

# Math 1F03 Assignment

Due on December 5

**Question 1:** Recall that a function  $f$  is even if  $f(-x) = f(x)$ , and odd if  $f(-x) = -f(x)$ .

- (a) Using the chain rule, show that the derivative of an odd function is an even function.
- (b) Using first principles (and a change of variables), show that the derivative of an even function is an odd function.

**Question 2:** A spotlight on the ground shines on the wall 12m away. If a man 2m tall walks from the spotlight toward the building at a speed of 1.6m/s, how fast is the length of his shadow on the building decreasing when he is 4m from the building?

**Question 3:** There are exactly two points on the curve  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 9$  where the tangent line has slope  $\frac{\sqrt{5}}{2}$ . Find the slope of the line joining those two points.

**Question 4:** Show that the following limit exists, and evaluate it:

$$\lim_{x \rightarrow e^+} (\ln x)^{\frac{1}{x-e}}$$

**Question 5:** Let  $a, b \in \mathbb{R}$  be constants. Given that

$$\lim_{x \rightarrow 1} \frac{ax^4 + bx^3 + 1}{(x-1)\sin(\pi x)}$$

exists and is a finite number, find its value.

**Question 6:** By applying the Mean Value Theorem to  $f(x) = \sqrt{x}$  on the interval  $[100, 102]$ , prove the following inequality:

$$\frac{1}{11} < \sqrt{102} - 10 < \frac{1}{10}.$$

**Question 7:** There is one straight line which is tangent to the curve  $4y = x^2$  at a point  $A$ , and at the same time tangent to the curve  $x = 2y^2$  at the point  $B$ . Find the distance between the points  $A$  and  $B$  (the distance between two points is discussed in Chapter 12).

**Question 8:** A cylindrical can is to be constructed to hold a fixed amount of liquid. The cost of the material used for the top and bottom of the can is 3 cents per square centimeter, and the cost of the material used for the curved side is 2 cents per square centimeter. Find the relationship between the radius and the height of the can that minimizes the cost of the can; show your answer actually corresponds to the absolute minimum.

**Question 9:** Consider the function  $f(x) = xe^{\frac{-x^2}{2}}$ .

- (a) Is it an even function, an odd function, or neither even nor odd?
- (b) Find the horizontal and vertical asymptotes of the function  $f$ .
- (c) Find the critical points and the intervals of increase and decrease of the function  $f$ .
- (d) Find the points of inflection and the intervals of concavity of the function  $f$ .
- (e) From steps (a),(b),(c) and (d), draw the graph of the function  $f$ . Label all important points.