Quiz 5A: Chapter 5

1. $\frac{-2x}{x-7} \geq \frac{2}{x-3}$

$= -2x(x-3) \geq 2(x-7)$

$= -2x^2 + 4x + 14 \geq 0$

$\Rightarrow \frac{-2x^2 + 4x + 14}{(x-7)(x-3)} \geq 0$

Solving for $-2x^2 + 4x + 14 \geq 0 \Rightarrow -2x^2 + 4x + 14 = 0$

$x = \frac{-4 \pm 8\sqrt{2}}{-4}$; $x = 1 - 2\sqrt{2}$, $x = 1 + 2\sqrt{2}$

$x - 7 = 0$; $x = 7$

$x - 3 = 0$; $x = 3$

$\Rightarrow \frac{-2(x - 1 - 2\sqrt{2})(x - 1 + 2\sqrt{2}) - 1}{(x-7)(x-3)} \leq -1$

$\frac{2(x - 1 - 2\sqrt{2})(x - 1 + 2\sqrt{2})}{(x-7)(x-3)} \leq 0$

Identify intervals:

$1 - 2\sqrt{2} \leq x \leq 3$ and $1 + 2\sqrt{2} \leq x \leq 7$

2. $4x - 3 < \frac{2x + 5}{2x - 3}$

$(2x - 3)(4x - 3) < 2x + 5$

$8x^2 - 18x + 9 < 2x + 5$

$8x^2 - 20x + 4 < 0$

3. $D: x \in \mathbb{R} \mid x \neq 3/4$

x – intercept: $(1, 0)$

y – intercept $(0, 2/3)$

$$y = \frac{-1/2}{x - 3/4} + x$$

Possible equation: $y = \frac{-1/2}{x - 3/4} + x$

f. $f(x) = \dfrac{x+6}{x-5}$

$$f'(x) = \frac{\left(\frac{d}{dx}(x+6)\right)\cdot(x-5) - \left(\frac{d}{dx}(x-5)\right)\cdot(x+6)}{(x-5)^2}$$

$$= \frac{x-5-x-6}{(x-5)^2} = \frac{-11}{(x-5)^2}$$

$\dfrac{-11}{(x-5)^2} = 0$ No sln.

$x \neq 5$

	$-\infty, 5$	$(5, \infty)$
$\frac{d}{dx}$	-	-

Intervals of decrease: $(-\infty, 5) \cup (5, \infty)$
Intervals of Increase : None

5. Pipe A fills in 16 minutes longer
Pipe B in y minutes

In 1 minute pipe B fills $\frac{1}{y}$

Pipe A: $(y+16)$ minutes

In 1 minute pipe A fills $\frac{1}{(y+16)}$

$$\frac{75}{y+16} + \frac{75}{y} = 1$$

$$75y + 75(y+16) = y(y+16)$$
$$75y + 75y + 1200 = y^2 + 16y$$

$$y^2 + (-134y) - 1200 = 0$$

$$y = \frac{134 \pm \sqrt{134^2 + 4 \times 1 \times 1200}}{2}$$

$$y = 142.425 \approx 142.4 \text{ minutes}$$

Pipe A: 158.4 minutes
Pipe B: 142.4 minutes.

6. $f(x) = \frac{2x}{2x-3}$

$$f'(x) = 2\left(\frac{\frac{d}{dx}(2x-3) - \frac{d}{dx}(2x-3)\,x}{(2x-3)^2}\right)$$

$$= \frac{-6}{(2x-3)^2}$$

$$f'(0) = \frac{-6}{(-3)^2} = \frac{-6}{9} = \frac{-2}{3}$$

$$f(1) = \frac{2}{-1} = -2 \qquad f(2) = \frac{4}{1} = 4$$

$$\frac{4-(-2)}{1} = 6 \text{ (average rate of change)}$$

7 $f(x) = \dfrac{1}{3x-7}$

VA: $x = 7/3$

HA: $y = 0$

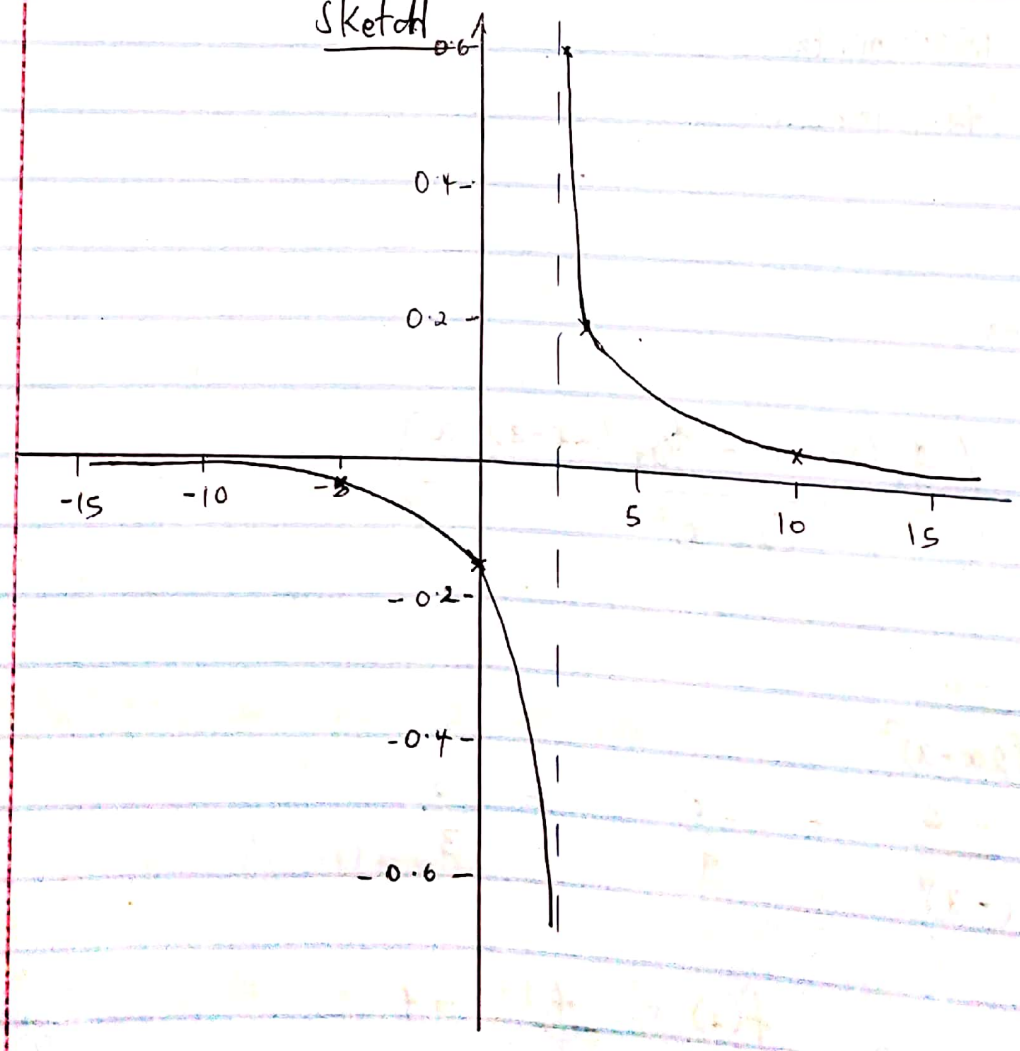
x - intercepts: None

y - intercept: $(0, -1/7)$

Intervals of increase: None

Intervals of decrease: $(-\infty, 7/3) \cup (7/3, \infty)$

Sketch



8 $f(x) = -(x+3)^2 + 2$ $\quad f^{-1}(x) = -3 - \sqrt{-x+2}$

Domain : $(-\infty, \infty)$

Range: $(-\infty, -2]$

Reciprocal function: Domain: $(-\infty, -2]$

Range: $(-\infty, \infty)$

x - intercept : None $(-3-\sqrt{2}, 0)$, $(-3+\sqrt{2}, 0)$

Y - intercept: $(0, -7)$

Reciprocal function: x - intercepts: None

y - intercepts: $(0, -3-\sqrt{2})$, $0(-3+\sqrt{2})$

Intervals of increase: $(-\infty, -3)$

Intervals of decrease: $(-3, \infty)$

9. $f(x) = \dfrac{(x-2)(x^2+7x+12)}{(x^2+4x-5)(x+4)}$

VA : $x = 1$, $x = -5$

HA : $y = 1$

x - intercepts : $(-3, 0)$ $(2, 0)$

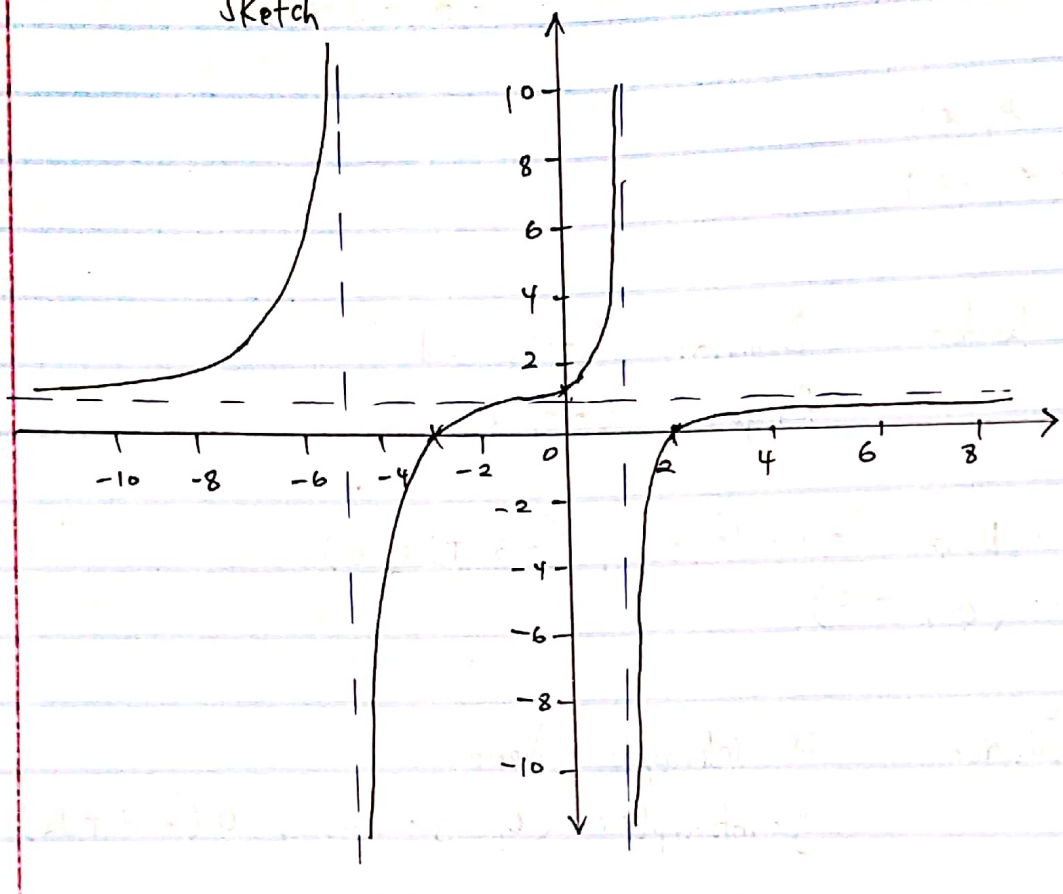
y - intercept : $(0, 6/5)$

End behaviour: : As $x \to \infty$, $y \to 1$ and as $x \to -\infty$, $y \to 1$

Intervals of Increase : $(-\infty, -5) \cup (-5, -4) \cup (-4, 1) \cup (1, \infty)$

Intervals of Decrease : None

Sketch



10. Sold at $ 2150

$2150 - 1000 = \$1150$

$\frac{1150}{10} = 115$

$= 115$ Pizzas

(b) Original prize

$\$1000$

(c) $\frac{2150}{215}$

$= \$10$

20 pizzas → feed players

x Pizzas → sold for $ 2150

$\frac{2150}{10x} = 1$

$x = 215$

$= 215 + 20$

$= 225$ Pizzas

(10) $f(x) = \dfrac{(x^2 - 2x - 3)}{x + 2}$

VA: $x = -2$

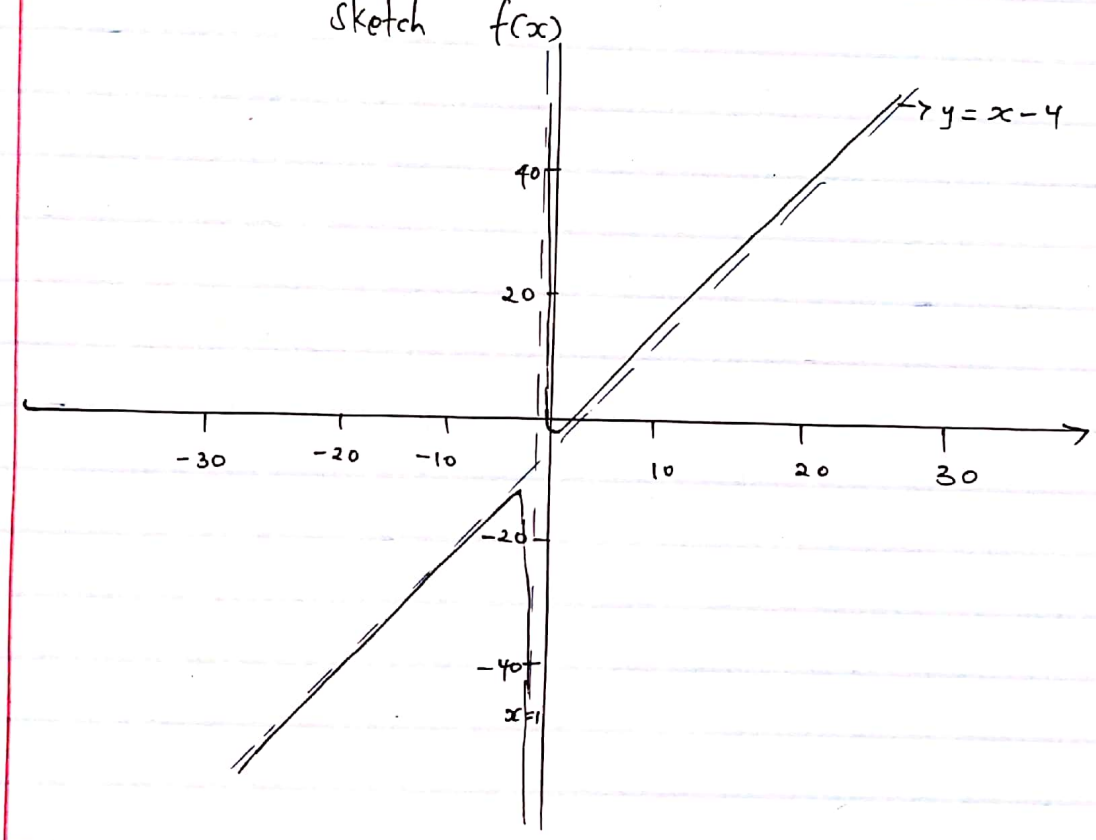
Behaviour: as $x \to -\infty$, $y \to -\infty$, as $x \to \infty$, $y \to \infty$

Asymptote: slant asymptote: $y = x - 4$

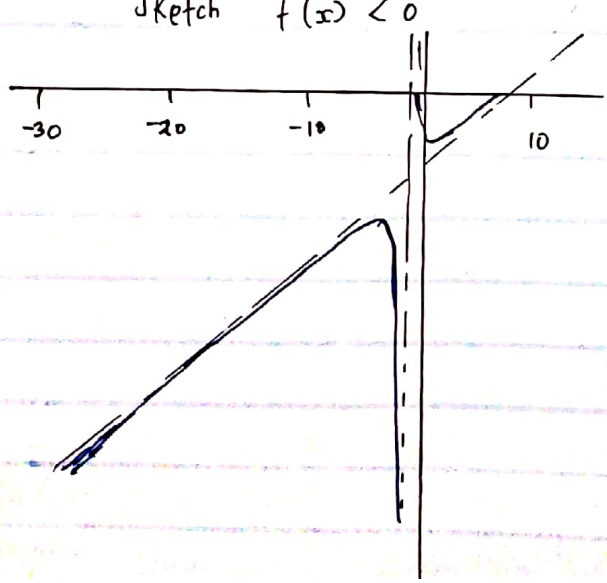
x-intercepts: $(-1, 0)$, $(3, 0)$

y intercept: $(0, -3/2)$

sketch $f(x)$



sketch $f(x) < 0$



12. Number: x

Reciprocal: $\frac{1}{x}$

$$\frac{1}{x} - \frac{1}{3x} \leq 3$$

$$\frac{1}{x} - \frac{1}{3x} - 3 < 0$$

$$\frac{2}{3x} - 3 < 0 \Rightarrow \frac{-9x+2}{3x} < 0$$

$$3x < 0; \quad x < 0$$

$$-9x + 2 < 0$$

$$-9x < -2$$

$$9x > 2$$

$$x > 2/9$$

Solution $x < 0$ or $x > 2/9$