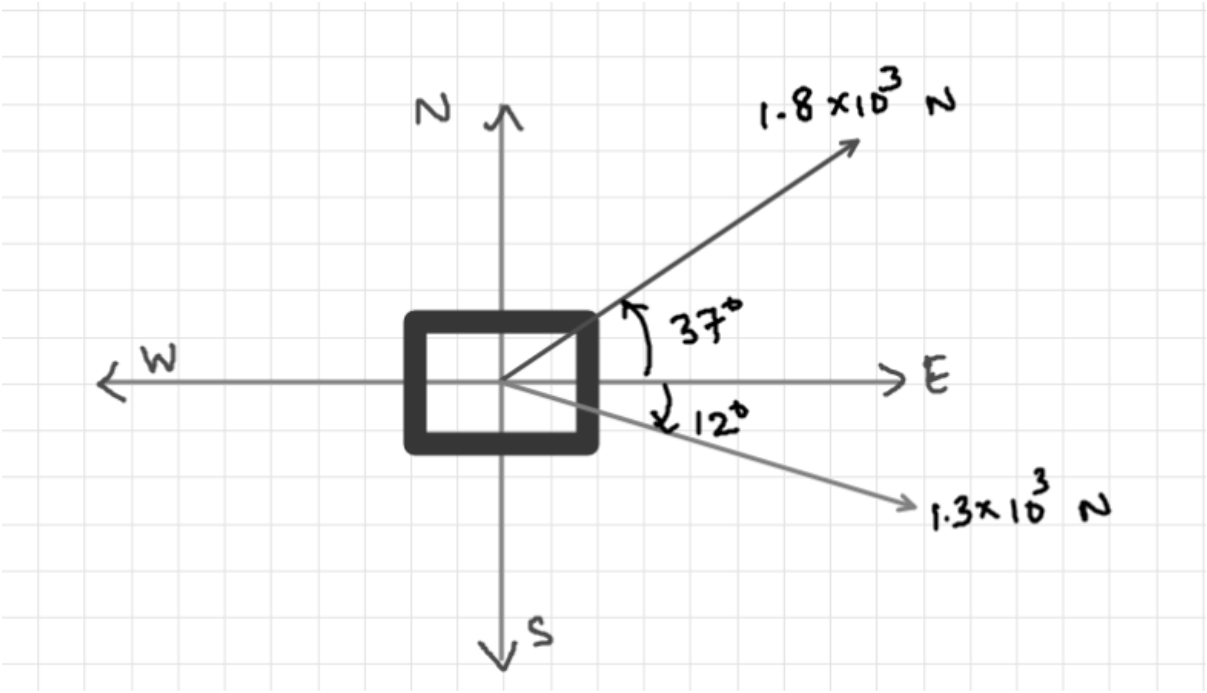
A pair of *Malibu* towboats are pulling a $4.23\ x\ 10^3$ kilogram canal boat into a harbour. The first towboat exerts a constant force of $1.8\ x\ 10^3\ N$ [E 37° N]. The second towboat exerts a constant force of $1.3\ x\ 10^3\ N$ [E 12° S]. What is the acceleration (magnitude & direction) of the canal boat? Assume that there is no friction acting on the canal boat.

Consider the following FBD of the boat –




Suppose east axis is positive x axis and North axis is positive y axis.

Thus, we can find x and y component of the forces.

X component of $1.8 \times 10^3$N force

$$\mathrm{F}\cos\theta = 1.8 \times 10^3 \cos 37^0 = 1437.5\mathrm{N}$$

y component of $1.8 \times 10^3$N force

$$\mathrm{F}\sin\theta = 1.8 \times 10^3 \sin 37^0 = 1083.3\mathrm{N}$$

X component of $1.3 \times 10^3$N force

$$\mathrm{F}\cos\theta = 1.3 \times 10^3 \cos 12^0 = 1271.6\mathrm{N}$$

y component of $1.3 \times 10^3$N force

$$-\mathrm{F}\sin\theta = -1.8 \times 10^3 \sin 12^0 = -270.3\mathrm{N}$$

{Note this component is negative because it is along negative y axis}

Thus, net force acting on the boat =

Along x axis : $1437.5 + 1271.6\ \text{N} = 2709.1\ \text{N}$

Along y axis = $1083.3 - 270.3\ \text{N} = 813\ \text{N}$

Now, to find acceleration along each axis, we will be using following formula –

$$a = \frac{F}{m}$$

In x direction

$$a_x = \frac{2709.1}{4.23 \times 10^3} = 0.64 \text{m/s}^2$$

In y direction

$$a_y = \frac{813}{4.23 \times 10^3} = 0.19\ \text{m/s}^2$$

Now, in order to find magnitude, we need to find the resultant of the two perpendicular vectors.

Magnitude

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{0.64^2 + 0.19^2} = 0.67\ \text{m/s}^2$$

Direction (i.e. angle above x axis in counter-clockwise direction) =

$$\rightarrow \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{0.19}{0.64}\right) = 16.5^0$$