Given that $\tan(\theta) = 3/4$, determine the following:

$\sin(\theta)$ $\csc(\theta)$ $\cot(\theta)$

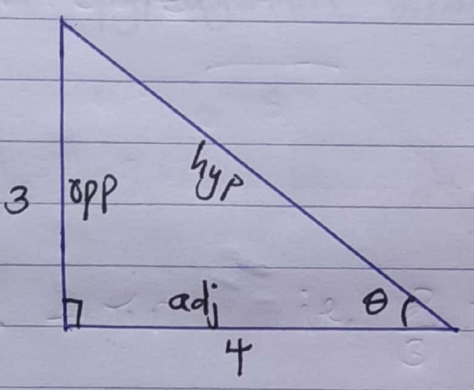
$\cos(\theta)$ $\sec(\theta)$

Solution:

The value of $\tan(\theta) = 3/4$. By defination of $\tan(\theta)$, it is given by the opposite divided by the adjacent

ie, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$.

This can be illustrated by a help of a right angled triangle as shown below.



$\tan\theta = \frac{3}{4} = \frac{\text{opp}}{\cancel{\text{hyp}}\,\text{adj}} \equiv$ opp = 3, adj = 4

Since it is a right angled triangle, the pythagorean theorem can be used to obtain the value of the hypotenuse, ~~&~~ The other two values are known, implying that the pythagorean theorem is applicable in this case.

From pythagoras' theorem,

$a^2 + b^2 = c^2$, where a and b are the two short sides of the triangle, and c is the longest side, ie hypotenuse.

$a = \text{opp}$, $b = \text{adj}$ or vice versa.

$\therefore (\text{opp})^2 + (\text{adj})^2 = (\text{hyp})^2$. Replacing the known values,

$$3^2 + 4^2 = (hyp)^2$$

$$9 + 16 = (hyp)^2$$

$\therefore (hyp)^2 = 25$. Square root both sides to obtain the value of the hypotenuse.

$hyp = \sqrt{25}$

$= \pm 5$ since length is a positive vector, only the positive value is used.

$= 5$

From the pythagorean theorem, the value of the hypotenuse is $\underline{5}$

Since the values of the three sides are now known, the values of ~~#~~ all other trigonometric functions can be obtained from them.

(i) $\sin(\theta)$

The value of $\sin(\theta)$ is given by:

$\sin\theta = \dfrac{\text{Opposite}}{\text{hypotenuse}}$. where Opposite $= 3$, hypotenuse $= 5$.

Replacing with the known values, we obtain

$\sin\theta = \dfrac{3}{5}$.

(ii) $\cos(\theta)$

The value of $\cos(\theta)$ is given by:

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ where adjacent $= 4$, hypotenuse $= 5$.

Replacing the formulae with the known values, we obtain:

$\cos\theta = \frac{4}{5}$.

(iii) $\csc(\theta)$

$\csc(\theta)$ can be evaluated in two different ways.

(a) by the formula $\csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}}$

Replacing the values we get:

$\csc\theta = \frac{5}{3}$.

(b) By definition, $\csc(\theta) = [\sin(\theta)]^{-1} = \frac{1}{\sin(\theta)}$

Since $\sin(\theta) = \frac{3}{5}$,

$\csc(\theta) = \left(\frac{3}{5}\right)^{-1} = \frac{5}{3}$.

(iv) $\sec\theta$

It can be evaluated in two ways:

(a) $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

$= \frac{5}{4}$

(b) $\sec(\theta) = (\cos\theta)^{-1} = \frac{1}{\cos(\theta)}$

Since $\cos(\theta) = \frac{4}{5}$,

$\csc(\theta) = \left(\frac{4}{5}\right)^{-1} = \frac{1}{4/5} = \underline{\underline{5/4}}$

(v) $\cot(\theta)$

It can be evaluated in two ways:

(a) $\cot(\theta) = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{4}{3}$

$\therefore \cot(\theta) = 4/3.$

(b) $\cot(\theta) = (\tan\theta)^{-1} = \dfrac{1}{\tan(\theta)}$

Since the value of $\tan(\theta) = 3/4$ (in the question), we replace it in the formula.

$\cot\theta = \left(\frac{3}{4}\right)^{-1} = \dfrac{1}{(3/4)} = 4/3.$

$\therefore \cot\theta = 4/3$