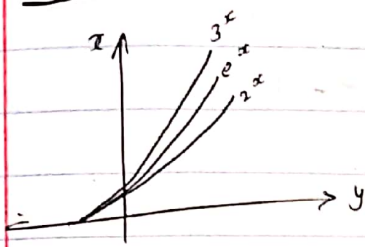


Multiple choice

Q1



Q2: $\ln(0) = \text{undefined}$

Q3: $f(x) = 10^x$
 $f'(x) = 10^x \ln(10)$

Q4: $f(0) = 0e^0$
 $= 0$

Q5: $f(x) = \frac{x}{e^x}$

$f'(x) = \frac{1-x}{e^x}$; Quotient rule.

$f'(0) = \frac{1}{e^0} = 1$

Q6 $f(x) = x2^x$
 $f'(x) = x \frac{d}{dx}(2^x) + 2^x \frac{d}{dx}(x)$
 $= x2^x \ln(2) + 2^x$

$f'(0) = 0 + 2^0 = 1$

Q7: $f(x) = 2x^2 e^x$
 $f'(x) = 2x^2 \frac{d}{dx} e^x + e^x \frac{d}{dx} 2x^2$
 $= e^x \cdot 2x^2 + e^x \cdot 4x$
 $f'(1) = e^1 \cdot 2 + e^1 \cdot 4$
 $= 6e$

Q8

$f(x) = x^2 e^x$
 $f'(x) = x^2 \frac{d}{dx} e^x + e^x \frac{d}{dx} x^2$
 $= e^x x^2 + 2x e^x$
 $f'(1) = e^1 + 2e^1$
 $= 3e$

Q9: $f(x) = e^x (e^x)^{1/2}$
 $f'(x) = e^x \frac{d}{dx} (e^x)^{1/2} + \sqrt{e^x} \frac{d}{dx} e^x$
 $= e^x \cdot \frac{e^x}{2\sqrt{e^x}} + \sqrt{e^x} \cdot e^x$
 $= e^2 \cdot \frac{e^2}{2\sqrt{e^2}} + e^3$
 $= \frac{3e^3}{2}$

Q10 $f(x) = e^{\sin x}$

$f'(x) = \cos(x) e^{\sin x}$
 $f''(x) = -\sin(x) e^{\sin x} + \cos^2(x) (e^{\sin x})$
 $f''(0) = 0 + 1(e^0) = 1$

Q11: $f(x) = \frac{1}{e^x}$

$$f'(x) = -e^{-x}$$

$$f''(x) = e^{-x}$$

$$e^{-(-1)} = e^1 = e$$

Q12 $f(x) = x^2 e^x$

$$f'(x) = 2x e^x + e^x \cdot x^2$$

$$f'(1) = 2e + e$$

$$= 3e$$

$$y - e = 3e(x - 1)$$

$$y = 3ex - 2e$$

Q13

$$y = x; m = 1$$

$$f(x) = e^x + x$$

$$f'(x) = e^x + 1$$

$$\cancel{e^x + 1} = 1 \quad m = 1;$$

$$\cancel{e^x} = 0 \quad e^x + 1$$

$$x = \quad e^0 + 1 = 1$$

$$= (0, 1)$$

Q14

$$y = 2e^x$$

$$m = 2e$$

$$y' = 2e^{2x+1}$$

$$\cancel{2} e^{2x+1} = \cancel{2} e$$

$$1 = 2x + 1$$

$$2x = 0$$

$$x = 0$$

$$= (0, e)$$

Q15

$$y' = e^x$$

$$e^0 = 1 \quad ; m = 1$$

$$y - 1 = 1(x - 0)$$

$$y = x + 1$$

Q16

$$y = 3(2^x)$$

$$y' = 3(2^x \ln(2))$$

$$m = 3(2^0 \ln(2))$$

$$= 3 \ln(2)$$

$$y - 3 = 3 \ln(2) (x - 0)$$

$$y - 3 = 3 \ln(2) x$$

$$y = 3 \ln(2) x + 3$$

Q17

$$y = 6e^x + 5 \quad ; m = 6e$$

$$y' = 3e^x + 3e$$

$$3e^x + 3e = 6e \quad ; x = 1$$

$$y(1) = 6e$$

$$y - 6e = 6e(x - 1)$$

$$y = 6ex$$

Q18

$$M(t) = M_0 \left(\frac{1}{2}\right)^{t/3.8}$$

$$M_0 = 80$$

$$t = 3$$

$$\begin{aligned} M(t) &= 80 \left(\frac{1}{2}\right)^{3/3.8} \\ &= 80 \cdot 0.579 \\ &= 46.9 \end{aligned}$$

Q19

$$M_0 = 200$$

12.5% of its initial mass.

$$1 - 0.125 = 0.875$$

$$0.875 = 0.5^{x/3.8}$$

$$\ln(0.875) = \frac{x}{3.8} \ln 0.5$$

$$x = \left(\frac{\ln 0.875}{\ln 0.5} \right) 3.8$$

$$0.125 = 0.5^{x/3.8}$$

$$x = \frac{\ln(0.125)}{\ln(0.5)} 3.8$$

$$11.4 \approx 11 \text{ days}$$

Q20

$$x(t) = 1.5 \cos(t) e^{-0.05t}$$

$$\begin{aligned} x'(t) &= 1.5 \left[\cos(t) \cdot \frac{d}{dt} e^{-0.05t} + e^{-0.05t} \frac{d}{dt} \cos(t) \right] \\ &= 1.5 \left(-\cos(t) e^{-0.05t} \cdot 0.05 + e^{-0.05t} -\sin(t) \right) \end{aligned}$$

$$V(t) = 1.5 \left(-e^{-0.05t} \sin t - 0.075 e^{-0.05t} \cos t \right)$$

$$a(t) =$$

$$\frac{d}{dx} \left(-e^{-0.05t} \sin t \right) = -e^{-0.05t} \cos t + (-\sin t) \cdot 0.05 e^{-0.05t}$$

$$\frac{d}{dx} \left(e^{-0.05t} \cos t \right) = -e^{-0.05t} \sin t + (-\cos t) \cdot 0.05 e^{-0.05t}$$

$$\Rightarrow a(t) = 1.5 \left(0.1 e^{-0.05t} \sin t - 0.0998 e^{-0.05t} \cos t \right)$$

$$a(t) = 0$$

$$3e^{-1/20} (2\sin x + x \cos x) = 0$$

$$2\sin x + x \cos x = 0$$

$$x = 0$$

$$x = 1.41087$$

$$V(0) = 1.5$$

$$V = -1.5$$

Thinking & Inquiry

1. $h(t) = 0.5 \cos 2t + 0.7 \sin t$

Max Velocity:

$$v(t) = -\sin(2t) + 0.7 \cos(t)$$

$$a(t) = -\cos(2t) - 1.4 \sin(t)$$

$$-\cos(2t) - 1.4 \sin(t) = 0$$

$$t = 0.9275$$

$$v(0.927) = -0.54$$

$$t = 0.9275$$

$$v = -0.54 \text{ m/s}$$

2. $t = 8$

$$M = 0.33$$

(a) half life

$$0.33 = 0.5^{t/8}$$

$$\ln(0.33) = \frac{t}{8} \ln(0.5)$$

$$t = \left(\frac{\ln 0.5}{\ln 0.33} \right) \cdot 8$$

$$= 5 \text{ days}$$

(b) $M = M_0 \left(\frac{1}{2}\right)^{t/5}$

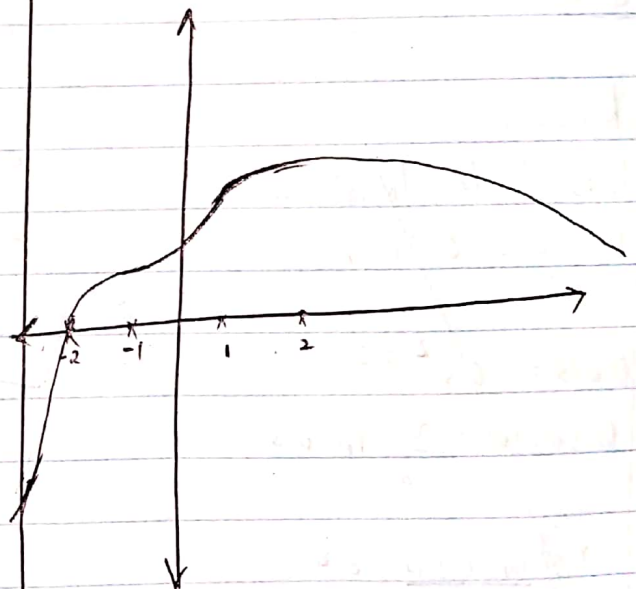
$$\frac{dM}{dt} = -0.13862 e^{-0.13862t} \cdot M$$

$$\frac{dM}{dt}(10) = -0.13862 e^{-1.3862} M$$

$$= 0.035$$

$$= 3.5\%$$

Communication



Increasing: $(-\infty, -2) \cup (-1, 1)$

Decreasing: $(-2, -1) \cup (1, \infty)$

Concave up: $(-\infty, -2) \cup (-1, 0)$

Concave down: $(0, \infty)$