1.

(a)

Since $\frac{8}{3}\pi > 2\pi$, the angle is greater than a full circle, so it is equivalent to $\frac{8}{3}\pi - 2\pi = \frac{2}{3}\pi$.

$\frac{1}{2}\pi < \frac{2}{3}\pi < \pi$, so the angle is **obtuse**.

(b)

$$\frac{8}{3}\pi \times \frac{180}{pi} = 480°$$

(c)

Considering $\alpha = \frac{2}{3}\pi$ (since an arc cannot span more than a circle):

$$s = r\alpha = 6 \cdot \frac{2}{3}\pi = 4\pi$$

$$A = \frac{1}{2}r^2\alpha = \frac{1}{2} \cdot 6^2 \cdot \frac{2}{3}\pi = 12\pi$$

(d)

$$\frac{8}{3}\pi - 2\pi = \frac{2}{3}\pi$$

$$\frac{2}{3}\pi - 2\pi = -\frac{1}{3}\pi$$

(e)

$$\left(2\cos\frac{2}{3}\pi, 2\sin\frac{2}{3}\pi\right) = \left(-1, \sqrt{3}\right)$$

(f)

$$\beta = 2\pi - \frac{8}{3}\pi = -\frac{2}{3}\pi$$

$$\left(2\cos\left(-\frac{2}{3}\pi\right), 2\sin\left(-\frac{2}{3}\pi\right)\right) = \left(-1, -\sqrt{3}\right)$$

2.

(a)

$H = 12$

$A = 3\sqrt{5}$

$O = \sqrt{12^2 - \left(3\sqrt{5}\right)^2} = 3\sqrt{11}$

$\sin\theta = \dfrac{3\sqrt{11}}{12} = \dfrac{\sqrt{11}}{4}$

$\cos\theta = \dfrac{3\sqrt{5}}{12} = \dfrac{\sqrt{5}}{4}$

$\tan\theta = \dfrac{3\sqrt{11}}{3\sqrt{5}} = \dfrac{\sqrt{55}}{5}$

$\csc\theta = \dfrac{4}{\sqrt{11}} = \dfrac{4\sqrt{11}}{11}$

$\sec\theta = \dfrac{4}{\sqrt{5}} = \dfrac{4\sqrt{5}}{5}$

$\cot\theta = \dfrac{5}{\sqrt{55}} = \dfrac{\sqrt{55}}{11}$

(b)

$\sin\gamma = \sin(\theta + \pi) = -\sin\theta = -\dfrac{\sqrt{11}}{4}$

$\cos\gamma = \cos(\theta + \pi) = -\cos\theta = -\dfrac{\sqrt{5}}{4}$

$\tan\gamma = \tan(\theta + \pi) = \tan\theta = \dfrac{\sqrt{55}}{5}$

$\csc\gamma = -\dfrac{4\sqrt{11}}{11}$

$\sec\gamma = -\dfrac{4\sqrt{5}}{5}$

$\cot\gamma = -\dfrac{\sqrt{55}}{11}$

3.

(a)

$O = 4\sqrt{5}$

$A = 2\sqrt{15}$

$$H = \sqrt{O^2 + A^2} = \sqrt{\left(4\sqrt{5}\right)^2 + \left(2\sqrt{15}\right)^2} = 2\sqrt{35}$$

$$\sin\theta = \frac{4\sqrt{5}}{2\sqrt{35}} = \frac{2\sqrt{7}}{7}$$

$$\cos\theta = \frac{2\sqrt{15}}{2\sqrt{35}} = \frac{\sqrt{21}}{7}$$

$$\tan\theta = \frac{4\sqrt{5}}{2\sqrt{15}} = \frac{2\sqrt{3}}{3}$$

$$\csc\theta = \frac{\sqrt{7}}{2}$$

$$\sec\theta = \frac{2\sqrt{15}}{2\sqrt{35}} = \frac{\sqrt{21}}{3}$$

$$\tan\theta = \frac{4\sqrt{5}}{2\sqrt{15}} = \frac{\sqrt{3}}{2}$$

(b)

(i)

$\alpha = 2\pi - \theta$

(ii)

$\frac{\pi}{2} < \theta < \pi$

Negating all sides of the inequality:

$-\pi < -\theta < -\frac{\pi}{2}$

Adding 2π to all sides of the inequality:

$2\pi - \pi < 2\pi - \theta < 2\pi - \frac{\pi}{2}$

$\pi < 2\pi - \theta < \frac{3\pi}{2}$

$\pi < \alpha < \frac{3\pi}{2}$

(iii)

$\cos \alpha = \cos (2\pi - \theta)$

Using the symmetry property $\cos(-\theta) = \cos \theta$:

$\cos \alpha = \cos (\theta - 2\pi)$

Using the periodic property $\cos (\theta \pm 2\pi) = \cos \theta$:

$\cos \alpha = \cos \theta$

4.

(a)

$$\cos\alpha = \frac{1}{\sec\alpha} = \frac{1}{-\frac{5}{\sqrt{5}}} = -\frac{1}{\sqrt{5}}$$

$$\tan^2\alpha = \sec^2\alpha - 1 = \left(-\frac{5}{\sqrt{5}}\right)^2 - 1 = 4$$

$$\tan\alpha = +\sqrt{4} = 2$$

(tan is positive in the 3rd quadrant)

$$\cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{2}$$

$$\sin\alpha = \cos\alpha\tan\alpha = -\frac{\sqrt{5}}{5}\cdot 2 = -\frac{2}{\sqrt{5}}$$

$$\csc\alpha = \frac{1}{\sin\alpha} = -\frac{\sqrt{5}}{2}$$

(b)

$$\sin\beta = -\sin\alpha = \frac{2}{\sqrt{5}}$$

$$\cos\beta = \cos\alpha = -\frac{1}{\sqrt{5}}$$

$$\tan\beta = -\tan\beta = -2$$

$$\csc\beta = -\csc\alpha = \frac{\sqrt{5}}{2}$$

$$\sec\beta = \sec\alpha = -\sqrt{5}$$

$$\cot\beta = -\cot\beta = -\frac{1}{2}$$

5.

(a)

Using $\cot\left(\frac{\pi}{2}-\theta\right) \equiv \tan\theta$:

$$\cot\left(\frac{\pi}{2}-\alpha\right) = \tan\alpha$$

Using $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$:

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\therefore \cot\left(\frac{\pi}{2}-\alpha\right) = \frac{\sin\alpha}{\cos\alpha}$$

(b)

Using $1+\cot^2\theta \equiv \csc^2\theta$:

$$\cot^2\theta = \csc^2\theta - 1$$

$$\cot\theta = -\sqrt{8^2-1} = -\sqrt{63} = -3\sqrt{7}$$

(cot is negative in the 2nd quadrant)

Using $\tan\theta \equiv \frac{1}{\cot\theta}$

$$\tan\theta = -\frac{1}{3\sqrt{7}} = -\frac{\sqrt{7}}{21}$$

(c)

$$\cos\alpha = \frac{\sqrt{3}}{3}$$

Using $\cos(\pi-\theta) \equiv \cos\theta$

$$\cos(\pi-\alpha) = \cos\alpha = \frac{\sqrt{3}}{3}$$

(d)

Using $\tan(\theta + 2\pi) \equiv \tan\theta$:

$$\tan(\theta + 2\pi) = \frac{17}{3}$$

Using $\tan\theta \equiv \frac{1}{\cot\theta}$

$$\cot(\theta + 2\pi) = \frac{3}{17}$$