Given that $\tan\theta = 34$ determine the following.

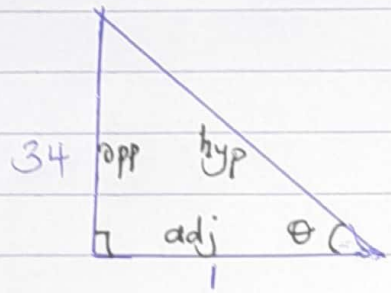
$\sin\theta$ $\quad$ $\csc\theta$ $\quad$ $\cot\theta$

$\cos\theta$ $\quad$ $\sec\theta$

Solution.

We are given $\tan\theta = 34$. From the trigonometric identities, the value of $\tan(\theta)$ is given by the opposite divided by the ~~hypotenuse~~ adjacent.

It can be illustrated below with the help of a right angled triangle.




$$\tan\theta = \frac{34}{1} = \frac{\text{opposite}}{\text{adjacent}} \quad \therefore \text{opp} = 34 \text{ and } \text{adj} = 1$$

Since it is a right angled triangle, we can use the pythagoras' theorem to obtain the value of the hypotenuse.

From pythagoras' theorem,

$$a^2 + b^2 = c^2.$$

$$(\text{opp})^2 + (\text{adj})^2 = (\text{hyp})^2 \quad \text{Replace the known values.}$$

$$\therefore (34)^2 + (1)^2 = (\text{hyp})^2.$$

$$1156 + 1 = \text{hyp}^2.$$

$$(\text{hyp})^2 = 1157 \quad \text{Square root both sides to get the length of the hypotenuse.}$$

$$\text{hyp} = \sqrt{1156}.$$

Therefore from pythagoras' theorem, the length of the hypotenuse is $\sqrt{1156}$

Next, since we have the values of the three sides, the value of the other trigonometric functions can be determined.

(i) $\sin\theta$.

The value of $\sin\theta$ can be given by:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}.$$

$$\text{opposite} = 34$$
$$\text{hypotenuse} = \sqrt{1157}$$

$$\therefore \sin\theta = \frac{34}{\sqrt{1157}}$$

Rationalize the denominator by multiplying by the conjugate.

$$\sin\theta = \frac{34\sqrt{1157}}{1157}$$

(ii) $\cos\theta$

The value of $\cos(\theta)$ is given by:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

but $\frac{\text{adjacent}}{\text{opposite}} = 1$, hypotenuse $= \sqrt{1157}$

$$\therefore \cos\theta = \frac{1}{\sqrt{1157}}$$

Rationalize the denominator by multiplying by the conjugate.

$$\cos\theta = \frac{\sqrt{1157}}{1157}$$

(iii) csc(θ)

By defination, $\csc\theta = \frac{1}{\sin\theta} = \frac{\text{hypotenuse}}{\text{opposite}}$.

$$\csc\theta = \frac{\sqrt{1157}}{34}.$$

iv) sec(θ)

The value of sec(θ) is given by:

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{\sqrt{1157}}{1}$$

$$\therefore \sec\theta = \sqrt{1157}$$

v) cot(θ)

The value of cot(θ) is given by:

$$\cot\theta = \frac{1}{\tan\theta}.$$

Since the value of tan θ is given as $\tan\theta = 34$, we replace it in the above formula to get:

$$\cot(\theta) = \frac{1}{34}.$$