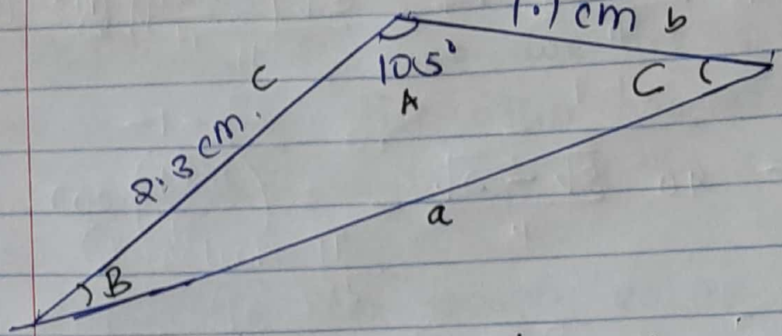
Quiz
Solve the following triangle.



Describe the methods, ~~and~~ processes and equations used to determine the missing sides and angles.

<u>Solution</u> (150 words)

First, use letters to denote the sides and the angles. In this case, the following letters are used:
a - denotes the length of the unknown side of the triangle.
b - denotes the length of one of the known ~~length~~ side of the triangle.
c - denote the remaining side of the triangle whose length is known.
A - the angle opposite to side a
B - the angle opposite to side b
C - the angle opposite to side c

Let's start by determining the length of the missing side, ~~(a)~~ labbled a.
Since the two other sides ~~are~~ and its opposite angle are known, the COSINE RULE is the most appropriate. It is used when relating the three sides of a triangle with one angle. In this case, we will need to use the cosine rule where a^2 is the subject of the formula:

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

Side a is the one we are trying to find. Sides b and c are the other two sides whose length are known, and angle A is the angle opposite to side a.

Substitute the known values to the formula.

$$a^2 = (1.1)^2 + (2.3)^2 - 2 \times 1.1 \times 2.3 \cos(105^\circ)$$

NB: It doesn't matter which way around the two sides b and c are put. Both will work.

Evaluate the right hand side and then find the square root to find the length.

$$\begin{aligned} a^2 &= 1.21 + 5.29 - 5.06 \times (-0.25882) \\ &= 1.21 + 5.29 + 1.3096. \\ &= 7.8096. \end{aligned}$$

$$\begin{aligned} \therefore a &= \sqrt{7.8096} \\ &= 2.795 \quad \text{(Round off to 2. d.p)} \\ &\approx 2.8. \end{aligned}$$

Lastly, let's determine the length of the missing angles. Since all the sides are known and one angle is known, the sine rule is used.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

The known angle will be our reference angle. It will help us to determine the ~~length~~ size of the unknown angles.

To determine the size of angle B:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

Replace the known values in the equation:

$$\frac{2.8}{\sin(105^\circ)} = \frac{1.1}{\sin(B)}$$

Make $\sin(B)$ the subject of the formula, then find the $\sin^{-1}$ of the solution to get the ~~value of B~~ size of angle B.

$$\sin(B) = \frac{\sin(105^\circ)}{2.8} \times 1.1$$

$$= 0.3795.$$

$$B = \sin^{-1}(0.3795)$$

$$= 22.3^\circ.$$

To determine the size of angle C:

$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)}$$

Replace the known values to the equation:

$$\frac{2.8}{\sin(105^\circ)} = \frac{2.3}{\sin(C)}$$

$$\sin(C) = \frac{\sin(105^\circ)}{2.8} \times 2.3.$$

$$= 0.7934$$

$$\sin^{-1}(0.7934) = 52.7^\circ$$

Since the angles of a triangle add up to 180°, angle C can also be determined by:

$$180 - (105 - 22.3) = 52.7^\circ.$$