Name Alena D.

MATH 124
TEST 3
Sec. 3.5, 4.1 – 4.4, 5.1 - 5.2

Directions: Show work whenever possible. Points may be given for partially correct answers. Points may be deducted for insufficient or nonsensical steps even if the answer is correct. DO NOT USE MATH APPS!

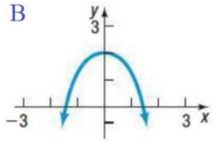
Q1 -12

In Problems 1-12, match each graph to one of the following functions:

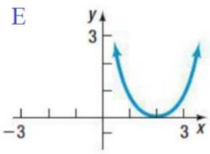
A. $y = x^2 + 2$	D. $y = -\lvert x\rvert + 2$	G. $y = \lvert x - 2\rvert$	J. $y = -2x^2$
B. $y = -x^2 + 2$	E. $y = (x-2)^2$	H. $y = -\lvert x + 2\rvert$	K. $y = 2\lvert x\rvert$
C. $y = \lvert x\rvert + 2$	F. $y = -(x+2)^2$	I. $y = 2x^2$	L. $y = -2\lvert x\rvert$

Write the letters below.

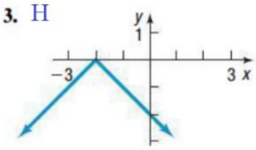
1. B



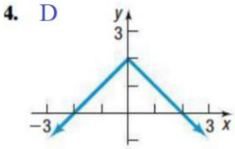
2. E



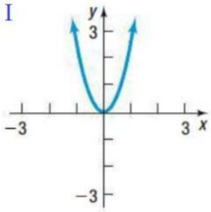
3. H



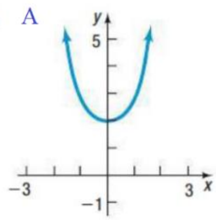
4. D



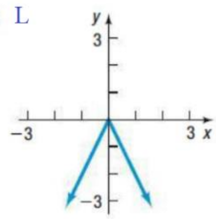
5. I



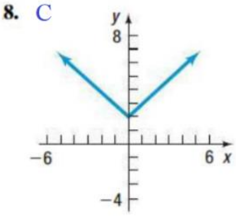
6. A



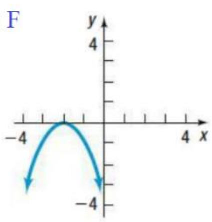
7. L



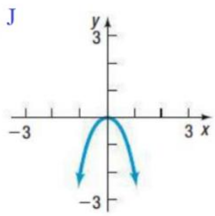
8. C



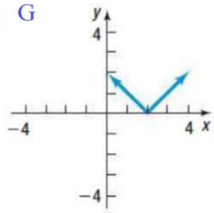
9. F



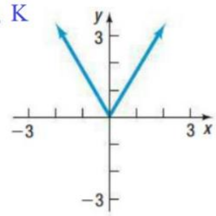
10. J



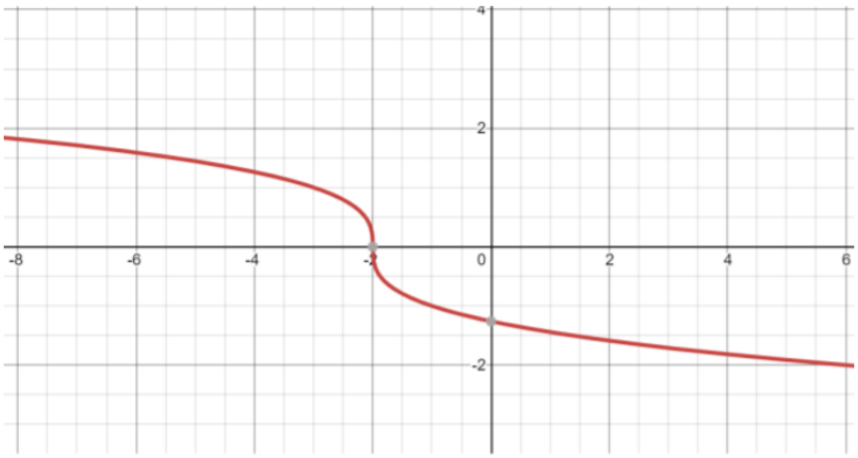
11. G



12. K

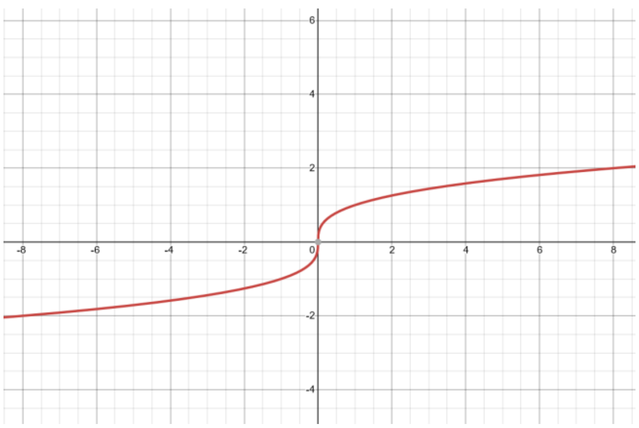


13. Write the function that matches this graph.

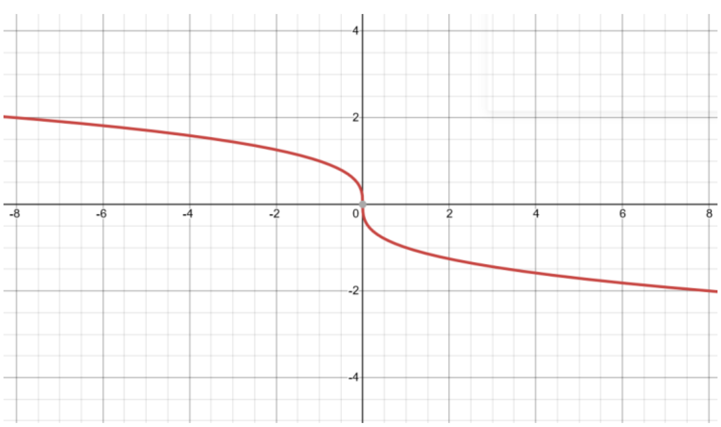


Solution

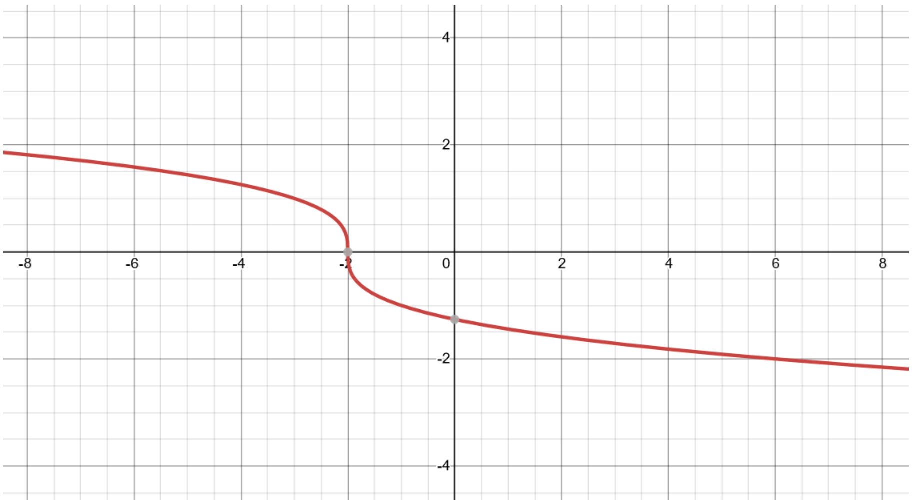
Graph of $x = y^3$is as follows



Now we plot the graph of $x = -y^3$, by mirroring the graph about y-axis

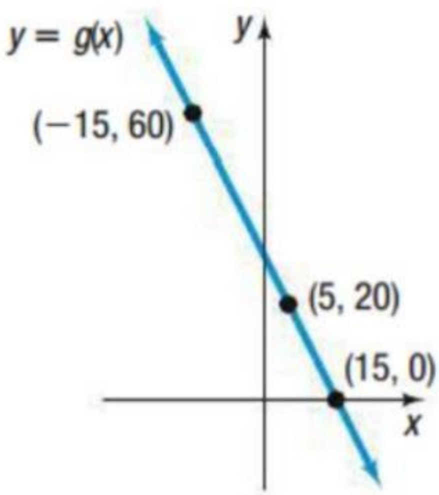


Now we shift the origin by 2 units left, so the equation becomes $x = -y^3 - 2$ and graph is as follows



So the equation is $x = -y^3 - 2$

14. In parts (a)–(f), use the figure below.



(a) Solve $g(x) = 20$.

From the graph it can be seen that g(x) = 20, for the value of x = 5

(b) Solve $g(x) = 60$.

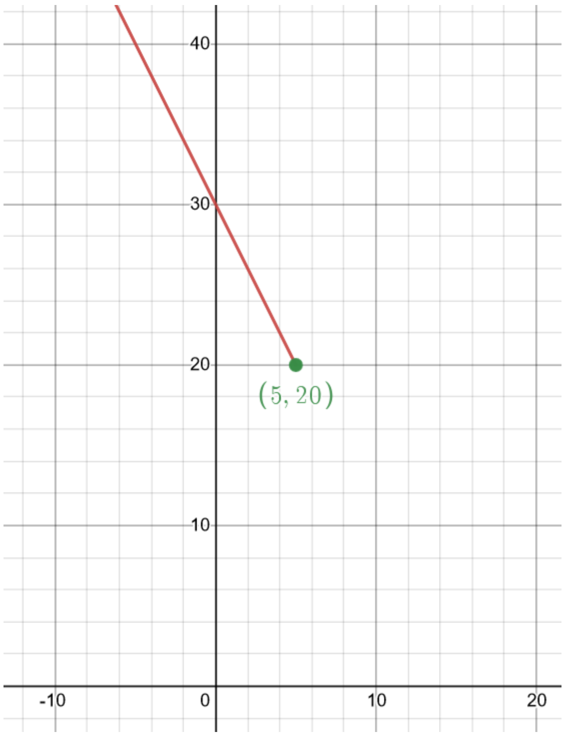
From the graph it can be seen that g(x) = 60, for the value of x = -15

(c) Solve $g(x) = 0.$

From the graph it can be seen that g(x) = 0, for the value of x = 15

(d) Solve $g(x) > 20.$

This region is shown where g(x) is greater than 20



Equation of the line through (5,20) and (15,0) is given by

$$\frac{y-0}{x-15} = \frac{0-20}{15-5} \rightarrow \frac{y}{x-15} = -2$$

$$y = 30 - 2x$$

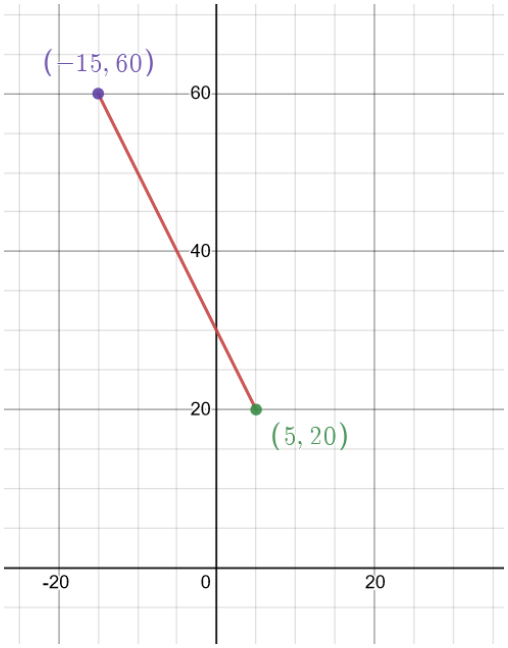
$$g(x) = 30 - 2x$$

$g(x) > 20$ is shown as

$$30 - 2x > 20 \rightarrow 10 > 2x \rightarrow x < 5$$

Also shown in the graph

(e) Solve $20 < g(x) < 60$



$20 < g(x) < 60$ is shown as

$$20 < 30 - 2x < 60 \rightarrow -15 < x < 60$$

Also shown in the graph

15. **Supply and Demand** Suppose that the quantity supplied S and the quantity demanded D of hot dogs at a baseball game are given by the following functions:

$$S(p) = -2000 + 3000p$$

$$D(p) = 10{,}000 - 1000p$$

where p is the price of a hot dog.

Find the equilibrium price for hot dogs at the baseball game. What is the equilibrium quantity?

Solution: We are given

$$S(p) = -2000 + 3000p$$

$$D(p) = 10000 - 1000p$$

To find equilibrium price for hot dogs at the baseball game

$$S(p) = D(p)$$

$$\rightarrow\ -2000 + 3000\text{p} = 10000 - 1000\text{p}$$

$$\rightarrow 4000\text{p} = 12000 \rightarrow \text{p} = 3$$

Therefore, the equilibrium price for the hot dog is $3

To find the equilibrium quantity we put p = 3 in S(p), we get

$$\text{S}(3) = \ -2000 + 3000 \times 3$$

$$\text{S}(3) = 7000 \text{ hot dogs}$$

16. **Candy** The following data represent the weight (in grams) of various candy bars and the corresponding number of calories.

Candy Bar	Weight, *x*	Calories, *y*
Hershey's Milk Chocolate®	44.28	230
Nestle's Crunch®	44.84	230
Butterfinger®	61.30	270
Baby Ruth®	66.45	280
Almond Joy®	47.33	220

A. Use the lightest (least weight) and heaviest (greatest weight) candy bars to write the equation of a line that goes through them. Round to 2 decimal places.

Candy with least weight is Hershey's Milk Chocolate weight, x = 44.28, calories, y = 230

Candy with greatest weight is Baby Ruth weight, x = 66.45, calorie, y = 280

So the equation if line through (44.28, 230) and (66.45,280) is given by

$$\frac{y - 280}{x - 66.45} = \frac{280 - 230}{66.45 - 44.28} \rightarrow \frac{y - 280}{x - 66.45} = \frac{50}{22.17}$$

Further solving,

$$22.17\,(y - 280) = 50(x - 66.45)$$

$$22.17y\ - 6207.6 = 50x - 3322.5$$

$$y = 2.2553x + 130.135$$

$$y\ = 2.26\,x + 130.14$$

B. Use your equation in A to estimate the number of calories in a candy bar that weighs 68.3 g.

In the above obtained equation, we put x = 68.3 to get the values of calories

$$y = 2.26 \times 68.3 + 130.14$$

$$y = 284.498 \sim 284.5$$

Estimated calories for candy bar that weighs 68.3g is 284.5

C. Use technology to calculate the line of best fit using the least squares method. Round to 2 decimal places.

Using technology, we get equation as

$$y = 2.53041x + 112.29322$$

Rounding to 2 digits

$$y = 2.53x + 112.29$$

D. Interpret the slope in part C in words.

By using least square method we get the line of best fit for a set of paired data allowing us to estimate the value of a dependent variable y from a given independent variable x.

The line of best fit is described by the equation $y = 2.53x + 112.29$, where slope of the equation obtained is 2.53 and 112.29 is the intercept

17. Given f(x) = -2(x – 1)^2 + 32, graph and identify the following.

Vertex

Comparing equation y = 4a(x-h)^2 + k

Vertex = (h,k) = (1,32)

Axis of symmetry

From graph, x = 1

y-intercept

At y intercept x = 0, substituting

$$y = -2(0-1)^2 + 32$$

$$y = -2 + 32 = 30$$

y-intercept (0,30)

Convert to standard form $\boldsymbol{ax^2 + bx + c}$

$$y = -2(x-1)^2 + 32$$

$$\rightarrow y = -2(x^2 - 2x + 1) + 32$$

$$\rightarrow y = -2x^2 + 4x - 2 + 32$$

$$\rightarrow y = -2x^2 + 4x + 30$$

x-intercept

At x-intercept y = 0, substituting

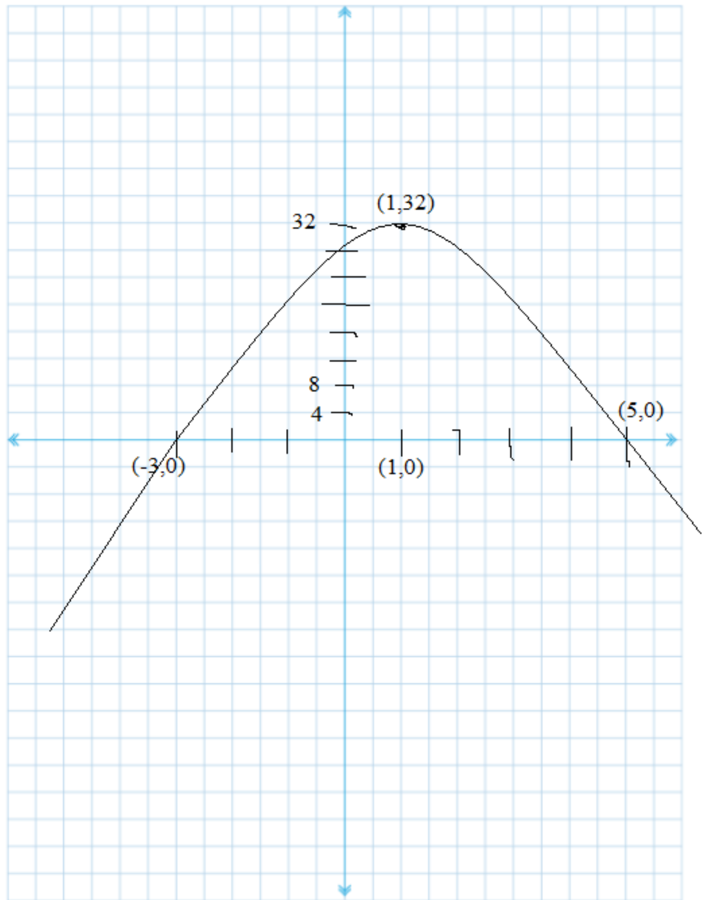
$$0 = -2(x-1)^2 + 32 \rightarrow 2(x-1)^2 = 32$$

$$\rightarrow (x-1)^2 = 16$$

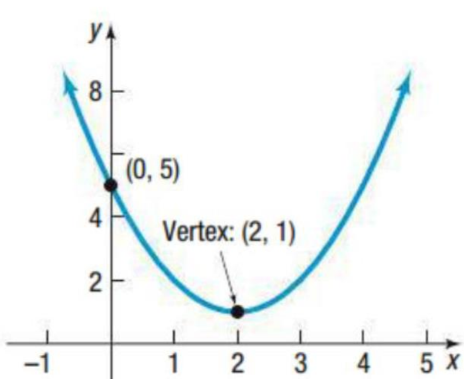
$$\rightarrow (x-1) = \pm 4$$

$(x-1) = 4$ $x = 5$ $x - intercept\ (5,0)$	$(x-1) = -4$ $x = -3$ $x - intercept\ = (-3,0)$

From above data we plot the graph as follows



18. Given the following graph of a quadratic function:



On what interval is the function increasing?

On what interval is the function decreasing?

Identify the function in either vertex or standard form.

Function is said to be increasing if for every increase in value of x there is increase in value of y,

From graph it is clearly seen that function is increasing from $(2, \infty)$

Function is said to decreasing if for every increase in value of x there is decrease in value of y

From graph it is clearly seen that function is decreasing from $(\infty, 2)$

Equation of the function

We are given vertex (2,1) and point (0,5)

The equation of parabola with vertex (h,k) is $y = a(x - h)^2 + k$

Thus, the equation of parabola is $y = a(x - 2)^2 + 1$

To find a, we know that parabola passes through (0,5), substituting

$$5 = 4a + 1 \rightarrow a = 1$$

Thus, the equation of parabola is $y = (x - 2)^2 + 1$

19. The price p (in dollars) and the quantity x sold of a certain product satisfy the demand equation

x = -20p + 500

(a) Find a model that expresses the revenue R as a function of p.

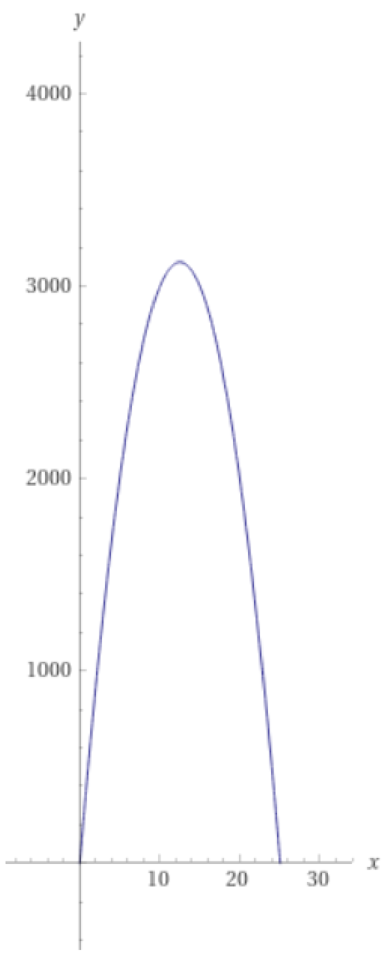
Revenue is given by

$R = xp$

$R = (-20p + 500)p$

$R = -20p^2 + 500p$

(b) Draw the graph of R(p).



(c) What price p maximizes the revenue?

Revenue R, is the quadratic equation with $a = -20, b = 500, c = 0$

Since a < 0

The vertex is the highest point on the parabola, Revenue R is maximum when the price

$$p = -\frac{b}{2a} = -\frac{500}{2(-20)} = \$12.5$$

(d) What is the maximum revenue?

$$R(12.5) = -20(12.5)^2 + 500 \times (12.5)$$

$$R(12.5) = -3125 + 6250 = 3125$$

(e) How many units are sold at this price?

No of products sold is given by demand equation

$$x = -20p + 500$$

$$\rightarrow -20 \times 12.5 + 500 = -25 + 500 = 475$$

20. Factor and find the real zeros.

f(x) = x^2 – 4x + 3

$$f(x) = x^2 - 4x + 3$$

$$\rightarrow x^2 - x - 3x + 3$$

$$\rightarrow x(x-1) - 3(x-1)$$

$$\rightarrow (x-3)(x-1)$$

21. Find a polynomial function of degree 3 with zeros -4, -2, and 3. Expand the polynomial so that it is in standard form.

So the zeroes of the polynomial is -4,-2,3

$$f(x) = (x - (-4))(x - (-2))(x - 3)$$

$$f(x) = (x + 4)(x + 2)(x - 3)$$

$$f(x) = (x^2 + 6x + 8)(x - 3)$$

$$f(x) = x^3 + 3x^2 - 10x - 24$$

22. Find the multiplicity of each zero and identify whether it touches or crosses the x-axis at that zero.

$F(x) = (x - 4)(x + 3)^2(x-2)^3$

zero	multiplicity	touches or crosses

$f(x)$ has zeroes of 4, -3, 2

In the given function, (x-4) is raised to the power 1, therefore, multiplicity of zero at x = 4 is 1 i.e. odd

(x + 3) are raised to the power 2, so, multiplicity of zero at x = -3 is 2 i.e. even.

(x-2) is raised to the power 3, therefore, multiplicity of zero at x = 2 is 3 i.e. odd

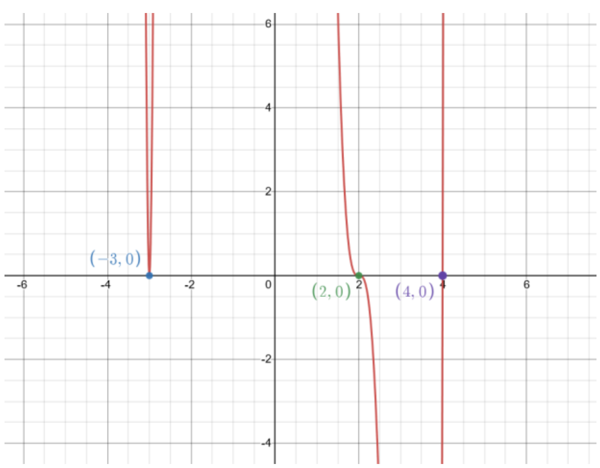
If the multiplicity of zero is odd, the graph crosses the x-axis and if the multiplicity is even , the graph touches the x-axis.

Zero 4 multiplicity of 1 crosses

Zero -3 multiplicity of 2 touches

Zero 2 multiplicity of 3 crosses

Sketch the graph. Label the intercepts.



23. Given f(x) = -3(x + 4)(x – 5)(x + 2)^2

(a) What is the first term?

First term is

$\rightarrow\ -3 \cdot x \cdot x \cdot x^2 =\ -3x^4$

(b) What is the degree of the polynomial?

Degree of polynomial

$1 + 1 + 2 = 4$

(c) What is the last term?

Last term is

$\rightarrow\ -3 \times 4 \times (-5) \times 2 = 120$

(d) How many turning points does the graph have?

Number of turning point = Number of degree - 1

$\rightarrow 4 - 1 = 3$

3 number of turning points

24. Given $f(x) = x^2(x - 6)$.

(a) Find the x and y-intercepts.

x-intercept

At x- intercept y = 0 , substituting

$$0 = x^2(x - 6)$$

$$\rightarrow x = 0 , x = 6$$

x-intercept $(0,0), (6,0)$

y -intercept

At y-intercept x = 0, substituting

$$y = 0^2(0 - 6) = 0$$

y-intercept (0,0)

(b) Use technology to identify the local max and local min. (Use ordered pairs)

Using technology

Local minima (4,-32)

Local maxima (0,0)