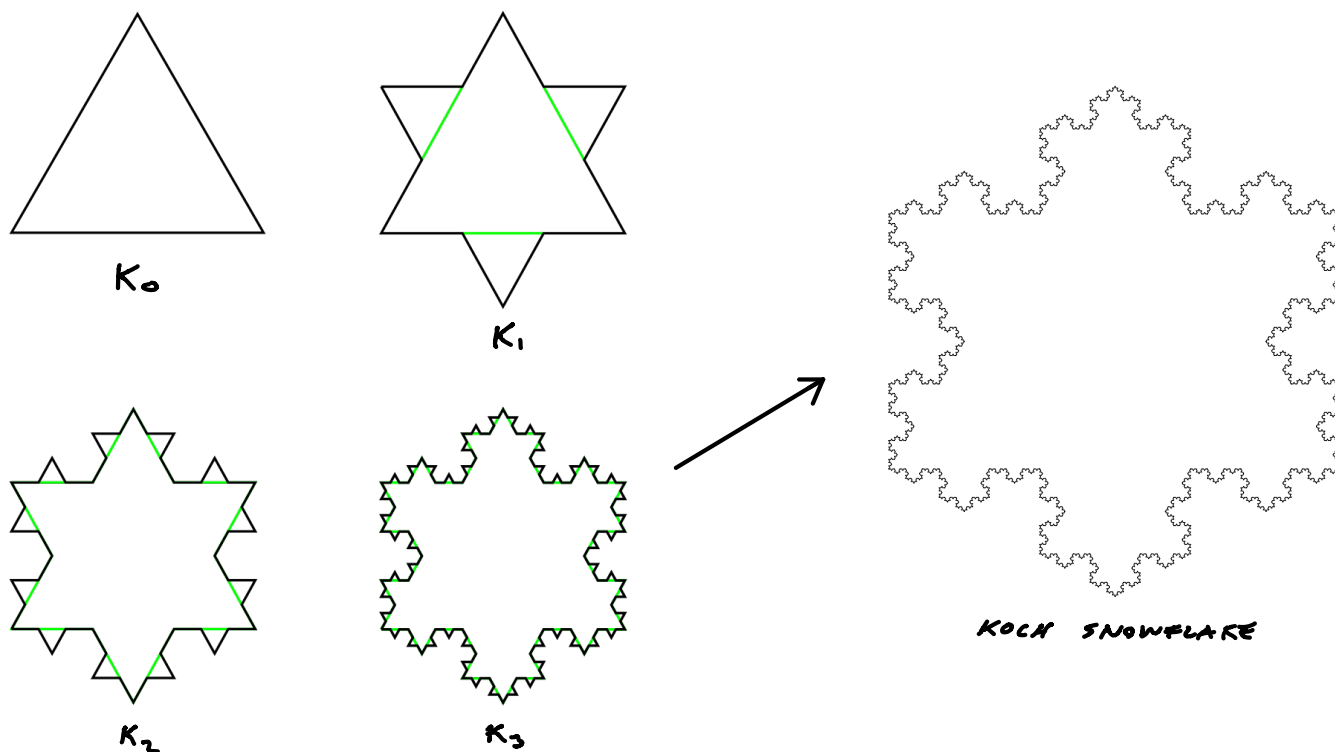
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In the late 19th and early 20th century, mathematicians began discovering geometric figures with a truly remarkable property: they exhibited the same elaborate structure at all scales. Today we call such shapes fractals. In 1904 Helge von Koch described one of the earliest known fractals, the so called Koch snowflake.

The Koch snowflake can be constructed by starting with an equilateral triangle with sides length one, then recursively altering each line segment as follows:

- Divide the line segment into three segments of equal length.
- Draw an equilateral triangle that has the middle segment from step 1 as its base and points outward.
- Remove the line segment that is the base of the triangle from step 2.



If we were to zoom in on the boundary we would see the same intricate structure repeating at every scale. That's what makes it an example of a fractal. Another remarkable property of the Koch snowflake is that it's boundary is continuous but nowhere differentiable (there are "corners" at every point). Very strange.

In this project we'll explore other strange properties of the Koch snowflake using our knowledge of sequences and geometric series.

1. Let N_n be the number of edges of K_n . Explain why

$$N_n = 3 \cdot 4^n.$$

Also explain why the length of any side of K_n is $\frac{1}{3^n}$

Solution:

2. For n any non-negative whole number, give a simple formula for the perimeter of K_n . Using this, prove the remarkable fact that the Koch snowflake has infinite perimeter.

Solution:

3. For n any non-negative whole number, give a simple formula for the area of K_n . Use this to find the area of the Koch snowflake.

Hint: Recall that the area of an equilateral triangle with side length s equals $\frac{\sqrt{3}}{4}s^2$. You should build up the area recursively from K_0 to K_1 to K_2

Solution: