## Section 3.5.

7 $P(x) = x^4 + 4x^2$

a) all zeros.

$$x^4 + 4x^2 = 0$$
$$x^2(x^2 + 4) = 0$$
$$x = 0, \; x^2 = -4$$
$$x = 2i, \; -2i$$

$$x = 0, \; 2i, \; -2i$$

b) factors:

$$x^2(x - 2i)(x + 2i)$$

11. $x^4 + 2x^2 + 1$

(a) All zeros.

$$x^4 + 2x^2 + 1 = 0$$

let $u = x^2$

$$\Rightarrow u^2 + 2u + 1 = 0$$

$$u = \frac{-2 \pm \sqrt{4 - 4}}{2} = -1$$

$$\therefore x^2 = -1$$
$$x = i, \; -i$$

(b) factors:

$$(x - i)^2 (x + i)^2$$

15. $x^3+8$

(a) all zeros:

$x^3+8=0$

$\Rightarrow x^3=-8$

$x=\sqrt[3]{-8}$

$\therefore x=-2,\ 1-\sqrt{3}i,\ 1+\sqrt{3}i$

(b) factor:

$(x+2)(x-1+\sqrt{3}i)(x-1-\sqrt{3}i)$

Complete factorization.

19. $P(x)=x^2+25$.

zeros: $x^2+25=0$

$\Rightarrow x^2=-25$.

$x=5i,\ -5i$

$x=5i$ of multiplicity 1

$x=-5i$ of multiplicity 1

23. $P(x)=x^3+4x$

zeros: $x^3+4x=0$

$x^3(x^2+4)=0$

$x=0$

$x^2+4=0$

$x^2=-4$

$x=2i,\ -2i$

$x=0$ of multiplicity 1

$x=2i$ of multiplicity 1

$x=-2i$ of multiplicity 1

27. $16x^4 - 81$.

$16x^4 - 81 = 0$.

$(4x^2+81)(4x^2-81) = 0$.

$4x^2 + 81 = 0$.

$x^2 = \frac{-81}{4}$ : $x = \frac{9}{2}i, \ -\frac{9}{2}i$

$4x^2 - 81 = 0$. :

$x^2 = \frac{81}{4}$ $x = \frac{9}{2}, -\frac{9}{2}$.

$x = \frac{9}{2}i$ of multiplicity 1

$-\frac{9}{2}i$ of multiplicity 1

$\frac{9}{2}$ of multiplicity 1

$-\frac{9}{2}$ of multiplicity 1

31. $Q(x) = x^4 + 2x^2 + 1$

let $u = x^2$

$u^2 + 2u + 1 = 0$.

$\frac{-2 \pm \sqrt{4-4}}{2} = -1$

$x^2 = -1 \Rightarrow x = \sqrt{-1}$

$x = i$ of multiplicity 2

$x = -i$ of multiplicity 2.

35 $x^5 + 6x^3 + 9x$

$x(x^4 + 6x^2 + 9) = 0$

$x = 0$

let $u = x^2$

$u^2 + 6u + 9 = 0$

$\frac{-6 \pm \sqrt{36 - 36}}{2} = -3$

$x^2 = -3$

$x = 0$ of multiplicity 1

$x = \sqrt{3}i$ of multiplicity 2.

$x = -\sqrt{3}i$ of multiplicity 2.

Finding polynomial with specified zeros.

39. P has degree 3 and zeros 3, 2i, -2i

$(x-3)(x-2i)(x+2i)$

$= (x+3)(x^2+4)$

$= x(x^2+4) + 3(x^2+4)$

$= x^3 + 4x + 3x^2 + 12$

$= x^3 + 3x^2 + 4x + 12$

43 R has degree 4 and zeros $1-2i$ and 1 with a zero of multiplicity 2.

$x^2(x-1+2i)(x-1)$

$(x^3 - x^2)(x - 1 + 2i)$

$= (x^4 - 2x^3 + x^2) + (2x^3 - 2x^2)i$

Finding complex zeros.

47 $P(x) = x^3 + 2x^2 + 4x + 8$

~~fact~~ $x^3 + 2x^2 + 4x + 8 = 0$

factor: $(x+2)(x^2+4) = 0$

$x = -2$.

$x^2 + 4 = 0 \Rightarrow x^2 = -4$

$x = 2i, -2i$

$x = -2, 2i, -2i$

53 $2x^3 + 7x^2 + 12x + 9 = 0$.

factor: $(2x+3)(x^2+2x+3) = 0$.

$\Rightarrow 2x + 3 = 0$

$x = -3/2$.

$x^2 + 2x + 3 = 0$

$$\frac{-2 \pm \sqrt{4-12}}{2} = \frac{-2 \pm \sqrt{-8}}{2} = \frac{-2 \pm 2\sqrt{2}i}{2}$$

$$= -1 \pm \sqrt{2}i$$

$x = -3/2, -1 + \sqrt{2}i, -1 - \sqrt{2}i$

57 $P(x) = x^5 - x^4 + 7x^3 - 7x^2 + 12x - 12$

$x^5 - x^4 + 7x^3 - 7x^2 + 12x - 12 = 0$

factor by grouping:

$(x-1)(x^2+3)(x^2+4) = 0$

$x = 1$

$x^2 + 3 = 0 \Rightarrow x = +\sqrt{3}i, -\sqrt{3}i$

$x^2 + 4 = 0 \Rightarrow x^2 = -4 \Rightarrow x = 2i, -2i$

$x = 1, \sqrt{3}i, -\sqrt{3}i, 2i, -2i$

63 $x^5 - 3x^4 + 12x^3 - 28x^2 + 27x - 9$

$\Rightarrow x^5 - 3x^4 + 12x^3 - 28x^2 + 27x - 9 = 0$

factor by grouping:

$\Rightarrow (x-1)^3(x^2+9) = 0$

$x^3 = 1 \Rightarrow (x-1)^3 = 0$

$x - 1 = 0$

$x = 1$ with multiplicity of 3.

$x^2 + 9 = 0 \Rightarrow x^2 = -9$

$x = \sqrt{3}i, -3i$

$x = 1, 3i, -3i$