Quiz 4A: Chapter 4

1. 1st Graph.

Roots: $x=-2$, $x=1$, $x=4$

Degree : 3

Standard form: $y=ax^3+bx^2+cx+d$

Points $(-2,0)$, $(1,0)$, $(4,0)$, $(3,-10)$

$0 = -8a + 4b - 2c + d$

$0 = a + b + c + d$

$0 = 64a + 16b + 4c + d$

$-10 = 27a + 9b + 3c + d$

$a = \frac{-4b - d + 2c}{8} \Rightarrow 0 = -\left(\frac{-4b - d + 2c}{8}\right) + b + c + d$

$0 = 64\left(\frac{4b + d - 2c}{8}\right) + 16b + 4c + d$

$-10 = 27\left(\frac{4b + d - 2c}{8}\right) + 9b + 3c + d$

Solving for a, b, c, d

$a = 1, b = -3, c = -6, d = 8$

$\therefore y = x^3 - 3x^2 - 6x + 8$

2nd Graph

Roots: $x = -5, \quad x = \frac{-3}{16}, \quad x = \frac{1}{8}, \quad x = \frac{5}{16}$

Degree: 4

Standard form: $y = ax^4 + bx^3 + cx^2 + dx + e$.

Points: $(-5, 0), \left(-\frac{3}{16}, 0\right), \left(\frac{1}{8}, 0\right), \left(\frac{5}{16}, 0\right), (0, 30)$

$0 = 625a - 125b + 25c - 5d + e$

$0 = \frac{81}{65536}a - \frac{27}{4096}b + \frac{9}{256}c - \frac{3}{16}d + e$

$0 = \frac{1}{4096}a + \frac{1}{512}b + \frac{1}{64}c + \frac{1}{8}d + e$

$0 = \frac{625}{65536}a + \frac{125}{4096}b + \frac{25}{256}c + \frac{5}{16}d + e$

$30 = e$

Solving for a, b and c;

$a = 4096/5 \approx 819$

$b = 19456/5 \approx 3891$

$c = -5296/5 \approx -1059$

$d = -170$

$e = 30$

$\therefore \quad y = \frac{1}{5}\left(4096x^4 + 19456x^3 - 5296x^2 - 34x + 6\right)$

2. height : (x) Now $= 3x$

width : $(x+1)$ Now $= (x+3)$

length : $(x+2)$ Now $= (x+4)$

Original volume:

$$x(x+1)(x+2) = x^3 + 3x^2 + 2x$$

New volume : $3x(x+3)(x+4) = 3x^3 + 21x^2 + 36x$

$$\Rightarrow 3x(x+3)(x+4) - x(x+1)(x+2) = 552$$

$$3x^3 + 21x^2 + 36x - (x^3 + 3x^2 + 2x) = 552$$

$$2x^3 + 18x^2 + 34x = 552$$

$$2x^3 + 18x^2 + 34x - 552 = 0$$

Factor $2x^3 + 18x^2 + 34x - 552$:

$$2(x^3 + 9x^2 + 17x - 276) = 0$$

$\Rightarrow x^3 + 9x^2 + 17x - 276$ is divisible by $x - 4$

Applying synthetic division: $\frac{x^3 + 9x^2 + 17x - 276}{x - 4} = x^2 + 13x + 69$

$$= 2(x-4)(x^2 + 13x + 69) = 0$$

$x = 4$ .. (i)

$x^2 + 13x + 69 = 0$ ... (ii) (gives imaginary roots)

using the 1st solution, $x = 4$

Height = 4 cm

Width , = 5cm

length , = 6cm

3. $-4x^4 + 5x^3 = 0$

$x^3(-4x + 5) = 0$

$x^3 = 0 \; ; \; x = 0$

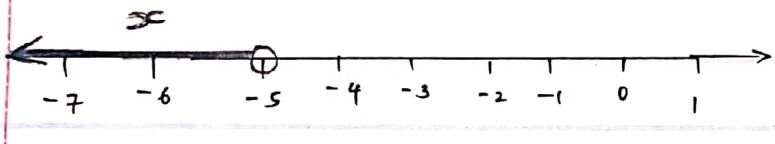
$-4x + 5 = 0$

$-4x = -5$

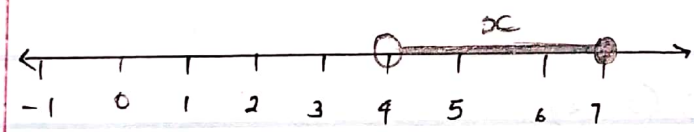
$x = 5/4$

$x = 0, \; x = 5/4$

4. $x < -5$



(b) $-4 < x \leq 7$



5. (a) $-3(x+4) < -21$

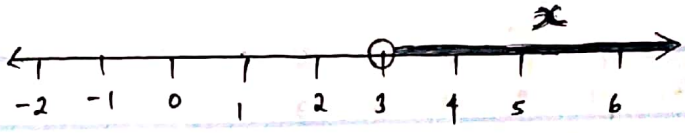
$-3x - 12 < -21$

$-3x < -21 + 12 \quad \Rightarrow -3x < -9$

$x > 3$

Interval notation: $(3, \infty)$

Number line:



(b) $4(x-6)+8 \geq 3-3(x-6)$

$4x-24+8 \geq 3-3x+18$

$4x-16 \geq -3x+21$

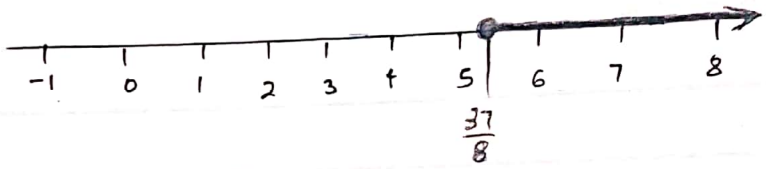
$7x \geq 37$

$x \geq \frac{37}{7}$

$\frac{37}{7} \approx 5.3$

Interval notation: $[\,37/8\,, \infty)$

Number line:



(6) $x^2 - x - 6 \leq 0$

Factor $x^2 - x - 6$

$x^2 + 2x - 3x - 6$

$x(x+2) - 3(x+2) = (x-3)(x+2)$

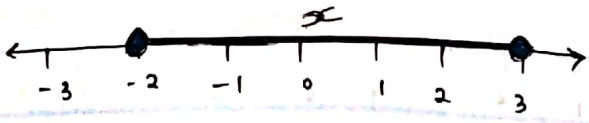
$x - 3 \leq 0$

$x \leq 3$

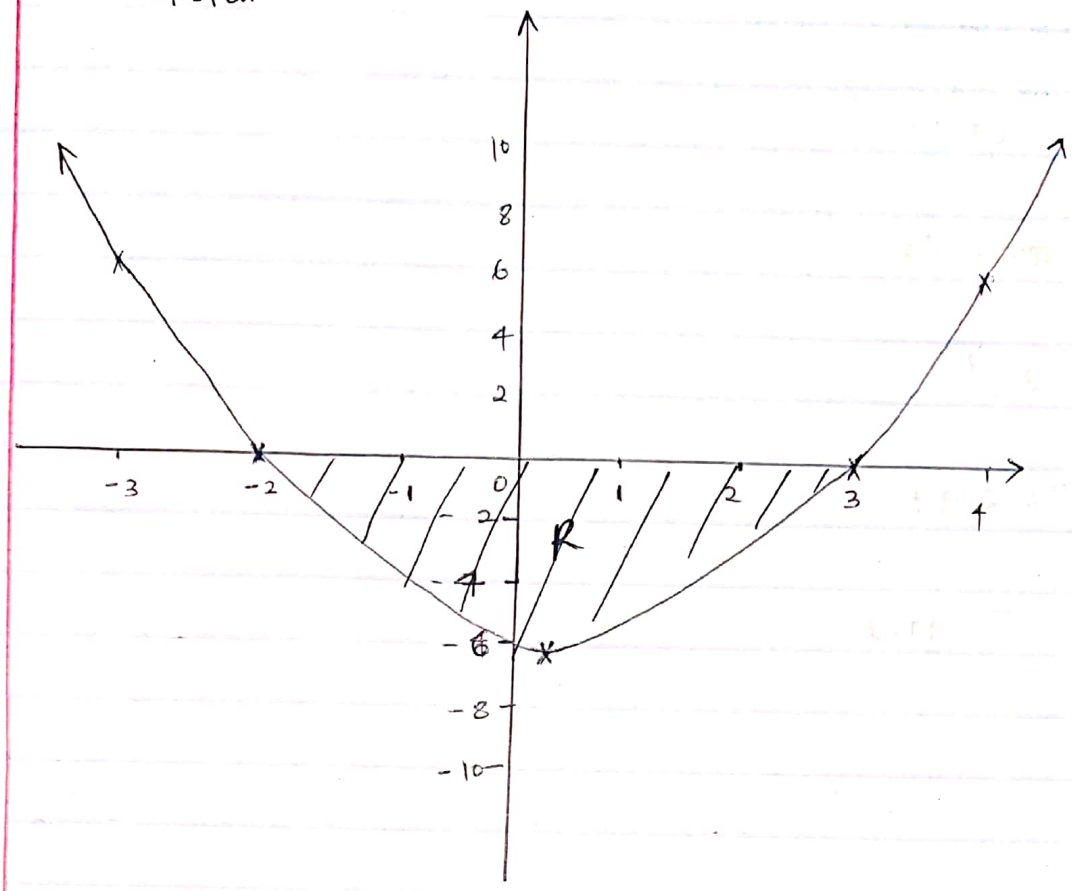
$x + 2 \leq 0$

$x \leq -2$

Solution $-2 \leq x \leq 3$



Sketch



7. (a) Positive: $(-3.75, -0.2) \cup (2.5, \infty)$
Negative: $(-\infty, -3.75) \cup (-0.2, 2.5)$

(b) Positive: $(-2.5, -20)$
Negative: $(1.25, 0)$
Zero: $(2.5, -30)$

8. $P(x) = 3(x-30)^3 + 30000, \quad 0 \leq x \leq 50$

(a) Rate between 1996 and 2012

$P(1996) = 3(26-30)^3 + 30,000$
$= 29808$

$P(2012) = 3(42-30)^3 + 30,000$
$= 35184$

$2012 - 1996 = 16$ Rate $= \frac{35184 - 29808}{16} = 336$

(b)

$$\frac{dp}{dx} = 9(x-30)^2$$

$2013 - 1910 = 43$

$= 9(43-30)^3$

$= 9 \times 2197$

$= 19773$