(i) Verify the Identity.
(ii) Determine if the identity is true for the given value of x.

① $\frac{\sec x}{\tan x} = \frac{\tan x}{\sec x - \cos x}$ $\quad x = \pi$.

soln.

(i) Manipulating the left side,

$\frac{\sec x}{\tan x}$, expressing in terms of sin and cos.

$\sec x = \frac{1}{\cos x}$ and $\tan x = \frac{\sin x}{\cos x}$.

$= \frac{1}{\cos x} \times \frac{\cos x}{\sin x} = \frac{1}{\sin x} = \operatorname{cosec} \pi$

On the right hand side,

$\frac{\tan x}{\sec x - \cos x}$, expressing in terms of sin and cos.

$= \frac{1}{\cos x} - \cos x = \sec x - \cos x$.

but $\tan x = \frac{\sin x}{\cos x}$

$\therefore = \frac{\sin x}{\cos x \left(\frac{1}{\cos x} - \cos x\right)} = \frac{\sin x}{1 - \cos^2 x}$

but $1 - \cos^2 x = \sin^2 x$

$= \frac{\sin x}{\sin^2 x} = \frac{1}{\sin x} = \operatorname{cosec} \pi$

Therefore, the identity is true.

(ii) Determine if the identity is true ~~for the give~~ when $x = \pi$.

soln.

On the left hand side,

$$\frac{\sec x}{\tan x}, \text{ when } x = \pi$$

$$= \frac{\sec \pi}{\tan \pi} = \frac{-1}{0} \text{ which is undefined.}$$

On the Right hand side,

$$\frac{\tan x}{\sec x - \cos x}, \text{ when } x = \pi$$

$$= \frac{\tan \pi}{\sec \pi - \cos \pi} = \frac{0}{-1+1} = \frac{0}{0} = 0$$

Therefore the identity is not true for $x = \pi$

But $\operatorname{cosec} \pi = \operatorname{cosec} \pi$

$\Rightarrow 1 = 1$

Since the identity must provide an equality for all allowable values of the variable, if the two expressions differ at one input, then the equation is not an identity.

Thereore the equation $\frac{\sec x}{\tan x} = \frac{\tan x}{\sec x - \cos x}$ is

Not an identity.

2. $\frac{\cot x - 1}{\cot x + 1} = \frac{1 - \tan x}{1 + \tan x}$, $x = \pi/4$

Solution

(i) verify the identity.

On the Left hand side,

$\frac{\cot x - 1}{\cot x + 1}$, Expressing in terms of sin and cos,

$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$.

and $\sin^2 x + \cos^2 x = 1$

In the numerator

$\frac{\cos x}{\sin x} - 1 = \frac{\cos x - \sin x}{\sin x}$.

and

~~cos~~ denominator

$\frac{\cos x}{\sin x} + 1 = \frac{\cos x + \sin x}{\sin x}$

$\therefore = \frac{\cos x - \sin x}{\sin x} \times \frac{\sin x}{\cos x + \sin x}$.

$= \frac{\cos x - \sin x}{\cos x + \sin x}$.

On the Right hand side,

$\frac{1 - \tan x}{1 + \tan x}$, expressing in terms of sin and cos.

$\tan x = \frac{\sin x}{\cos x}$.

Numerator.

$$1 - \frac{\sin x}{\cos x} = \frac{\cos x - \sin x}{\cos x}.$$

denominator:

$$1 + \frac{\sin x}{\cos x} = \frac{\cos x + \sin x}{\cos x}.$$

$$\Rightarrow \frac{\cos x - \sin x}{\cos x} \times \frac{\cos x}{\cos x + \sin x}.$$

$$= \frac{\cos x - \sin x}{\cos x + \sin x}.$$

Since the left hand side is equal to the right hand side, the identity is true.

(ii) Determine whether the identity is true fore $x = \pi/4$.

On the left hand side,

$$\frac{\cot x - 1}{\cot x + 1} \quad \text{at } x = \pi/4$$

$$= \frac{\cot(\pi/4) - 1}{\cot(\pi/4) + 1} = \frac{1-1}{1+1} = \frac{0}{2} \quad \text{Undefined.}$$

On the Right hand side,

$$\frac{1 - \tan x}{1 + \tan x} \quad \text{at } x = \pi/4$$

$$= \frac{1 - \tan(\pi/4)}{1 + \tan(\pi/4)} = \frac{1 - 1}{1 + 1} = \frac{0}{2}, \text{ Undefined.}$$

Therefore, the identity is true for $x = \pi/4$.