Exercise0 Show that the set of all dilation-translations $v \mapsto \lambda v + b$ with $\lambda \neq 0$ of an affine plane form a group under composition.

Exercise1 Let $D_1 v = 2\left(v - \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) + \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ be a dilation in $\mathbb{R}^2$. Find another dilation $D_2 v = \lambda(v - p) + p$ such that $(D_2 \circ D_1)v = v + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Exercise 2 In this exercise, we will study the affine plane $\mathbb{F}^2$, where $\mathbb{F}$ is the field $\mathbb{F} = \{(a, b) : a, b \in \mathbb{F}_3\}$ with $(a, b) + (a'b') = (a + a', b + b')$ and $(a, b) \cdot (a', b') = (aa' - bb', ab' + a'b)$. Note that this field has 9 elements.

1. How many point are in $\mathbb{F}^2$?
2. How many lines are in $\mathbb{F}^2$?
3. How many lines are in $\mathbb{F}^2$ that pass through the origin $(0, 0)$?
4. How many points lie on each line?

Exercise 3 Consider the triangle with vertices $p_1 = (1, 2)$, $p_2 = (2, 3)$, and $p_3 = (4, 1)$ in the affine plane $\mathbb{F}_5^2$ over the field $\mathbb{F}_5$ with 5 elements. The side $p_i p_j$ is being divided by the points t_{ij} in the proportions $\tau_{12} = 1$, $\tau_{23} = 2$, and $\tau_{31} = 3$. Are the three lines $p_1 t_{23}$, $p_2 t_{31}$, and $p_3 t_{12}$ concurrent? If yes, determine their common intersection.