

1. Single-Server Model

M/M/1 (exponential service times) with;

- $\lambda = 9$ (trucks per hour)
- $\mu = 12$ (trucks per hour)
- $s = 1$ server

1. a. Average service rate per hour of the weigh station.

5 mins = 1 truck
60 mins X

$$\frac{60 \times 1}{5} = \underline{\underline{12 \text{ trucks/hour}}}$$

b. Average number of trucks in the queue at the weigh station.

$$L_q = \frac{\rho^2}{1-\rho}$$

$$\rho = \frac{\lambda}{\mu} = \frac{9}{12} = 0.75$$

$$L_q = \frac{0.75^2}{1-0.75}$$

$$= \underline{\underline{2.25 \text{ trucks.}}}$$

c. Average number of trucks in the entire queuing system.

$$L = \frac{\rho}{1-\rho} = \frac{0.75}{1-0.75} = \frac{0.75}{0.25} = \underline{\underline{3 \text{ trucks.}}}$$

d. Average number of minutes a truck waits to be weighed.

$$W_q = \frac{\rho}{\mu - \lambda} = \frac{0.75}{12 - 9} = \frac{0.75}{3} = 0.25 \text{ hrs}$$

$$\underline{\underline{0.25 \times 60 = 15 \text{ minutes.}}}$$

e. Average number of minutes a truck spends in the entire queuing system.

$$W = \frac{1}{\mu - \lambda} = \frac{1}{12 - 9} = \frac{1}{3} \text{ hrs.}$$
$$\frac{1 \times 60}{3} = \underline{20 \text{ minutes.}}$$

f. Probability that a truck will arrive at the weigh station and not wait.

$$1 - \rho$$
$$1 - 0.75 = \underline{0.25}$$

g. Percent of arriving trucks expected to wait.

$$1 - 0.25 = 0.75$$
$$0.75 \times 100 = \underline{75\%}$$

h. Goal that a truck should spend on average 10 mins or less in the queuing system.

No, the goal is not met.

A truck spends an average of 20 minutes in the queuing system, which is more than the goal.

QM for windows results table for M/M/1 (exponential service times)

- $\lambda = 9$ (trucks per hour)
- $\mu = 12$ (trucks per hour)
- $s = 1$ server

QM for Windows - [Data] Results						
(untitled) Solution						
Parameter	Value		Parameter	Value	Minutes	Seconds
M/M/1 (exponential service times)			Average server utilization	.75		
Arrival rate(lambda)	9		Average number in the queue(Lq)	2.25		
Service rate(mu)	12		Average number in the system(L)	3		
Number of servers	1		Average time in the queue(Wq)	.25	15	900
			Average time in the system(W)	.33	20	1200

2. Multiple-Server Model

M/M/s with;

- $\lambda = 9$ (trucks per hour)
- $\mu = 12$ (trucks per hour)
- $s = 2$ servers

2. a. Goal that a truck should spend an average 10 mins or less in the queuing system.

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \left(\frac{s\mu}{s\mu - \lambda}\right) \right]^{-1}$$

$$\left[\sum_{n=0}^1 \frac{0.75^n}{n!} + \frac{1}{2!} (0.75)^2 \left(\frac{24}{24-9}\right) \right]^{-1}$$

$$\left[\frac{1 + 0.75 + \frac{1}{2} (0.75)^2 \cdot \left(\frac{8}{5}\right)}{1} \right]^{-1}$$

$$\left[1.75 + 0.28125 \left(\frac{8}{5}\right) \right]^{-1} = 2.2^{-1}$$

$$P_0 = 0.4545$$

$$W = \left[\frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(\lambda\mu - \lambda)^2} \right] P_0 + \frac{1}{\mu}$$

$$= \left(\frac{1}{1} \cdot \frac{(0.75)^2 \cdot 108}{(24-9)^2} \right) 0.4545 + \frac{1}{12}$$

$$= \left(\frac{0.27}{9} \right) 0.4545 + \frac{1}{12} = 0.097 \text{ hrs.}$$

$$0.097 \times 60 = \underline{\underline{5.82 \text{ minutes}}}$$

Yes, the goal is met.

A truck spends an average of 5.82 minutes in the queuing system; This meets the goal of 10 mins or less.

b. Average minutes per truck reduced by using two weigh stations instead of one.

1 weigh station = 20 minutes

2 weigh stations = 5.82 minutes

$$20 - 5.82 = \underline{\underline{14.18 \text{ minutes}}}$$

c. Operation runs 8 hours per day for 5 days per week. Average trucks ~~per week~~ expected to arrive per week.

$\lambda = 9 \text{ trucks/hour}$

9 trucks = 1 hr

$\times 40 \text{ hrs}$

$$\frac{40 \times 9}{1}$$

$$= 360 \text{ trucks}$$

$8 \times 5 = 40 \text{ hrs per week}$

$$360 \times 2 = 720$$

$$= \underline{\underline{720 \text{ trucks}}}$$

d. Average time spent by all trucks at the weigh stations would be reduced by a total of how many minutes per week?

720 trucks.

$$20 \text{ minutes} \times 720 = 14400 \text{ minutes.}$$

$$5.82 \text{ minutes} \times 720 = 4190.4 \text{ minutes.}$$

$$14400 - 4190.4 = 10209.6$$

$$= \underline{\underline{10209.6 \text{ minutes per week}}}$$

QM for Windows M/M/s

- $\lambda = 9$ (trucks per hour)
- $\mu = 12$ (trucks per hour)
- $s = 2$ servers

QM for Windows - [Data] Results						
(untitled) Solution						
Parameter	Value		Parameter	Value	Minutes	Seconds
M/M/s			Average server utilization	.38		
Arrival rate(lambda)	9		Average number in the queue(Lq)	.12		
Service rate(mu)	12		Average number in the system(L)	.87		
Number of servers	2		Average time in the queue(Wq)	.01	.82	49.09
			Average time in the system(W)	.1	5.82	349.09